

# Moving Forward in the Era of Quantitative QCD

## Transversity 2014 Closing Remarks

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**Abstract.** After the development of QCD in the last quarter of the 20th century, we are now in the early years of an exciting new era in which much more quantitative QCD calculations can be tested against increasingly sophisticated experimental measurements. While there is more than one way to keep pushing forward our understanding, the proton, as a fundamental bound state of QCD, can serve as an excellent laboratory in which to probe the complexities of the strong force as we learn more about the very matter of which we ourselves are made.

## 1 Introduction

The arc of many fields of scientific endeavor can be delineated into an initial period of discovery and development, followed by a period of quantitative basic research, finally arriving at an era in which applications are developed. One can view the 2004 Nobel Prize awarded to Gross, Politzer, and Wilczek for their 1973 discovery of asymptotic freedom in the theory of the strong interaction as symbolically marking the closure of the initial discovery and development phase of quantum chromodynamics (QCD). Research in QCD can now be considered to be in the early years of the second phase: quantitative basic research.

### 1.1 Theoretical advances

With all experimental work involving color-neutral hadrons, the primary challenge in QCD has been to relate observations in the laboratory to calculations in terms of colored quarks and gluons in order to interpret measurements and derive a deeper understanding of hadronic and nuclear matter and its interactions. There have been several broad approaches since the very early years of QCD, including low-energy modeling, effective field theories, perturbative QCD (pQCD), and lattice QCD. However, in just the last 15–20 years, great advances have been made within these approaches.

While a complete discussion of the history, development, and status of the various topics touched upon below is well beyond the scope of these proceedings, I will sometimes select a few illustrative examples. Let me also state here explicitly that these proceedings are based on the closing talk I gave for the Transversity 2014 workshop,

which was meant to give a very broad overview of the topics discussed, as well as an outlook for the future. The perspectives presented here are my own personal ones, and the (im)balance of material will surely reflect some of my own interests and expertise.

In perturbative QCD (pQCD), since the 1990s we have been starting to consider detailed internal quark and gluon dynamics, parting from traditional parton-model ways of looking at hadrons and hadronization. And—crucially—we have been learning how to perform *phenomenological calculations* using these new ideas and tools, allowing us to compare to and make predictions for a wider and wider range of experimental measurements. Some examples of theoretical techniques and ideas that have emerged in pQCD since the mid-1990s include:

- various resummation techniques;
- non-linear evolution at small momentum fractions;
- non-collinearity of partons with parent hadron;
- spatial distributions of partons in hadrons.

*Resummation.* Since the 1990s various resummation techniques in QCD have come into use. Resummation techniques can generally be applied when there are two scales in a problem, and logarithms of their ratios are summed to all orders in  $\alpha_s$ . Threshold resummation [1–3] is one such example. Next-to-leading log threshold resummation calculations for hadron production in hadronic collisions at fixed-target energies (tens of GeV center-of-mass energy) have transformed earlier, qualitative descriptions of experimental measurements to quantitative ones, as illustrated for example in [4]. Previously, NLO calculations without resummation were able to describe the slope of the data correctly, but with a magnitude  $\sim 2$ – $10$  times lower than the experimental data. The resummed calculations are instead quantitatively consistent with the measurement

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within the theoretical uncertainty due to the choice of scale, and this scale uncertainty is in turn reduced compared to that of a pure NLO calculation. As a further illustration of how the theoretical community has been honing its methods to produce ever better phenomenological calculations for comparison to experimental data, a 2009 paper uses threshold resummation to describe dihadron production in hadronic collisions at fixed-target energies [5]. It goes beyond earlier resummed work to calculate cross sections involving cuts on individual hadron  $p_T$  and the rapidity of the pair, matching in an unprecedented way the cuts implemented in the experiment.

*Non-linear evolution at small momentum fraction  $x$ .* While many of the theoretical ideas related to non-linear evolution at small  $x$  were developed in 1997–2000 [6–8], it was only in 2006–07 that a first-principle calculation of the running coupling corrections to the evolution kernel was performed [9–11]. These corrections opened up the way for much more quantitative phenomenological calculations for comparison to a variety of experimental data at low  $x$ , again highlighting the significant effort it typically takes to go from theoretical ideas in QCD to robust phenomenology. The novelty of such calculations just a few years ago is captured well by the bold title of a 2009 *Physical Review D* article, "Non-linear QCD meets data: A global analysis of lepton-proton scattering with running-coupling BK evolution" [12].

