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η and η' transition form factors from Padé approximants

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Abstract. We employ a systematic and model-independent method to extract, from space- and time-like data, the η and η' transition form factors (TFFs) obtaining the most precise determination for their low-energy parameters and discuss the $\Gamma_{\eta\to\gamma\gamma}$ impact on them. Using TFF data alone, we also extract the $\eta - \eta'$ mixing parameters, which are compatible to those obtained from more sophisticated and input-demanding procedures.

1 Introduction

The hadronic structure of neutral pseudoscalar mesons may be probed via the two-photon mechanism. The most general matrix element for such process is given by

$$\mathcal{M}_{P\gamma^*\gamma^*} = ie^2\varepsilon^{\mu\nu\rho\sigma}q_{1,\mu}\epsilon_{1,\nu}q_{2,\rho}\epsilon_{2,\sigma}F_{P\gamma^*\gamma^*}(q_1^2,q_2^2), \tag{1}$$

where $q_i(\epsilon_i)$ stands for the i-*th* photon momentum (polarization) and $F_{P\gamma^*\gamma^*}(q_1^2,q_2^2)$ is the pseudoscalar transition form factor (TFF) encoding all the strong-interaction effects. Of particular interest is the single virtual TFF $F_{P\gamma^*\gamma}(Q^2) \equiv F_{P\gamma^*\gamma^*}(-q^2,0)$ for which many measurements are available. At low energies, the TFF can be expressed in terms of its low-energy parameters (LEPs) $b_P, c_P, d_P, ...$

$$F_{P\gamma^*\gamma}(Q^2) = F_{P\gamma\gamma}(0)\left(1 - b_P\left(\frac{Q^2}{m_P^2}\right) + c_P\left(\frac{Q^4}{m_P^2}\right) - d_P\left(\frac{Q^6}{m_P^2}\right) + ...\right). \tag{2}$$

However, due to the non-perturbative behavior of QCD at low energies, neither the TFF, nor its LEPs, can be calculated from first principles. Only its low- and high-energy limits are known from the axial anomaly [1, 2] and perturbative QCD [3], respectively. Remarkably, both limits depend on the same parameters. In this work [4], we focus on the η and η' TFFs. Using the flavor basis to describe the $\eta - \eta'$ mixing, these limits read

$$F_{\eta\gamma\gamma}(0) = \Big((\hat{c}_q/F_q)\cos\phi - (\hat{c}_s/F_s)\sin\phi\Big)/4\pi^2\ , \tag{3}$$

$$F_{\eta'\gamma\gamma}(0) = \Big((\hat{c}_q/F_q)\sin\phi + (\hat{c}_s/F_s)\cos\phi\Big)/4\pi^2\ , \tag{4}$$

$$\lim_{Q^2\to\infty} Q^2F_{\eta\gamma^*\gamma}(Q^2) = 2(\hat{c}_qF_q\cos\phi - \hat{c}_sF_s\sin\phi)\ , \tag{5}$$

$$\lim_{Q^2\to\infty} Q^2F_{\eta'\gamma^*\gamma}(Q^2) = 2(\hat{c}_qF_q\sin\phi + \hat{c}_sF_s\cos\phi)\ , \tag{6}$$

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Table 1. LEPs extraction from a fit to SL data using P_1^N and P_N^N sequences.

	η TFF				η' TFF			
	N	b_η	c_η	χ^2/dof	N	$b_{\eta'}$	$c_{\eta'}$	χ^2/dof
P_1^N	5	0.58(6)	0.34(8)	0.80	6	1.30(15)	1.72(47)	0.70
P_N^N	2	0.66(10)	0.47(15)	0.77	1	1.23(3)	1.52(7)	0.67
Final		0.60(6)	0.37(10)			1.30(15)	1.72(47)	

