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A dispersive treatment of $K_{\ell 4}$ decays

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Abstract. The $K_{\ell 4}$ decay is one of the best sources of information on some low-energy constants of chiral perturbation theory (χPT). We present a dispersive approach to $K_{\ell 4}$ decays, which takes rescattering effects fully into account. The dispersion relation treats both experimentally accessible form factors simultaneously and also describes the dependence on the dilepton invariant mass. We apply isospin-breaking corrections before fitting the data of NA48/2 and E865 experiments and extract the values of the low-energy constants L_1^r, L_2^r and L_3^r from a matching to χPT.

1 Introduction

$K_{\ell 4}$ is the semileptonic decay of a kaon into two pions and a lepton pair. Due to the two-pion final state, it is a clean source of information on pion dynamics [1]. Furthermore, $K_{\ell 4}$ is one of the best sources for the determination of some of the low-energy constants (LECs) of χPT [2–4]. The process happens at lower energies than e.g. $K\pi$ scattering, which would give access to the same LECs. As an expansion in the masses and momenta, χPT is expected to give a better description of $K_{\ell 4}$ than $K\pi$ scattering. However, due to the strong final-state rescattering effects, even two-loop χPT is not able to reproduce the measured curvature of one of the $K_{\ell 4}$ form factors.

Here, we present a new dispersive description of two experimentally accessible $K_{\ell 4}$ form factors [5]. The current framework represents a substantial improvement and extension of our former treatment [6, 7]. The dispersion relation provides a resummation of $\pi\pi$- and $K\pi$-rescattering effects. We correct the NA48/2 and E865 data by isospin-breaking effects computed in [8]. By matching the dispersion relation to χPT, we extract the LECs L_1^r, L_2^r and L_3^r.

2 Dispersion relation for $K_{\ell 4}$

2.1 $K_{\ell 4}$ form factors

We consider the decay mode ($\ell = e, \mu$)

$$K^+(p) \to \pi^+(p_1)\pi^-(p_2)\ell^+(p_\ell)\nu_\ell(p_\nu) \tag{1}$$

as well as its charge conjugate mode. Experimental data is available on the electron mode. The hadronic part of the matrix element can be written in terms of four form factors F, G, R and H:

$$\begin{aligned}
\langle \pi^+(p_1)\pi^-(p_2)|V_\mu(0)|K^+(p)\rangle &= -\frac{H}{M_K^3}\epsilon_{\mu\nu\rho\sigma}L^\nu P^\rho Q^\sigma, \\
\langle \pi^+(p_1)\pi^-(p_2)|A_\mu(0)|K^+(p)\rangle &= -i\frac{1}{M_K}(P_\mu F + Q_\mu G + L_\mu R),
\end{aligned} \tag{2}$$

where $P = p_1 + p_2$, $Q = p_1 - p_2$, $L = p - p_1 - p_2$. The form factors are functions of the usual Mandelstam variables s, t and u. In K_{e4} experiments, R is not accessible. H gets a first contribution only at $O(p^4)$ due to the chiral anomaly. We focus on the form factors F and G.

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2.2 Reconstruction theorem and integral equations

Based on fixed-$s/t/u$ dispersion relations, one can derive a decomposition of the form factors into functions of only one Mandelstam variable, known as 'reconstruction theorem' [9, 10]. The derivation neglects the imaginary parts of D- and higher partial waves, an $O(p^8)$ effect:

$$\begin{aligned} F(s,t,u) &= M_0(s) + \frac{u-t}{M_K^2} M_1(s) + (\text{terms involving functions of } t \text{ or } u) + O(p^8), \\ G(s,t,u) &= \tilde{M}_1(s) + (\text{terms involving functions of } t \text{ or } u) + O(p^8), \end{aligned} \tag{3}$$

where the functions of one variable $M_0, \ldots$ are defined to contain only the right-hand cut of the partial waves of the form factors F and G in the three channels. E.g. the function M_0 contains the right-hand cut of the s-channel S-wave f_0 of the form factor F:

$$M_0(s) = P(s) + \frac{s^2}{\pi} \int_{4M_\pi^2}^{\infty} ds' \frac{\mathrm{Im} f_0(s)}{(s'-s-i\epsilon)s'^2}, \tag{4}$$

where $P(s)$ is a subtraction polynomial. Eight more functions $M_1, \ldots$ take care of the right-hand cuts of S- and P-waves in all channels, such that all the discontinuities are divided up into functions of a single variable. They satisfy inhomogeneous Omnès equations with the solution

$$M_0(s) = \Omega_0^0(s) \left\{ \tilde{P}(s) + \frac{s^3}{\pi} \int_{4M_\pi^2}^{\Lambda^2} ds' \frac{\hat{M}_0(s') \sin \delta_0^0(s')}{|\Omega_0^0(s')|(s'-s-i\epsilon)s'^3} \right\}, \tag{5}$$

where $\tilde{P}(s)$ is a new subtraction polynomial and the Omnès function is given by

$$\Omega_0^0(s) = \exp\left\{ \frac{s}{\pi} \int_{4M_\pi^2}^{\infty} ds' \frac{\delta_0^0(s')}{(s'-s-i\epsilon)s'} \right\}. \tag{6}$$

In total, 9 subtraction constants appear. We need the following elastic $\pi\pi$ and $K\pi$ phase shifts as input, which we assume to reach a multiple of π at the cut-off Λ^2:

- δ_0^0, δ_1^1: elastic $\pi\pi$-scattering phase shifts [11, 12],
- $\delta_0^{1/2}$, $\delta_1^{1/2}$, $\delta_0^{3/2}$, $\delta_1^{3/2}$: elastic $K\pi$-scattering phase shifts [13, 14].

The inhomogeneities $\hat{M}_0, \ldots$ contain the left-hand cuts of the partial waves and are given as angular averages of all the functions $M_0, \ldots$. Hence, we face a set of coupled integral equations: the functions $M_0, \ldots$ are defined by dispersion integrals involving the inhomogeneities $\hat{M}_0, \ldots$, which are again defined as angular integrals of the functions $M_0, \ldots$.

3 Solution of the dispersion relation

We note that the integral equations are linear in the subtraction constants. Therefore, for each subtraction constant we construct a basic solution, which we obtain by solving numerically the integral equations in an iterative procedure. The final result is a linear combination of these basic solutions.

We determine the subtraction constants using three sources of information: first, we fit the experimental data on the form factors F and G from the high-statistics experiments NA48/2 [1, 15] and E865 [16, 17]. Secondly, we use the soft-pion theorem, which establishes relations between F, G and f_+, the $K_{\ell 3}$ vector form factor, as an additional constraint. Finally, we fix the subtraction constants that are not well determined by the data with chiral input.

4 Results

The matching of the dispersion relation to χPT is performed directly at the level of the subtraction constants in the Omnès representation. Thus, it avoids a mixing with the resummation of $\pi\pi$- and $K\pi$-rescattering effects. From the matching to one-loop χPT, we obtain the values of three LECs:

$$\begin{aligned} 10^3 \cdot L_1^r(\mu) &= 0.51(02)(05), \\ 10^3 \cdot L_2^r(\mu) &= 0.86(05)(08), \\ 10^3 \cdot L_3^r(\mu) &= -2.77(10)(07), \end{aligned} \tag{7}$$

where $\mu = 770$ MeV. The first error indicates the statistical uncertainty stemming from the form factor data. The second error is the systematic uncertainty due to the phase shift input and chiral input on L_4^r, L_5^r and L_9^r. The obtained values are close to recent results of a global two-loop χPT fit [18]. Compared to our former treatment [6, 7], the new determination of the LECs has a much better precision.

The matching at two-loop level shows a rather strong dependence on the input values for the NNLO LECs C_i^r. We hope to reduce this instability by imposing constraints on the chiral convergence.

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