*Dropping the simplifying assumption of collinearity.* For decades in pQCD, calculations to describe inclusive cross sections have been performed assuming that the partons within a hadron move collinearly with the parent hadron (integrated over parton transverse momentum  $k_T$ ), using collinear parton distribution functions (pdfs). The 1990s saw the development of transverse-momentum-dependent (TMD) pdfs as well as fragmentation functions (FFs), many of which describe explicit spin-momentum correlations. Early work was done in the first half of the decade [13, 14], and in 1995, all the various possible tree-level functions contributing to polarized deep-inelastic scattering were laid out, including transverse momentum dependence [15]. Since then, and in particular after a theoretical breakthrough related to naive T-odd TMD pdfs in 2002 discussed below, the QCD subfield of spin-momentum correlations and TMD functions has exploded with activity. Theory and experiment have largely been advancing hand-in-hand, with experimental measurements in proton-proton collisions, semi-inclusive deep-inelastic scattering, and electron-positron annihilation all revealing a variety of large spin-momentum correlations in nature. Very recent advances include a more rigorous formal definition of TMD functions in 2011 [16], and ongoing work to perform phenomenology including the evolution of TMD functions, discussed below.

*Spatial distributions of partons in hadrons.* The vast majority of nucleon structure studies since the first deep-inelastic scattering experiments in the late 1960s have focused on the one-dimensional structure of the nucleon in terms of the longitudinal parton momentum fraction  $x$ . While the 1990s saw the development of parton distribu-

tions that explicitly contained transverse momentum dependence, it also saw proposals for experimental investigation of the spatial distributions of partons within the nucleon via generalized parton distributions (GPDs) [17–21]. Thus we are now squarely in an era where multi-dimensional imaging of the nucleon is possible. Adding further interest to the study of GPDs is their relation to the total quark orbital angular momentum to the spin of the nucleon [20]. The study of spatial distributions of partons within the nucleon is still in its early stages, but a number of experimental measurements of relevant exclusive reactions have now been successfully made.

Lattice QCD has additionally made great advances just in the last few years. The development of more sophisticated algorithms as well as higher-speed computers has improved the accuracy of many predictions, and in 2010 the first-ever calculations were performed at the physical pion mass for certain observables [22, 23], eliminating the need for any extrapolation. The first-ever exploratory TMD pdf calculations on the lattice were performed in 2011 and 2012 [24, 25]. In 2013, a paper by Ji [26] laid out a method for calculating the  $x$  dependence of parton distribution functions (pdfs) on the lattice, rather than simply moments of pdfs, which is what could be calculated previously. It also permitted the first direct calculation of pdfs for sea quarks, another exciting development. While the lattice community is only at the beginning of calculating  $x$ -dependent pdfs, this development represents a tremendous step forward for the study of nucleon structure on the lattice. Previously, with lattice calculations only able to access the full moments of pdfs, with  $x$  integrated from 0 to 1, they could never be compared directly to experimental measurements, which always have finite  $x$  coverage. It will be exciting to see how these calculations advance over the next several years.

Also since the 1990s, there has been the development of new effective field theories as useful approximations within certain regimes of QCD. QCD exhibits different behavior at different scales, and effective field theories can often be useful approximations within these different regimes. While there have been numerous effective field theories in use for quite a long time such as Non-Relativistic QCD or ones involving chiral symmetry breaking, there are several that have been developed during the 1990s or after the year 2000. Heavy Quark Effective Theory [27] provides an approach to QCD problems involving a heavy quark, the Color Glass Condensate provides a means of performing calculations at high energies and high densities [28], and Soft-Collinear Effective Theory [29, 30] has offered new insights into performing complicated perturbative calculations very quickly.

In terms of theoretical developments within the last two decades, it is additionally worth mentioning the anti-de Sitter/conformal field theory (AdS/CFT) correspondence, first proposed in 1997 [31], and its relationship to QCD. It is certainly not providing quantitative calculations within QCD right now, and QCD is not even a conformal field theory. However, it offers a powerful set of tools for the study of strongly coupled (conformal) quantum field

theories, and if nothing else, it is exciting as the first completely new handle on trying to tackle QCD in decades. It will be interesting to see how much and what exactly can be learned from this approach in upcoming years.

