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Improvement of $\pi\pi$ amplitudes and correct position of the σ pole

V.Nazari[1, a], Petr Bydžovský[2], and Robert Kamiński[1]

[1] *The Henryk Niewodniczański Institute of Nuclear Physics PAN, Radzikowskiego 152, 31-342 Kraków, Poland*
[2] *Nuclear Physics Institute, ASCR, Rež/Prague, Czech Republic*

Abstract. Meson-meson interaction amplitudes especially $\pi\pi$ scattering amplitudes are often used incorrectly. It also applies to parameters of the σ meson. These causes two significant problems: threshold behavior and position of the σ pole which is suspicious. We modified multichannel S- and P- wave amplitudes for the $\pi\pi$ scattering, using dispersion relations with imposed crossing symmetry condition. The amplitudes are modified in the low-energy region to improve their consistency with experimental data and the dispersion relations. Agreement with data is achieved for both amplitudes from the threshold up to 1800 MeV and with dispersion relations up to 1100 MeV. Consequences of the applied modifications, e.g. changes of the S-wave lowest-pole positions, are presented.

1 Introduction

A model independent analysis of the $\pi\pi$ scattering is an important tool in getting information about the spectrum of light mesons. A reliable description of the process is therefore desirable to allow us to learn more on nature and parameters of the mesons. Quite often (like in [1] for example) phenomenological multichannel $\pi\pi$ amplitudes for the S- and P- waves were constructed without any specific assumptions about dynamics of the process, only requiring analyticity and unitarity of the S-matrix and applying the uniformization procedure. This procedure can be applied exactly in the two-channel case. However, in the three-channel case, simplifying approximations have to be done resulting in a very poor description of experimental data in the threshold region.

The crossing symmetry condition, which relates the S- and P- waves and which is an important below the inelastic threshold, was not taken into account in the construction of those amplitudes [1]. Since the crossing symmetry is properly included in the Roy-like dispersion relations [2], it is possible and desired to improve the low-energy behavior of the three-channel amplitudes of [1] and to check their consistency with the dispersion relations (DR).

Poles are singularities in the amplitudes and can be related with resonances. There are many poles in the $\pi\pi$ interaction amplitudes and the most important is the lightest one which is called σ and known as $f_0(500)$ PDG 2012 [3]. In the multichannel uniformizing (MI) approach used in [1] a heavy and broad σ meson is predicted, with $M = 829 \pm 10$ MeV and $\Gamma = 1108 \pm 22$ MeV, in disagreement (by many standard deviations) with results from [4] and values recommended by Particle Data Group

a. e-mail: vnazari@ifj.edu.pl

[3]. It is therefore interesting to show how much the modifications of the three-channel amplitudes affect position of the pole connected with the σ meson.

In this note we present an example of using the DR for improving the low-energy behavior of the phenomenological three-channel S- and P-wave $\pi\pi$ amplitudes and for imposing the crossing symmetry condition on the amplitudes below 1100 MeV.

2 Method

In order to obtain a precise description of $\pi\pi$ amplitudes we have applied once subtracted Roy-like GKPY dispersion relations [2] with imposed crossing symmetry for the S- and P-wave amplitudes. These new dispersion relations impose quite strong constraints in our fits to the data on the analyzed $\pi\pi$ interactions. For amplitudes $f_l^I(s)$ with spin $l = 0, 1, 2, 3$ and isospin $I = 0, 1, 2$:

$$\mathrm{Re} f_l^I(s)^{OUT} = \sum_{I'=0}^{2} C_{st}^{II'} a_0^{I'} + \sum_{I'=0}^{2} \sum_{l'=0}^{3} \int_{4m_\pi^2}^{S_{max}} ds' K_{ll'}^{II'}(s, s') \mathrm{Im} f_{l'}^{I'}(s')^{IN} + d_l^I(s). \tag{1}$$

The difference between $\mathrm{Re} f_l^I(s)^{out}$ and $\mathrm{Re} f_l^I(s)^{in}$ demonstrates a consistency of the amplitudes with the dispersion relations (i.e. with crossing symmetry). The smaller the difference the better consistency with crossing symmetry. Full amplitude in the s channel $T_s(s, t)$ and in the t channel $T_t(t, s)$ should be similar and differ only by a crossing matrix factor C_{st}

$$T_s(s, t) = C_{st} T_t(t, s). \tag{2}$$

Figure 1 shows the phase shifts for the $\pi\pi$ interaction in the S-wave versus energy from the $\pi\pi$ threshold up to 900 MeV i.e. focuses on the threshold region. Looking at results of the original amplitude [1] from $\pi\pi$ threshold up to 900 MeV, one sees that there is a significant problem with its threshold behavior. We replaced low energy part of the amplitudes for S- as well as P-wave by a polynomial functions from $\pi\pi$ threshold up to the "matching point", where momentum $k = \sqrt{s/4 - M_\pi^2}$.

