EPJ Web of Conferences **85**, 02013 (2015)
DOI: 10.1051/epjconf/20158502013

A_N in proton-proton collisions and the role of twist-3 fragmentation

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Abstract. We review and give an update on the current status of what causes transverse single-spin asymmetries (TSSAs) in semi-inclusive processes where a single hadron is detected in the final state, especially those involving proton-proton (pp) collisions. In particular, we provide a new analysis within collinear factorization of TSSAs in high transverse momentum charged and neutral pion production in pp collisions at the Relativistic Heavy Ion Collider (RHIC). This study incorporates the so-called twist-3 fragmentation term and shows that one can describe RHIC data through this mechanism. Moreover, by fixing other non-perturbative inputs through extractions of transverse momentum dependent functions in $e^+e^- \to h_1 h_2 X$ and semi-inclusive deep-inelastic scattering (SIDIS), we provide for the first time a consistency between certain spin/azimuthal asymmetries in all three reactions (i.e., pp, e^+e^-, and SIDIS).

1 Introduction

Transverse single-spin asymmetries (TSSAs) have received much attention from both the experimental and theoretical side starting in the mid-1970s. Initially, large effects were seen in transversely polarized Λ production in proton-beryllium collisions [1]. These results were thought to contradict perturbative Quantum Chromodynamics (pQCD) because for such asymmetries (denoted by A_N) one should have $A_N \sim \alpha_s m_q / P_{h\perp}$, with $\alpha_s = g^2/4\pi$ (g is the strong coupling constant), m_q the mass of the quark, and $P_{h\perp}$ the transverse momentum of the outgoing hadron/particle [2]. Later it was shown how twist-3 quark-gluon-quark correlations in the nucleon could cause significant asymmetries [3], with benchmark calculations first performed in [4, 5] using collinear factorization. Experimental measurements of TSSAs in single-inclusive hadron production in proton-(anti)proton collisions also continued to show a sizable A_N [6–10]. This led to much (still ongoing) theoretical work on these and similar observables — see, e.g., [4, 5, 11–17]. In Sec. 2 we summarize the theoretical formalism used to describe TSSAs, namely collinear twist-3 factorization. We also review attempts to explain this effect and why it has remained a puzzle for close to 40 years. We next show in Sec. 3 how the fragmentation mechanism could play a crucial role in TSSAs for single-inclusive pion production from proton-proton (pp) collisions. This is the main content of the manuscript and is based on the work in [18]. In particular, we demonstrate that one can obtain a very good fit of all high transverse momentum RHIC data in $p^\uparrow p \to \pi X$ by including this fragmentation term while also fixing certain non-perturbative inputs from transverse momentum dependent (TMD) functions extracted in $e^+e^- \to h_1 h_2 X$ and semi-inclusive deep-inelastic scattering (SIDIS). Thus we provide for the first time in pQCD a simultaneous description of certain spin/azimuthal asymmetries in all three reactions (i.e., pp, e^+e^-, and SIDIS). In Sec. 4 we summarize our results and provide an outlook on the much fruitful work that lies ahead in order to fully understand TSSAs.

2 Collinear twist-3 formalism

We consider a process of the type $A(P, \vec{S}_\perp) + B(P') \to C(P_h) + X$, where the 4-momenta and polarizations of the incoming protons A, B and outgoing hadron C are specified. In collinear twist-3 QCD factorization one has

$$
\begin{aligned}
d\sigma(\vec{S}_\perp) = {} & H \otimes f_{a/A(3)} \otimes f_{b/B(2)} \otimes D_{C/c(2)} \\
& + H' \otimes f_{a/A(2)} \otimes f_{b/B(3)} \otimes D_{C/c(2)} \\
& + H'' \otimes f_{a/A(2)} \otimes f_{b/B(2)} \otimes D_{C/c(3)} \,, \qquad (1)
\end{aligned}
$$

where $f_{a/A(t)}$ denotes the twist-t distribution function for parton a in proton A, and similarly for the other distribution function $f_{b/B(t)}$ and fragmentation function $D_{C/c(t)}$ for hadron C in parton c. The hard factors are given by H, H', and H'', which are different for each term, and the symbol $\otimes$ represents convolutions in the appropriate momentum

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fractions. In Eq. (1) a sum over partonic channels and parton flavors in each channel is understood.

