Sivers Effect in Dihadron Electroproduction

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Abstract. The Sivers effect in polarized SIDIS can be measured in two hadron production as sine modulations involving the azimuthal angles $\varphi_T$ and $\varphi_R$ of both the total and the relative transverse momenta of the hadron pair, complementary to the conventional single hadron studies. In this talk we briefly present the results obtained in our recent work \cite{1} and \cite{2}. We also present the leading order parton model expression for the two hadron SIDIS cross section for different choices of the relative transverse momentum that dismiss the seeming contradiction of our results with previous work. Finally, we show the numerical predictions for the corresponding single spin asymmetries in the kinematics of COMPASS experiment obtained using the modified version of the LEPTO Monte Carlo event generator that includes the Sivers effect.

1 Introduction

One of the most interesting phenomena related to internal 3D structure of polarized nucleons is the Sivers effect \cite{3}. The corresponding SSA in one hadron SIDIS have been measured in HERMES \cite{4}, COMPASS \cite{5}, and JLab \cite{6} experiments and allow to extract the Sivers PDF, e.g. \cite{7}.

Recently, the two hadron production in polarized SIDIS (2h SIDIS) and in the jets from $e^+e^-$ annihilation attracted a lot of interest both in experimental \cite{8–10} and theoretical \cite{11–13} studies. The main accent has been on studies of transversity PDFs and so called interference fragmentation functions.

Here we present the phenomenology of Sivers effect in 2h SIDIS. We also discuss the different choices for the relative transverse momentum of the hadrons pair and their implications on the observable Sivers asymmetry.

2 The Sivers effect in 2h SIDIS

\textbf{Figure 1.} Two hadron production in the current fragmentation region of SIDIS.

\textbf{Figure 2.} $\gamma^* - N$ center of mass frame.

It is easy to see that in this system $k_T' = k_T$ and (similar to single hadron SIDIS \cite{7}) in the DIS limit for small transverse momenta of the produced hadrons

\begin{equation}
P_{1\perp} \approx P_{1T} - z_1k_T, \quad P_{2\perp} \approx P_{2T} - z_2k_T.
\end{equation}

2.1 Cross section expression

The process of two hadron electro-production in the current fragmentation region of SIDIS is schematically depicted in Fig. 1. In this work we adopt the $\gamma^* - N$ center of mass frame, where the $z$ axis is along the direction of the virtual photon momentum $q$. The $x$-$z$ plane is the lepton scattering plane. In this frame, we will define the transverse components of the momenta with respect to the fragmenting quark's direction with subscript $\perp$, as demonstrated in Fig. 2. We use that standard SIDIS kinematical variables as described in Ref. \cite{7}.
We assume that the cross section of the process of interest factorizes into convolutions of transverse polarization dependent TMD PDFs, \( f_1^T(x, k_T) \), unpolarized dihadron fragmentation functions (DiFFs), \( D_{q_h}^{h/z}(z_1, z_2, P_{1\perp}, P_{2\perp}) \) and a hard lepton-quark scattering cross section. The polarized PDF \( f_1^T \) is composed of contributions from the unpolarized \( f_1^T(x, k_T) \) and the Sivers term describing the correlation between the active quark’s transverse momentum, \( k_T \) and the transverse polarization of the nucleon, \( S_T \),

\[
 f_1^T(x, k_T) = f_1^S(x, k_T) + \frac{[S_T \times k_T]}{M} f_1^{uq}(x, k_T),
\]

where \( f_1^{uq}(x, k_T) \) denotes the Sivers PDF, \( M \) is the nucleon mass and the subscript 3 denotes the \( z \) component of the vector. Then, using Eq. (1), the 2h SIDIS cross section can be separated into the usual unpolarized part, \( \sigma_U \), and a spin-dependent part induced by the Sivers effect, \( \sigma_S \):

\[
 d\sigma^{h/z} \over dx \over dq^2 \over dz_1 \over dz_2 \over dp_{1T} \over dp_{2T} = C(x, Q^2) (\sigma_U + \sigma_S),
\]