where $\hat{c}_q(\hat{c}_s) = 5/3(\sqrt{2}/3)$, $F_{q(s)}$ are decay constants, and ϕ is the mixing angle [4–6]. As an attempt to achieve a unified description for the whole energy regime, vector meson dominance (VMD) models, which find inspiration in the large-N_c limit of QCD, have been extensively used. However, these models contain potential systematic errors coming from simplifying assumptions and large-N_c corrections, which should not be ignored when calculating precision observables such as the hadronic light by light contribution to the $(g-2)_\mu$. This uncertainty may be observed when comparing the different determinations from space-like (SL) and time-like (TL) data for b_η. The result, which is obtained after a fit to data using the most simple VMD parametrization $F_{\eta\gamma^*\gamma}(Q^2) = F_{\eta\gamma^*}(0)/(1+Q^2/\Lambda^2)$, is significantly different when using SL or TL data. Such result may be taken as the crudest one in a systematic expansion in terms of Padé approximants (PA) as suggested in Ref. [7]. Only when taking into account the systematic error from this expansion, the different determinations agree. In this work [4], we extend the PA description for the π^0-TFF in Ref. [8] to the η and η' cases.

2 Method and results for the TFF

Padé approximants $P_M^N(x)$ are rational functions of two polynomials $P_M^N(x) = R_N(x)/Q_M(x)$ of degree N and M respectively, which coefficients are related to the original function $f(x)$ to be approximated through the condition $f(x) - P_M^N(x) = \mathcal{O}(x^{N+M+1})$ [9]. They are known to converge for meromorphic and Stieltjes functions [9], which has proven useful in QCD. For applications, see [10] and references therein.

In our case of study [4], the Q^2-dependence for $F_{P\gamma^*\gamma}(Q^2)$ as well as its analytic structure is unknown, and therefore, convergence theorems cannot be applied. Instead, we check the excellent performance of PA for different well-motivated physical models. Furthermore, since the LEPs are unknown, we extract them from a fitting procedure to the published $Q^2F_{\eta(\eta')\gamma^*\gamma}(Q^2)$ data using sequences of PAs. Particularly, we use the P_1^N and P_N^N sequences, from which, the VMD ($N=1$) parametrization, is the crudest approximation. Having a finite amount of data, these sequences must be truncated at some finite N. The systematic error this implies for the LEPs determination is estimated from the models. Averaging over the different sequences and including this last error allows for a model-indepedent determination of the LEPs, which may be used later to systematically reconstruct the TFF through the use of PAs.

Our results for the LEPs from a fit to the available SL data are shown in Tab. 1, while the slope (b_P) convergence pattern is illustrated in Fig. 1. Due to the systematic error, only the first two LEPs determination are meaningful. Our final result for $b_{\eta'}$ is the most precise to date and that of b_η is comparable to the most precise experimental extraction obtained by A2 Coll. $b_\eta = 0.585(54)$ [11] based on low-energy TL data. A detailed comparison to different results may be found in Ref. [4].

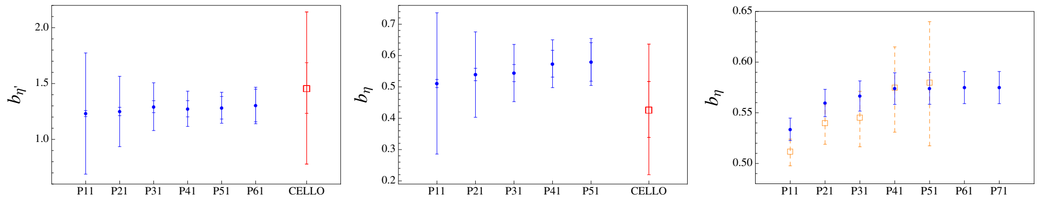

Figure 1. Slope (b_P) determination from the P_1^N sequence. The left and central panels show the results for η' and η from SL data with stat. (inner band) and combined stat. and syst. (outer band) errors. The right panel shows the η SL+TL result (solid line) together with the old SL result (dashed line). Only stat. errors are shown.

In Ref. [4], we suggested that our results may be used to predict the low-q^2 TL region, accessible through the $P \to \gamma^*\gamma \to \bar{\ell}\ell$ Dalitz decay. Such measurement was recently performed by the A2 Coll. for $P = \eta$ [11], and found an excellent agreement with our prediction. Their results encouraged us to include the TL data in our fitting procedure [12]. Our new determination for the LEPs is shown in Tab. 2. The advantages of including the TL data are clear: we reduce significantly the systematic errors by going to higher PAs, being able to obtain up to the third derivative d_η, and we improve both, on convergence (reducing the systematic error), and statistical errors as shown in Fig. 1, right panel.