The first term in (1), where collinear twist-3 correlators appear for the transversely polarized proton, has been studied extensively [5, 12–14, 17, 19]. It contains three pieces that involve different matrix elements: i) quark-gluon-quark (qgq) soft-gluon pole (SGP) [5, 12, 14, 19]; ii) qgq soft-fermion pole (SFP) [13]; and iii) tri-gluon (ggg) SGP [17]. The second term in (1), which involves collinear twist-3 functions for the unpolarized proton, was shown to be small [20]. For the third term in (1), in which collinear twist-3 functions enter for the outgoing (unpolarized) hadron, the complete analytical result was obtained only recently [21].

For quite some time it was assumed that the first term in (1) dominates A_N in $p^\uparrow p \to hX$ for the production of light hadrons (see, e.g., Refs. [5, 12, 14]), where the qgq SGP matrix element, called the Qiu-Sterman (QS) function T_F [4, 5], is generally considered the most important part. The QS function can be related to the TMD Sivers function $f_{1T}^\perp$ [22, 23]. One has [24]

$$T_F^q(x,x) = -\int d^2\vec{p}_\perp \, \frac{\vec{p}_\perp^{\,2}}{M} \, f_{1T}^{\perp q}(x, \vec{p}_\perp^{\,2})\big|_{\rm SIDIS}\,, \qquad (2)$$

where q is the quark flavor, and M is the nucleon mass. Because of the relation in (2), one should be able to extract $f_{1T}^\perp$ from the Sivers TSSA $A_{\rm SIDIS}^{\rm Siv}$ in SIDIS, evaluate the r.h.s. of (2) and obtain the QS function on the l.h.s. that has been extracted directly from A_N in pp collisions. It was quite a shock, then, when such an attempt failed, i.e., one could not simultaneously explain both A_N and $A_{\rm SIDIS}^{\rm Siv}$ [25]. Rather, what was found in Ref. [25] was that the two extractions for T_F differ in sign. This "sign-mismatch" puzzle could not be resolved by more flexible parameterizations of $f_{1T}^\perp$ [26]. Also tri-gluon correlations are unlikely to fix this issue [17], while SFPs may play some role [13].

At this point one may start to question the assumption that the QS function is the dominant source of A_N in $p^\uparrow p \to \pi X$. In fact, data on the TSSA in inclusive DIS [27, 28] seems to support this point of view, i.e., that the first term in (1) is not the main cause of A_N. This is seen clearly through the analysis in [15], where one obtains the wrong sign for the neutron target TSSA when using T_F extracted directly from A_N data on pp collisions. Therefore, in the next section we analyze the impact of the third term in (1), i.e., the collinear twist-3 fragmentation part, to see if one can describe charged and neutral pion RHIC A_N data through this mechanism.

However, before we proceed to the phenomenology, let us first recall the important details of the analytical result for this fragmentation term. Its contribution to the cross section in (1) reads [21]

$$\begin{aligned}\frac{P_h^0 d\sigma(\vec{S}_\perp)}{d^3\vec{P}_h} &= -\frac{2\alpha_s^2 M_h}{S}\epsilon_{\perp,\alpha\beta}\, S_\perp^\alpha P_{h\perp}^\beta \sum_i \sum_{a,b,c}\int_{z_{min}}^1 \frac{dz}{z^3} \\ &\times \int_{x'_{min}}^1 \frac{dx'}{x'}\frac{1}{x}\frac{1}{x'S+T/z}\frac{1}{-x'\hat{t}-x\hat{u}}\, h_1^a(x)\, f_1^b(x') \\ &\times \Bigg\{\left[\hat{H}^{C/c}(z) - z\frac{d\hat{H}^{C/c}(z)}{dz}\right] S_{\hat{H}}^i + \frac{1}{z}H^{C/c}(z)\, S_H^i \\ &+ 2z^2\int_z^\infty \frac{dz_1}{z_1^2}\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\,\hat{H}_{FU}^{C/c,\Im}(z,z_1)\,\frac{1}{\xi}\,S_{\hat{H}_{FU}}^i\Bigg\}. \qquad (3)\end{aligned}$$