\[
 \sigma_U = \sum_q e_q^2 \int \frac{d^2k_Td^2P_{1\perp}d^2P_{2\perp}}{P_{1T} - P_{1\perp} - z_1k_T} \times \delta^2(P_{2T} - P_{2\perp} - z_2k_T) f_1^S(x, k) D_{q}^{h/z}(z_1, z_2, P_{1\perp}, P_{2\perp}),
\]

\[
 \sigma_S = \sum_q e_q^2 \int \frac{d^2k_Td^2P_{1\perp}d^2P_{2\perp}}{P_{1T} - P_{1\perp} - z_1k_T} \times \delta^2(P_{2T} - P_{2\perp} - z_2k_T) \frac{[S_T \times k_T]}{M} f_1^{uq}(x, k) D_{q}^{h/z}(z_1, z_2, P_{1\perp}, P_{2\perp}),
\]

where \( C(x, Q^2) = \frac{a_s}{\alpha} (1 + (1 - y)^2) \) and \( \alpha \) is the fine-structure coupling constant.

It is easy to see using rotational and parity invariance, that the most general dependence of \( \sigma_S \) on the azimuthal angles \( \varphi_{1,2} \) (of the transverse momenta \( P_{1,2T} \)) and \( \varphi_S \) is given by two “Sivers-like” terms

\[
 \sigma_S = S_T \left( \sigma_1 \frac{P_{1T}}{M} \sin(\varphi_1 - \varphi_S) + \sigma_2 \frac{P_{2T}}{M} \sin(\varphi_2 - \varphi_S) \right),
\]

where \( \sigma_1, \sigma_2 \) and \( \sigma \) depend on \( x, Q^2, z_1, z_2, P_{1T}, P_{2T} \) and \( P_{1T} \cdot P_{2T} \) (or \( \cos(\varphi_1 - \varphi_2) \)).

The explicit expressions for structure functions \( \sigma_U, \sigma_1 \) and \( \sigma_2 \) are presented in Ref. [2] using Gaussian parameterizations for PDFs, similar to Ref. [7], and the following form for DiFFs:

\[
 D_{q}^{h/z}(z_1, z_2, P_{1\perp}, P_{2\perp}) = D_{q}^{h/z}(z_1, z_2, P_{1\perp}, P_{2\perp}) = \frac{1}{\pi^2 \gamma_{12}^2} e^{-\gamma_{12}^2/2} (1 + c P_{1\perp} \cdot P_{2\perp}),
\]

where the term \( c P_{1\perp} \cdot P_{2\perp} \) takes into account the experimentally established transverse momentum correlations (see, e.g. Ref. [14], where \( a^2 \) coefficient is used instead of \( c \)) in unpolarized 2h SIDIS.

### 2.2 Choice of the relative transverse momentum

It is often convenient to use linear combinations of \( P_{1T} \) and \( P_{2T} \) as independent transverse momentum variables. One vector is chosen as the transverse component of the pair’s total momentum,

\[
 P = P_{1T} + P_{2T},
\]

\[
 P_T = P_{1T} + P_{2T}.
\]

For the second vector, the following choices have been considered

1. In Refs. [1, 2, 9] – the transverse component of the relative momentum

\[
 R = \frac{1}{2} (P_{1T} - P_{2T}),
\]

\[
 R_T = \frac{1}{2} (P_{1T} - P_{2T}).
\]

2. In Refs. [10, 11] – the weighted difference

\[
 R_{TA} = \xi_2 P_{1T} - \xi_1 P_{2T},
\]

where

\[
 \xi_i = z_i/z, \quad z = z_1 + z_2.
\]

3. In Ref. [8, 9, 12] – the transverse to \( y^\ast \)-direction component of the transverse to pair total momentum, \( P \), component of \( R \):

\[
 R_{T,P} = R_{P} - (R_{P} \cdot \hat{q}) \hat{q},
\]

where

\[
 R_{P} = R - (R \cdot \hat{P}) \hat{P},
\]

and \( \hat{q} \) denotes the unit vector in the direction of vector \( v \).