Table 2. New determination of the η LEPs from a fit to SL+TL data [12].

	N	b_η	c_η	d_η	χ^2/dof
P_1^N	7	0.575(16)	0.338(22)	0.198(21)	0.7
P_N^N	2	0.576(15)	0.340(20)	0.201(19)	0.7
Final		0.576(11)	0.339(15)	0.200(14)	

Our results in Tab. 2 are, by far, the most precise to date. Particularly, we believe that the precision achieved for b_η will be hard to improve even if new data becomes available. Nevertheless, it must be stressed that the values obtained mildly depend on $F_{\eta\gamma\gamma}(0,0)$. For instance, if we would have used the value implied by the Primakoff $\Gamma_{\eta\to\gamma\gamma}$ decay width omitted in the PDG average [13], we would find $b_\eta = 0.57(6)$ and $b_\eta = 0.570(13)$ for the SL an SL+TL extractions respectively, which is relevant at the obtained precision. Therefore, clarifying this experimental situation would be important.

3 $\eta - \eta'$ mixing parameters

Using the $P_N^N(Q^2)$ sequence results, which has the correct asymptotic behavior implemented, we can extract the TFF asymptotic value (5). With this information, as well as $F_{\eta\gamma\gamma}(0)$ we can obtain the $\eta - \eta'$ mixing parameters from Eqs. (3-6). With three unknowns (F_q, F_s, ϕ), we must drop one of the equations. We discard the η' asymptotics (6) since its determination, coming from the first ($N = 1$) element, is the less reliable. Using the SL dataset alone, we obtained [4]

$$F_q/F_\pi = 1.06(1)\,, \quad F_s/F_\pi = 1.56(24)\,, \quad \phi = 40.3(1.8)^\circ\,, \tag{7}$$

where $F_\pi = 92.21(14)$ MeV is the pion decay constant [13]. From the SL+TL dataset, we obtain [12]

$$F_q/F_\pi = 1.07(1)\,, \quad F_s/F_\pi = 1.39(14)\,, \quad \phi = 39.3(1.2)^\circ\,, \tag{8}$$

which is a significant improvement compared to (7). This translates to $F_8/F_\pi = 1.29(10)$, $F_0/F_\pi = 1.19(6)$, $\theta_8 = -22.1(2.8)^\circ$, $\theta_0 = -8.1(3.2)^\circ$ in the octet-singlet basis [5, 6]. In Fig. 2 we compare our determination (8) with different phenomenological results [5, 6] and find a very good agreement even though we use a much smaller amount of inputs than Refs. [4–6]. Our determination is in tension with BaBar results at high TL q^2 values [14]. This urges for a second measurement of high-Q^2 data-points both for η and η' which may be accessed by the Belle collaboration.

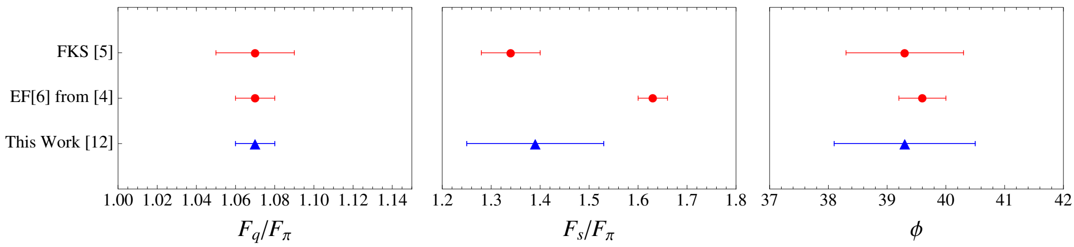

Figure 2. Our mixing parameters determination (triangles) compared to other phenomenological results (circles).

4 Conclusions

We have presented an easy and model-independent approach to describe and extract information of the η and η' transition form factors, such as the low energy parameters and the asymptotic behavior. Moreover, for the η case, we have extended its application from the space-like region, where it was originally intended, to the time-like region, obtaining the most precise extraction of the low-energy parameters. Additionally we comment on the impact of the $\Gamma_{\eta\to\gamma\gamma}$ decay width on the slope parameter. Finally, we have used the information on the transition form factor to extract the $\eta - \eta'$ mixing parameters with an excellent compromise between precision and predictivity.

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