Here i denotes the channel, $x = -x'(U/z)/(x'S + T/z)$, $x'_{min} = -(T/z)/(U/z + S)$, $z_{min} = -(T + U)/S$, and $\xi = (1-z/z_1)$. The Mandelstam variables are $S = (P+P')^2$, $T = (P - P_h)^2$, and $U = (P' - P_h)^2$, which on the partonic level give $\hat{s} = xx'S$, $\hat{t} = xT/z$, and $\hat{u} = x'U/z$. Relevant kinematic quantities also include Feynman-x $x_F = 2P_{hz}/\sqrt{S}$, where P_{hz} is the longitudinal momentum of the hadron, as well as pseudo-rapidity $\eta = -\ln\tan(\theta/2)$, where θ is the scattering angle. The variables x_F, η are further related by $x_F = 2P_{h\perp}\sinh(\eta)/\sqrt{S}$, where $P_{h\perp}$ is the transverse momentum of the hadron.

There are several non-perturbative functions that enter into Eq. (3). They are the transversity distribution h_1, the unpolarized parton density f_1, and the three (twist-3) fragmentation functions (FFs) $\hat{H}$, H, and $\hat{H}_{FU}^{\Im}$, with the last one being the imaginary part of a 3-parton correlator. In Ref. [21] one can also find the definition of those functions and the results for the hard scattering coefficients S^i for each channel i. (An alternative notation of the relevant FFs can also be found in Ref. [29], where twist-3 fragmentation effects in SIDIS were computed.)

Similar to the relation between T_F and the Sivers function $f_{1T}^\perp$ in Eq. (2), the function $\hat{H}$ can be written in terms of the TMD Collins function $H_1^\perp$ [30] according to [21, 31]

$$\hat{H}^{h/q}(z) = z^2\int d^2\vec{k}_\perp \, \frac{\vec{k}_\perp^{\,2}}{2M_h^2}\, H_1^{\perp h/q}(z, z^2\vec{k}_\perp^{\,2})\,. \qquad (4)$$

Exploiting the universality of the Collins function [32], one can simultaneously extract (see [33] and references therein) $H_1^\perp$ and h_1 from data [34, 35] on the Collins TSSA $A_{\rm SIDIS}^{\rm Col}$ in SIDIS [36] and data [37, 38] on the $\cos(2\phi)$ modulation $A_{e^+e^-}^{\cos(2\phi)}$ in $e^+e^- \to h_1h_2X$ [39]. Such information for $H_1^\perp$ and h_1, as well as that for the $f_{1T}^\perp$ [40, 41], will be useful when describing A_N. The FFs $\hat{H}$, H, and $\hat{H}_{FU}^{\Im}$ are not independent, but rather satisfy [21]

$$H^{h/q}(z) = -2z\hat{H}^{h/q}(z) + 2z^3\int_z^\infty \frac{dz_1}{z_1^2}\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\hat{H}_{FU}^{h/q,\Im}(z,z_1)\,, \qquad (5)$$

implying that in the collinear twist-3 framework one has two independent FFs. It is important to realize that this is different from the so-called TMD approach for A_N, where only $H_1^\perp$ enters the fragmentation piece [42]. We note that the Sivers effect in the TMD formalism has also been applied to A_N in $p^\uparrow p \to hX$ [41]. However, given that for single-inclusive processes there is only one large scale, using TMD factorization (which requires two different scales) in such reactions can only be considered a phenomenological model. In that sense, the collinear twist-3 formalism is the more rigorous theoretical framework.