The explicit calculations in the leading order approximation where we neglect mass terms and for small transverse momenta compared to large \( Q \)

\[
 P_1^T = E_1^T - m_1^2 \approx \xi_1^2 k^2,
\]

then

\[
 R_P \approx R - \frac{\xi_1^2 - \xi_2^2}{(z_1 + z_2)^2} P = \xi_2 P_1 - \xi_1 P_2,
\]

and

\[
 R_{T,P} \approx R_{T,A}.
\]

It is useful to define the total and relative transverse momenta also in terms of transverse momenta acquired in fragmentation:

\[
 P_{1\perp} \pm P_{1\perp} + P_{2\perp}, \quad R_{\perp} \pm \frac{1}{2} (P_{1\perp} - P_{2\perp}),
\]

\[
 R_{\perp A} \pm \xi_2 P_{1\perp} - \xi_1 P_{2\perp}.
\]
The remarkable feature of the $R_{TA}$ is that at the leading order approximations it is independent on the quark intrinsic transverse momentum, $k_T$, due to relations Eq. (1):

$$R_{TA} = \xi_2 (P_{\perp} + z_1 k_T) - \xi_1 (P_{\perp} + z_2 k_T) = R_{\perp A},$$

whereas

$$P_T = P_{\perp} + z k_T, \quad R_T = R_{\perp} + \frac{1}{2} (z_1 - z_2) k_T$$

depend on $k_T$.

2.3 Cross section in terms of relative and total hadron momenta

Let us first rewrite Eq. (3) in terms of $P_T$ and $R_{TA}$

$$\frac{d^2\sigma_{h2}}{d\phi_s d^2P_T d^2R_{TA}} = \int d^2k_T d^2P_{\perp} d^2R_{\perp A} \times \delta^2(\xi_1 (P_T - P_{\perp}) + R_{TA} - R_{\perp A} - z_1 k_T) \times \delta^2(\xi_2 (P_T - P_{\perp}) - R_{TA} + R_{\perp A} - z_2 k_T) \times \left[f_1^q(k) + \frac{[S_x \times k_T]}{M} f_{1T}^q(k)\right] D_{qA}^{h2}(P_{\perp}, R_{\perp A}),$$

where we defined

$$D_{qA}^{h2}(P_{\perp}, R_{\perp A}) \equiv D_{qA}^{h2}(\xi_1 P_{\perp} + R_{\perp A}, \xi_2 P_{\perp} - R_{\perp A}).$$

Note that, the product of $\delta$-functions in Eq.(20) can be written as $\delta^2(P_T - P_{\perp} - z k_T) \delta^2(R_{TA} - R_{\perp A})$. Then, after integrating over $R_{\perp A}$, we can write Eq.(20) as

$$\frac{d^2\sigma_{h2}}{d\phi_s d^2P_T d^2R_{TA}} = \int d^2k_T d^2P_{\perp} \delta^2(P_T - P_{\perp} - z k_T) \times \left[f_1^q(k) + \frac{[S_x \times k_T]}{M} f_{1T}^q(k)\right] D_{qA}^{h2}(P_{\perp}, R_{TA}).$$

The $P_T$-integrated cross section becomes

$$\frac{d\sigma_{h2}}{d\phi_s d^2R_{TA}} = \int d^2k_T d^2P_{\perp} \times \left[f_1^q(k) + \frac{[S_x \times k_T]}{M} f_{1T}^q(k)\right] D_{qA}^{h2}(P_{\perp}, R_{TA}) \int d^2P_{\perp} D_{qA}^{h2}(P_{\perp}, R_{TA}),$$

and does not depend on target polarization. This is what we have expected for the second choice of relative transverse momentum, since for SIDIS it is not sensitive to quark intrinsic transverse momentum. This feature of Sivers effect in the third choice was well known [12] and can be easily understood by considering Eq. (16).