## 1.2 Experimental advances, in broad strokes

While a rich variety of novel experimental measurements in QCD has been performed over the last 10–20 years, and I will not attempt to list recent measurements here, aspects of experimental advances can be placed into several broad categories: more sophisticated experimental observables, multidifferential measurements, and new regimes opened up periodically as new facilities come online.

Experimentalists are now studying increasingly sophisticated observables such as angular distributions, spin dependence, and multiparticle final states, many of which are sensitive in particular to parton *dynamics*. This is in contrast to a greater focus in earlier decades on single-inclusive measurements in processes involving hadron scattering and/or hadron production.

The high luminosities of modern facilities often provide the power to perform not only inclusive measurements with very small uncertainties, but also multidifferential measurements with reasonably small uncertainties, for example simultaneously in  $x$ ,  $Q^2$ ,  $p_T$ , and  $z$ , depending on the process. A single experimental result may now comprise hundreds to thousands of new data points.

New facilities often open up the investigation of new regimes. When HERA turned on in the early 1990s, the high-energy lepton-proton collisions it provided enormously expanded knowledge of proton structure over a huge range of  $x$  and  $Q^2$ . With the advent of the Relativistic Heavy Ion Collider (RHIC) in the early 2000s came the first polarized proton-proton collisions up to hundreds of GeV in center-of-mass energy. RHIC furthermore offers ultra-relativistic heavy ion collisions enabling the study of hot, dense QCD systems, along with systematic comparisons to heavy ion collisions with varying geometry, heavy ion collisions in collider kinematics down to energies low enough to search for the QCD critical point, and cold nuclear matter studied via a hadronic probe. The Large Hadron Collider (LHC), which turned on just a few years ago, is at the energy frontier of (unpolarized) proton-proton collisions, and provides ultra-relativistic heavy ion collisions at even higher energies leading to higher temperatures, as well as proton-nucleus collisions.

All of these experimental advances in turn demand more of theory: the ability to describe more complex observables, description as a function of many kinematic variables simultaneously, and calculations appropriate for new regimes. It is worth emphasizing once again that knowing the theory does not mean understanding how to do the phenomenology to make valid comparisons to these experimental measurements. Similar to the case above where running coupling BK evolution was a necessary ingredient to advance phenomenology within the regime of non-linear QCD at low  $x$ , pinning down the evolution of TMD pdfs and fragmentation functions is currently an ex-

remely active area of study in order to move toward rigorous global fits of TMD functions. There were in fact nine presentations on the subject just at this workshop, compared to only one presentation at the last Transversity workshop three years ago. The issue in this particular example is that for TMD functions, the evolution equations themselves depend on a nonperturbative component, unlike the collinear case where DGLAP evolution is completely perturbatively defined. While many different voices suddenly weighing in on an issue may make it feel like the community has descended into confusion where there did not use to be any, it is in fact an indication of scientific progress! We have now advanced to the point of worrying about things we did not even consider several years ago.

## 2 Fundamental issues coming to the fore: Gauge links have physical consequences

Some of the advances of the last 15 years or so have brought to the fore deep, fundamental issues within QCD. While it was originally claimed by Collins in the early 1990s that the Sivers TMD pdf had to vanish due to time-reversal invariance [14], in 2002 Brodsky, Hwang, and Schmidt pointed out to the community that the Sivers function, in fact naive T-odd, could be non-vanishing in semi-inclusive deep-inelastic scattering (SIDIS) because of final-state phase interference effects due to gluon exchange between the scattered quark and the remnant of the nucleon from which it was removed [32]. Shortly thereafter, Collins identified observable consequences of these interference effects, predicting that the Sivers TMD pdf should have the same magnitude but opposite sign when measured in SIDIS versus Drell-Yan [33], leading to *modified universality* of the Sivers TMD pdf, as well as the Boer-Mulders TMD pdf, also naive T-odd. In Drell-Yan, gluon exchange is instead possible in the initial state, between the scattering quark or antiquark and the remnant of the *other* nucleon (or pion).

The predicted modified universality of naive T-odd TMD pdfs is a direct physical consequence of gauge links. The original 1992 prediction by Collins that the Sivers function vanish in fact stated explicitly, "We have ignored here the subtleties needed to make this a gauge-invariant definition" [14]. It has been pointed out by Pijlman [34] and Sivers [35] that these quantum mechanical, universality-modifying effects are, in fact, QCD analogs of similar famous quantum mechanical effects that can arise in certain situations in quantum electrodynamics (QED), such as in the Aharonov-Bohm phenomenon [36]. Namely, color charges can experience effects *even in regions of space where the chromo-electric and chromo-magnetic fields are zero*. The Aharonov-Bohm effect in QED arises from the recasting of Maxwell's classical electromagnetic theory as a quantum gauge theory. While QED was the only quantum gauge theory known to describe nature when Aharonov and Bohm made their original prediction in 1959, it was stressed that the prediction

should hold universally for any gauge-invariant quantum field theory [37]. So the QCD community has in effect "rediscovered" the intriguing physical properties of quantum mechanics when applied to gauge field theories.