3 Phenomenological fit of pion data

We analyze A_N data for $p^\uparrow p \to \pi X$ in the forward region of the polarized proton, which has been studied by the STAR [8], BRAHMS [9], and PHENIX [10] collaborations at RHIC. The data at $\sqrt{S} = 200\,\text{GeV}$ typically has $P_{h\perp} > 1\,\text{GeV}$, and we therefore focus on those measurements in order to safely apply pQCD. Throughout we use the GRV98 unpolarized parton distributions [43] and the DSS unpolarized FFs [44]. The GRV98 functions were also used in Refs. [33, 40, 41] for extracting the Sivers function and the transversity, which we take as input in our calculation, so we adhere with this choice as a matter of consistency. The qgq SGP contribution to (1) is computed by fixing T_F through Eq. (2) with two different inputs for the Sivers function — SV1: $f_{1T}^{\perp}$ from Ref. [40], obtained from SIDIS data on $A_{\rm SIDIS}^{\rm Siv}$ [45, 46]; and SV2: $f_{1T}^{\perp}$ from Ref. [41], "constructed" such that, in the TMD approach, the contribution of the Sivers effect to A_N is maximized while maintaining a good fit of $A_{\rm SIDIS}^{\rm Siv}$. The input SV1 has a flavor-independent large-x behavior, while SV2 in that region has a flavor dependence and also falls off slower. To compute the fragmentation contribution we take h_1 and $H_1^{\perp}$ (which fixes $\hat{H}$ through (4)) from [33]. For favored fragmentation into π^+ we make for $\hat{H}_{FU}^{\Im}$ the ansatz

$$\frac{\hat{H}_{FU}^{\pi^+/(u,\bar{d}),\Im}(z,z_1)}{D^{\pi^+/(u,\bar{d})}(z)\,D^{\pi^+/(u,\bar{d})}(z/z_1)} = \frac{N_{\rm fav}}{2I_{\rm fav}J_{\rm fav}}\, z^{\alpha_{\rm fav}}(z/z_1)^{\alpha'_{\rm fav}} \times (1-z)^{\beta_{\rm fav}}\,(1-z/z_1)^{\beta'_{\rm fav}}\,, \quad (6)$$

with the parameters $N_{\rm fav}$, $\alpha_{\rm fav}$, $\alpha'_{\rm fav}$, $\beta_{\rm fav}$, $\beta'_{\rm fav}$ and the unpolarized FF D. This parameterization follows the standard procedure of modifying the small and large "x" behavior of twist-2 unpolarized functions when trying to fit an unknown function. Note that z and z/z_1 are chosen as our "variables" because their allowed range is [0, 1] [47] and that our ansatz satisfies the constraint $\hat{H}_{FU}(z,z) = 0$ [47, 48]. With the use of DSS FFs [44], the factor $I_{\rm fav}$ reads $I_{\rm fav} \equiv I_{u+\bar{u}} - I_{\bar{u}}$ where I_i ($i = u+\bar{u},\, \bar{u}$) is defined as

$$I_i = \frac{N_i(K_{1,\rm fav} + \gamma_i K_{2,\rm fav})}{B[2+\alpha_i, \beta_i + 1] + \gamma_i B[2+\alpha_i, \beta_i + \delta_i + 1]}\,,$$
$$\text{with}\;\; K_{1,\rm fav} = B[\alpha'_{\rm fav} + \alpha_i + 1, \beta'_{\rm fav} + \beta_i]\,, \quad (7)$$
$$K_{2,\rm fav} = B[\alpha'_{\rm fav} + \alpha_i + 1, \beta'_{\rm fav} + \beta_i + \delta_i]\,,$$

and $B[a,b]$ the Euler β-function. The parameters N_i, α_i, β_i, γ_i, and δ_i come from D FFs at the initial scale and are given in Table III of [44]. Note that $D^{\pi^+/u}$ in Ref. [44] differs from $D^{\pi^+/\bar{d}}$. $J_{\rm fav}$ in (6) is similarly defined as $J_{\rm fav} \equiv J_{u+\bar{u}} - J_{\bar{u}}$, where J_i ($i = u+\bar{u},\, \bar{u}$) follows from I_i through $\alpha'_{\rm fav} \to (\alpha_{\rm fav}+4)$, $\beta'_{\rm fav} \to (\beta_{\rm fav}+1)$. The factor $1/(2I_{\rm fav}J_{\rm fav})$ in (6) is convenient and implies $\int_0^1 dz\, z\, H_{(3)}^{\pi^+/u}(z) = N_{\rm fav}$ at the initial scale, where $H_{(3)}$ represents the entire second term on the r.h.s. of (5). For the disfavored FFs $\hat{H}_{FU}^{\pi^+/(d,\bar{u}),\Im}$ we make an ansatz in full analogy to (6), introducing the additional parameters $N_{\rm dis}$, $\alpha_{\rm dis}$, $\alpha'_{\rm dis}$, $\beta_{\rm dis}$, $\beta'_{\rm dis}$. ($I_{\rm dis}$ and $J_{\rm dis}$ are calculated using $D^{\pi^+/d} = D^{\pi^+/\bar{u}}$ from [44].) The π^- FFs are then fixed through charge conjugation, and the π^0 FFs are given by the average of the FFs for π^+ and π^-. The FFs $H^{\pi/q}$ are computed by means of (5). All parton correlation functions are evaluated at the scale $P_{h\perp}$ with leading order evolution of the collinear functions.