In our studies [1, 2] of the Sivers effect in 2hSIDIS we have chosen the first definition of the relative transverse

\[1\]For brevity, in the following we only consider the variables related to transverse components of vectors and omit in r.h.s. $C(x, Q^2) \sum_{q} \epsilon_q$.
In this section we present numerical results obtained using a modified version (mLEPTO) \cite{15, 16} of LEPTO \cite{17} unpolarized event generator. The Sivers effect is introduced as a modulation of the struck quark momentum’s azimuthal angle according to the empirical parametrizations of Sivers function of Ref. \cite{7}, that were fitted to the experimental data \cite{4, 5}.

In mLEPTO simulations we generated $10^{11}$ DIS events in the kinematical region of the the COMPASS experiment \cite{5}, where $E_\mu = 160$ GeV. The following kinematic cuts were applied: $Q^2 > 1$ GeV$^2$, $0.1 < y < 0.9$, $0.03 < x < 0.7$, $W > 5$ GeV. The single hadron SSAs calculated in these simulations then can be directly compared with those measured in Ref. \cite{5}.

We present the results of our mLEPTO simulations with only a single \( \sin(\phi_S - \phi) \) modulation kept (see Eqs. (29,30)), where \( \phi = \phi_h \) for one hadron production, and \( \phi = \phi_R \) or \( \phi = \phi_T \) for two hadron production (see Eqs. (29,30)). Then the cross section can be written as

\[
\frac{d\sigma}{d\phi} \propto \sigma_U + S T \tilde{\sigma}_S \sin(\phi - \phi_S),
\]

(31)

where \( \sigma_U \) and \( \tilde{\sigma}_S \) are the corresponding unpolarized and Sivers terms, and the asymmetry is defined as

\[
A_{Siv} = \frac{\tilde{\sigma}_S}{\sigma_U}.
\]

(32)

In Fig. 3 we compare mLEPTO results for the SSAs in single hadron SIDIS production off transversely polarized proton target with the measurements by the COMPASS collaboration \cite{5}, with the additional kinematic cuts \( P_T > 0.1 \) GeV and \( z > 0.2 \) used in the experiment. This allows us to validate mLEPTO by demonstrating that it provides a good description of the data used in extracting the parametrizations of Sivers PDFs.

In mLEPTO we can determine the flavor of the struck quark in each SIDIS event. This proves to be a very useful information for understanding the underlying phenomenology of our results. In Fig. 4 the results for the relative rates for the struck quark flavor in events that produce a given type hadron satisfying the kinematic cuts imposed in our analysis are depicted. We see that the production of the positively charged hadrons, \( h^+ \), is dominated by the \( u \) quarks, while in production of the negatively charged hadrons, \( h^- \), both \( u \) and \( \bar{u} \) quark contribute almost equally,

3 Modified LEPTO (mLEPTO) results

In this section we present numerical results obtained using a modified version (mLEPTO) \cite{15, 16} of LEPTO \cite{17} unpolarized event generator. The Sivers effect is introduced as a modulation of the struck quark momentum’s azimuthal angle according to the empirical parametrizations of Sivers
We extract the SSAs corresponding to both $\sin(\phi)$ and $\sin(\phi_T)$ asymmetries integrated over $P_T$ and $R$, respectively. The bands labeled "Cut" are the results with the additional cut on the positively charged hadron’s momentum.

Figure 5. mLEPTO predictions for the dependence of the Sivers asymmetry on: (a) $x$, and (b) the total energy fraction $z$, in oppositely charged hadron pair production off proton target for both $\varphi_R$ and $\varphi_T$ asymmetries integrated over $P_T$ and $R$, respectively. The results for $h^+h^-$ pairs are larger than those for $h^+h^-$ pairs. This expectation is confirmed by analyzing the relative rates of the different hadron pair production and their dependence on the struck quark’s flavor from the plots in Fig. 6. Both for the pairs with no additional cuts depicted in Fig. 6(a) and for those with asymmetric cuts on the momenta of the hadrons in the pair in Fig. 6(b), the oppositely charged pair production channel is predominant. Nevertheless, $h^+h^-$ pairs, produced predominantly by the $u$ quark, have a rate comparable to that of $h^-h^-$ pairs. On the other hand, the rate for $h^+h^-$ pairs is significantly smaller compared to the others, which impact strongly on the statistical errors for the extracted SSAs. This, compounded by the small values of the extracted asymmetries compared to other pairs, led us to omit the results for $h^-h^-$ pairs from this article.