It will be tremendously exciting to verify the universality of Aharonov and Bohm's general observations regarding gauges theories for the one other quantum gauge theory we have within the Standard Model at this point. I feel it would serve our community well, rather than talking about trying to measure the "sign change of the Sivers function," to instead emphasize the extremely fundamental nature of this measurement.

However, in QCD as a *non-Abelian* quantum gauge theory, the physical consequences of gauge invariance lead to even more spectacular effects, not present in QED. SIDIS and Drell-Yan are both QED rather than QCD processes at tree level. While the phase effects described are QCD effects, due to gluon exchange between a scattering quark and a hadron remnant, they are not sensitive to the fact that QCD is non-Abelian. To provide sensitivity to the non-Abelian nature of QCD, one needs to study hadron production in hadronic collisions, where gluon exchange can take place in both the initial and final state. Based on this, in 2010 Rogers and Mulders predicted the novel phenomenon of *color entanglement* of partons *across*(!) hadrons in processes involving hadroproduction of hadrons in which partonic transverse momentum is explicitly taken into account [38]. Thus in the case for example of proton-proton scattering to hadrons, the partons in the two initial-state protons become correlated with one another and can no longer be treated as independent pdfs. Rogers has furthermore predicted that this will lead to extra spin-dependent asymmetries in these processes [39]. Exploring this direction unique to QCD as a non-Abelian gauge-invariant quantum field theory will open up a completely new area of research.

### 3 Future directions

I was asked to give an outlook in this presentation, so I chose four areas to highlight as ones that I believe will be of increasing focus in upcoming years:

- sea quarks;
- multiparton correlations;
- partonic structure of nuclei;
- hadronization.

#### 3.1 Sea quarks

We now have increasing evidence, as well as a wide variety of further hints, that the nucleon sea is much more complex and rich than we often assume. Measurements of  $\frac{d}{u}$  up to  $\sim 1.6$  in the proton by CERN NA51 [40] and Fermilab E866 [41] indicate non-perturbative origins of light antiquarks. Measurements of sea quark helicity distributions via  $W$  production at RHIC suggest that the helicity distributions are not the same for antidown and antiup quarks [42]. A harder transverse momentum spectrum for

Drell-Yan pairs generated by proton-tungsten collisions than by antiproton-tungsten collisions hints at a larger mean partonic transverse momentum for sea than valence quarks [43]. A non-zero  $\cos 2\phi$  modulation has been measured by E866 for Drell-Yan pairs produced in proton-deuterium and proton-proton collisions, implying a non-vanishing Boer-Mulders TMD pdf for sea quarks [44, 45]. Transverse single-spin asymmetries in  $p + p$  collisions measured by BRAHMS for positive and negative kaons have both the same sign and the same magnitude despite the  $K^-$  ( $\bar{u}s$ ) being comprised of only sea quarks with respect to the proton, suggesting non-zero spin-momentum correlations for sea quarks in the proton [46]. Similarly,  $\cos 2\phi$  modulations in kaon production in unpolarized SIDIS measurements from HERMES show approximately the same magnitudes for positive and negative kaons [47]. There are also hints from Fermilab E704 [48], STAR [49], and PHENIX [50] that the transverse single-spin asymmetry for  $\eta$  mesons is larger in magnitude than that of neutral pions, suggesting that strange quarks may play a role. Very recent lattice QCD calculations furthermore suggest a non-zero transversity distribution for sea quarks [51]. In my opinion, within the community we currently tend to treat sea quarks inconsistently, at times neglecting them and at other times doing dedicated, stand-alone studies of sea quarks; I expect we will move toward more comprehensive and consistent treatments in upcoming years. It will additionally be interesting to consider in the future the sea within other hadrons such as pions. Further measurements sensitive in particular to the *dynamics* of sea quarks should provide an especially sharp probe of the various mechanisms by which they are generated, leading us to a better understanding of what the sea in fact is.