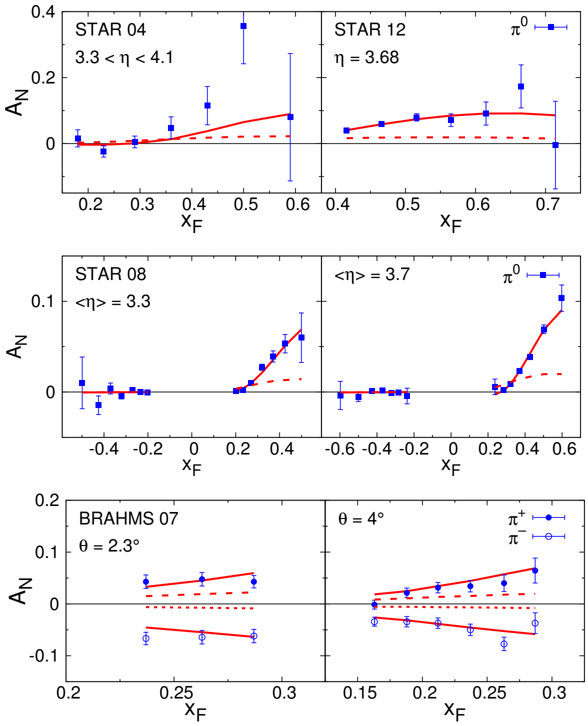

Figure 1. Fit results for $A_N^{\pi^0}$ (data from [8]) and $A_N^{\pi^\pm}$ (data from [9]) for the SV1 input. The dashed line (dotted line in the case of π^-) means $\hat{H}_{FU}^{\Im}$ switched off.

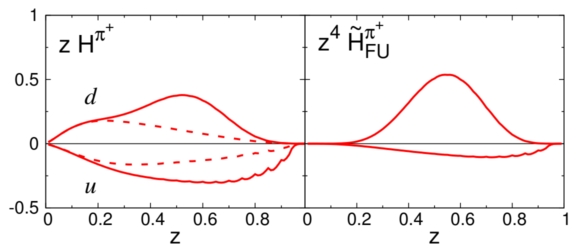

Figure 2. Results for the FFs $H^{\pi^+/q}$ and $\tilde{H}_{FU}^{\pi^+/q}$ (defined in the text) for the SV1 input. Also shown is $H^{\pi^+/q}$ without the contribution from $\hat{H}_{FU}^{\Im}$ (dashed line).

Using the MINUIT package we fit the fragmentation contribution to data for $A_N^{\pi^0}$ [8] and $A_N^{\pi^\pm}$ [9]. In order to limit the number of free parameters, we only keep 7 of them free in $\hat{H}_{FU}^{\pi^+/q,\Im}$. Since the large-$x$ behavior of h_1 is mostly unconstrained by current SIDIS data, we also allow the β-parameters $\beta_u^T = \beta_d^T$ of the transversity to vary within the error range given in [33]. All integrations are done using the Gauss-Legendre method with 250 steps.

For the SV1 input the result of our 8-parameter fit is shown in Tab. 1. For the SV2 input the values of the fit parameters are similar, with an equally successful fit (χ^2/d.o.f. = 1.10).