We can also generate non zero "R" type SSAs for same-charged pairs by ordering the hadrons in the pairs according to their energy fraction: the hadron $h_1$ would be the one with the larger value of $z$ (i.e. $z$-order the hadrons in a pair). The SSAs for the pairs with the same charge involve convolutions of the Sivers function with a different set of dihadron fragmentation functions than those used in SSAs of oppositely charged hadrons. These differences can be significant, analogous to the results of Ref. [18] for the NJL-jet calculations of dihadron fragmentation functions that depend on $z_h$ and $M_h$. This expectation is confirmed by analyzing the relative rates for the different hadron pair production and their dependence on the struck quark’s flavor from the plots in Fig. 6. Both for the pairs with no additional cuts depicted in Fig. 6(a) and for those with asymmetric cuts on the momenta of the hadrons in the pair in Fig. 6(b), the oppositely charged pair production channel is predominant. Nevertheless, $h^+h^-$ pairs, produced predominantly by the $u$ quark, have a rate comparable to that of $h^-h^-$ pairs. On the other hand, the rate for $h^+h^-$ pairs is significantly smaller compared to the others, which impact strongly on the statistical errors for the extracted SSAs. This, compounded by the small values of the extracted asymmetries compared to other pairs, led us to omit the results for $h^-h^-$ pairs from this article.

The results for $h^+h^-$ pairs are depicted in the Figs. 7, where we see that the SSAs for the $h^+h^-$ pairs are larger than those for $h^+h^-$ pairs. This will counteract the negative impact of the larger statistical errors stemming for the smaller rate of the $h^-h^-$ pairs compared to $h^+h^-$ in the experimental measurements.
Figure 7. mLEPTO predictions for the dependence of the Sivers asymmetry on: (a) $x$, and (b) the total energy fraction $z$, in positively charged hadron pair production off a proton target for both $\phi_R$ and $\phi_T$ asymmetries integrated over $P_T$ and $R$, respectively. The bands labeled “Cut” are the results with the additional cut on the first hadron’s momentum, as described in the text.

4 Conclusions

In our recent work [1, 2], we proposed a new approach for measuring the Sivers function in the 2h SIDIS process. We presented the results for the relevant cross section calculation that involve the Sivers PDF, with two non-vanishing SSAs with respect to sin($\phi_R - \phi_S$) and sin($\phi_T - \phi_S$), respectively. In contrast, the leading twist expression for the cross-section of the two-hadron SIDIS presented in Ref. [12] with different choice of the relative transverse momentum contains only a sin($\phi_T - \phi_S$) modulation term. The sin($\phi R_P - \phi_S$) modulation term is absent there, as well as in the subleading twist expression of Ref. [13] when integrated over the total transverse momentum of the pair.

Here we have presented the detailed explanation of origin of the ‘unexpected’ nonzero sin($\phi R_P - \phi_S$) modulation in 2h SIDIS. We showed that, using the leading order approximations the sin($\phi R_P - \phi_S$) type modulations with the choice of the relative transverse momenta of Refs. [12, 13] should indeed vanish, while they should be in general nonzero with our definition of $\phi_R$. The actual measurement of sine modulations in terms of above two different choices of the relative transverse momentum can serve as a good test of the applicability of factorization formalism and various leading order approximation in 2h SIDIS at the energy scale of that experiment.

We conclude that the magnitude of Sivers SSAs in the dihadron SIDIS process should be comparable to those for the single hadron SIDIS. Thus the experimental measurements of the Sivers SSAs for various hadron pairs using the data collected at COMPASS and the future SIDIS experiments, such as those planned at JLAB12 and EIC, are feasible and would contribute a large amount of new information for extracting the Sivers PDFs.

References

[8] A. Airapetian et al. (HERMES Collaboration), JHEP 0806, 017 (2008), 0803.2367