$$\mathrm{Re} f_l^I(s) = \frac{\sqrt{s}}{4k} \sin 2\delta_l^I = m_\pi k^{2l} [a_l^I + b_l^I k^2 + c_l^I k^4 + d_l^I k^6 + O(k^8)] \tag{3}$$

3 Results

Applying the modifications, we have achieved the "extended" S- and P-wave amplitudes for the $\pi\pi$ scattering with $\chi^2/n.d.f = 16.9$ for the S-wave (for details see [5]). When we performed fit to the data and to the DR, the re-fitted parameters significantly improved the result for which $\chi^2/n.d.f. = 605.5/n.d.f. = 1.14$. The biggest and most important improvement was that for the DR contribution (from $\chi^2_{DR} = 478.0$ to 35.6) which suggests a significant improvement of consistency of the amplitudes with the crossing symmetry. Figure 2 shows how strongly position of the pole related with $f_0(500)$ resonance changed from $829 - i554.0$ MeV for the original amplitude to $449^{+14}_{-14} - i289^{+14}_{-14}$ MeV for the re-fitted one. Note that the new pole position accords well with the result from the analysis based on ChPT and Roy-like equations ($441^{+16}_{-8} - i272^{+9}_{-13}$) [6] and the result from the analysis based on GKPY equations ($445^{+25}_{-25} - i278^{+22}_{-18}$) [4].

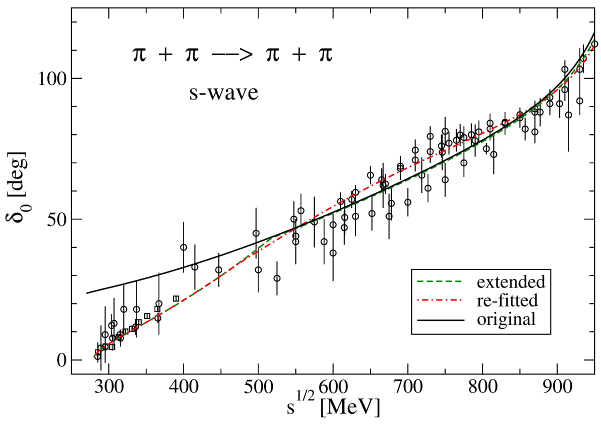

Figure 1. Phase shifts for the $S0$ wave amplitude as a function of the effective two pion mass $s^{1/2}$ for the original (solid line), extended (dashed line) and re-fitted (dash-dotted line) amplitude considered in the text. It shows how the *extended* amplitude solved the threshold behavior problem of low-energy S-wave $\pi\pi$ interaction amplitude. Above the "matching point "(350 - 650 MeV) the *original* and *extended* amplitudes are equivalent. Data are taken from [1].

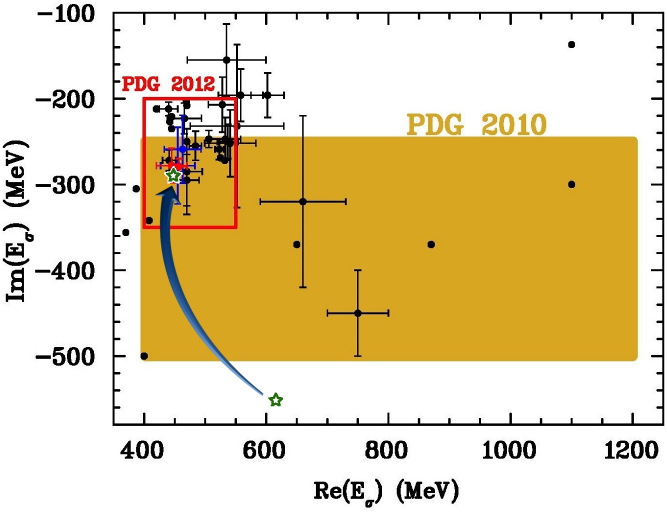

Figure 2. Shift of the σ pole after fitting to the GKPY equations. The big (PDG2010) and small (PDG2012) rectangles show area allowed by the Particle Data Tables in 2010 and 2012, respectively. Black points are positions of the poles listed in the Particle Data Tables published in 2010.

4 Conclusion

Agreement of the phase shifts with low-energy data was improved for the new re-fitted S- and P-wave $\pi\pi$ scattering amplitudes. The amplitudes are calculated with the scattering lengths and slope (effective-range) parameters consistent with results of calculations based on ChPT. Consistency of the three-channel amplitudes with the dispersion relations was improved significantly for the energies from the threshold up to 1100 MeV which means that the amplitudes better fulfill the crossing symmetry condition. The lowest pole in S-wave is strongly shifted to lower energy and nearer to the real axis which results in smaller values of the mass and width for the σ meson.

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