The very good description of the RHIC A_N data is explicitly evident in Fig. 1. We emphasize this is a non-trivial outcome if one keeps in mind the constraint in (5) and the need to simultaneously fit data for $A_N^{\pi^0}$ and $A_N^{\pi^\pm}$. Results for the FFs $H^{\pi^+/q}$ and $\tilde{H}_{FU}^{\pi^+/q} \equiv \int_z^\infty \frac{dz_1}{z_1^2}\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\frac{1}{1-\frac{z}{z_1}}\hat{H}_{FU}^{\pi^+/q,\Im}(z,z_1)$

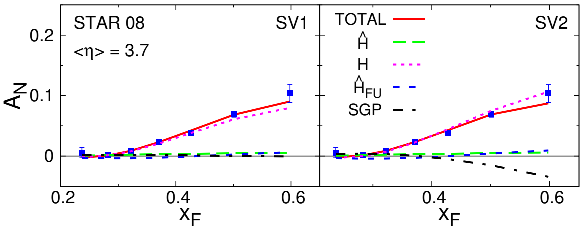

Figure 3. Individual contributions to $A_N^{\pi^0}$ (data from [8]) for SV1 and SV2 inputs.

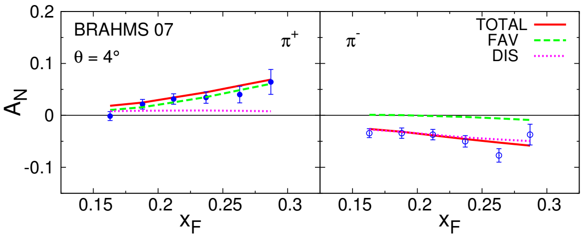

Figure 4. Individual contributions to $A_N^{\pi^\pm}$ from favored and disfavored fragmentation (data from [9]) for SV1 input.

Table 1. Fit parameters for SV1 input.

χ^2/d.o.f. = 1.03	
$N_{\rm fav} = -0.0338$	$N_{\rm dis} = 0.216$
$\alpha_{\rm fav} = \alpha'_{\rm fav} = -0.198$	$\beta_{\rm fav} = 0.0$
$\beta'_{\rm fav} = \beta'_{\rm dis} = -0.180$	$\alpha_{\rm dis} = \alpha'_{\rm dis} = 3.99$
$\beta_{\rm dis} = 3.34$	$\beta_u^T = \beta_d^T = 1.10$

are displayed in Fig. 2. Similar to the Collins function $H_1^\perp$, in either case the favored and disfavored FFs have opposite signs. Such reversed signs are actually "preferred" by the Schäfer-Teryaev (ST) sum rule $\sum_h \sum_{S_h} \int_0^1 dz\, z\, M_h \hat{H}^{h/q}(z) = 0$ [49]. Note that the ST sum rule, in combination with (5), implies a constraint on a certain linear combination of $H^{h/q}$ and (an integral of) $\hat{H}_{FU}^{h/q,\Im}$. In view of that, one benefits from favored and disfavored FFs having opposite signs like in Fig. 2. Also depicted in Fig. 2 is $H^{\pi^+/q}$ when $\hat{H}_{FU}^{\pi^+/q,\Im}$ is switched off. One sees $\hat{H}_{FU}^{\pi^+/q,\Im}$ causes a reasonable increase from this scenario. As shown in Fig. 1, when the 3-parton FF is turned off, one has difficulty describing the data for A_N. According to Fig. 3, the $\hat{H}$ term (including its derivative) contributes only very little to A_N. Also the (qgq) SGP pole term is small, except for the SV2 input at large x_F, where its contribution is opposite to the data. Note that with a Sivers function similar to SV2, there would definitely be serious issues with trying to match the A_N data without the 3-parton FF. Clearly A_N is governed by the H-term. (Recall from (5) that this function involves both $\hat{H}^{\pi/q}$ and $\hat{H}_{FU}^{\pi/q,\Im}$.) This result can mainly be traced back to the hard scattering coefficients: e.g., for the dominant $qg \to qg$ channel one has $S_H \propto 1/\hat{t}^3$, but $S_{\hat{H}} \propto 1/\hat{t}^2$ [21] in the forward region where $\hat{t}$ is small. Note also $S_{\hat{H}_{FU}} \sim 1/\hat{t}^3$ for that channel, but it is suppressed by a color factor of $1/(N_c^2 - 1)$. Next, Fig. 4 shows the breakdown of $A_N^{\pi^\pm}$ into favored and disfavored fragmentation contributions. One can see that

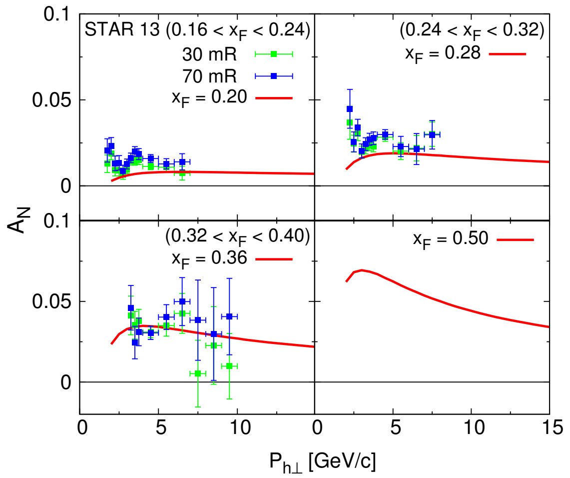

Figure 5. A_N as function of $P_{h\perp}$ for SV1 input at $\sqrt{S} = 500$ GeV (data from [50]).

$A_N^{\pi^+}$ ($A_N^{\pi^-}$) is dominated by favored (disfavored) fragmentation. Finally, Fig. 5 shows the $P_{h\perp}$-dependence of A_N for $\sqrt{S} = 500$ GeV. Preliminary data from STAR, extending to almost $P_{h\perp} = 10$ GeV, shows that A_N is rather flat [50]. Oftentimes it is stated that the collinear twist-3 calculation cannot reproduce this flat $P_{h\perp}$ dependence of A_N due to the naïve expectation that $A_N \sim 1/P_{h\perp}$ for a subleading twist effect. However, as was first argued in [5] and later shown in [14], this does not have to be the case. Our calculation indeed does lead to a flat $P_{h\perp}$ dependence, and also the magnitude of A_N is in line with the data. Note that the data of Ref. [50] were not included in our fit and that only statistical errors are shown in Fig. 5 [50].

4 Summary and outlook

For many years it was unclear what mechanism causes large TSSAs in hadron production from proton-proton collisions. Collinear twist-3 QCD factorization can be considered the most natural and rigorous approach to describe this observable, yet the sign-mismatch issue [25] threatened the validity of this formalism. Here we have shown for the first time that the fragmentation contribution in twist-3 factorization actually can describe high-energy RHIC data for A_N^π very well. By using a Sivers function fully consistent with SIDIS, we have demonstrated that this mechanism could also resolve the sign-mismatch crisis. We used the TMD Sivers, Collins, and transversity functions, which were extracted through spin/azimuthal asymmetries in SIDIS and $e^+e^- \to h_1 h_2 X$, to fix certain non-perturbative inputs in our calculation. Together with the collinear 3-parton FF, these functions allowed for a very good fit of $p^\uparrow p \to \pi X$ data. Thus we have shown that at present a simultaneous description of all three observables is possible (i.e., pp, e^+e^-, and SIDIS). We leave an analysis of A_N for kaons and etas and incorporation of SFPs for future work.

Ultimately in order to truly determine what mechanism underlies TSSAs, one must obtain information from other reactions in order to independently determine the relevant collinear twist-3 functions and/or verify that previously extracted functions are consistent with other measurements. In this context, one already has data on A_N in $p^{\uparrow}p \to jet\,X$ available from the A$_N$DY Collaboration [51]. Experiments to determine A_N for Drell-Yan and direct photon production would also be beneficial. Even measurements of the Sivers and Collins asymmetries at large $P_{h\perp}$ would be helpful and could be performed at Jefferson Lab (JLab) 12, COMPASS, or a future Electron-Ion Collider. In addition, data on TSSAs for single-inclusive hadron production from lepton-nucleon collisions is currently available from JLab [52] and HERMES [53]. This reaction was also recently analyzed in [54] using the collinear twist-3 approach and in [55] within the TMD framework. The main question then becomes if one can find a formalism that can consistently describe TSSAs in all of these processes. Much work is left to be done on both the theoretical and experimental side in order to answer this.

Acknowledgments

This work has been supported by the Grant-in-Aid for Scientific Research from the Japanese Society of Promotion of Science under Contract Nos. 24.6959 (K.K.), 23540292 and 26287040 (Y.K.), the National Science Foundation under Contract No. PHY-1205942 (A.M.), and the RIKEN BNL Research Center (D.P.).

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