#### 3.2 Multiparton correlations

Multiparton correlations in hadrons and in the process of hadronization can describe single-spin asymmetries without relying explicitly on partonic transverse momentum. They effectively extend our ideas about (single-parton) pdfs to correlation functions that cannot be associated with a single parton; as twist-3 collinear nonperturbative functions, we currently know a great deal less about them than we do about traditional twist-2 collinear pdfs and FFs. They can describe nonperturbative structure in QCD as matrix elements involving the quantum mechanical interference between scattering off of a (quark+gluon) and scattering off of a single quark (of the same flavor and effectively at the same  $x$ ). Similarly, one can also have interference between scattering off of a (gluon+gluon) and a single gluon. On the hadronization side, multiparton correlations describe the interference between a (quark+gluon) hadronizing and only a quark, or between a (gluon+gluon) hadronizing and only a single gluon. While early work on multiparton correlations was performed by Efremov and Teryaev in the first half of the 1980s [52, 53], multiparton correlations are arguably another theoretical development within QCD that was worked out primarily over the 1990s [54–56].

### 3.3 Partonic structure of nuclei

It has been known since the 1980s that the partonic structure of nuclei is not a simple superposition of the partonic structure of the constituent protons and neutrons, with several interesting effects evident from data. For example in nuclear DIS data taken at SLAC, four distinct regions are visible in the ratio of the  $F_2$  structure function for a variety of nuclei with respect to deuterium as a function of  $x$  [57]. The ratio is below 1 for  $x \lesssim 0.1$ , rises above 1 until  $\sim 0.3$ , dips back below 1 until  $\sim 0.8$ , and then rises above one for even larger  $x$ . Several of these effects remain poorly understood, but there has recently been a renaissance in trying to understand the partonic structure of nuclei, driven in part by the need to understand initial-state cold nuclear matter in order to interpret ultra-relativistic heavy ion collisions at RHIC and the LHC. The focus so far has largely been on the one-dimensional momentum structure of nuclei, but as this subfield matures and collects more experimental data, we will be ready to investigate modifications of the transverse momentum as well as spatial distributions in nuclei as well. Finally understanding how to go from the quarks and gluons of our fundamental field theory to the nuclei comprising everyday matter would be a great triumph.

### 3.4 Hadronization

A great deal more focus has been placed over the last 40 years of QCD on the structure of hadrons in terms of their colored constituents than on the process of a colored quark or gluon hadronizing to a color-neutral bound state. Our current knowledge and treatment of hadronization are correspondingly less developed. We have mainly tried to understand hadronization through "vacuum" fragmentation functions, parameterized from data in terms of the one-dimensional longitudinal momentum fraction for a single produced hadron type. In terms of recent advances, 2007 was the first year that FF fits included world data from  $e^+e^-$ , SIDIS, and  $p + p$  collisions, thanks to de Florian, Sassot, and Stratmann [58]. Discussions of TMD FFs and spin-momentum correlations in hadronization began in the 1990s, as discussed above, although similar to the case for collinear FFs compared to collinear pdfs, TMD FFs are not nearly as well studied as TMD pdfs have been thus far. Ideas regarding a dihadron interference FF [59, 60], where the fragmentation of a single parton into two observed hadrons is studied, have also been developed in the last two decades. Much of the interest in the dihadron interference FF, perhaps ironically, has in fact been because it can be used as a tool to probe nucleon structure. As a chiral-odd FF, it offers the possibility of measuring chiral-odd distribution functions such as transversity or Boer-Mulders.

Despite recent steps going beyond one-dimensional FFs in which a single parton hadronizes into a single observed hadron, even the state-of-the-art traditional FFs do not reproduce ratios of abundant particle species very well. For example, recent data from the PHENIX experiment [61] on the negative-to-positive pion ratio in  $p + p$  collisions fall  $\sim 20\%$  below next-to-leading-order pQCD calculations using DSS [58, 62] or AKK [63] FFs for

pions. Perturbative QCD calculations using the AESSS FFs for eta mesons [64] similarly overpredict the eta-to-neutral-pion ratio measured by PHENIX [65]. One way to improve the traditional collinear FF parameterizations would be to fit ratios whenever available from experiments. In the ratio, systematic uncertainties are significantly reduced on both the measurement and the theoretical calculation, so fitting ratios should provide greatly improved constraints. In the long term, one would eventually want to extend fits from ratios of two particle species to reconstructing full jets with identified particles and creating parameterizations of hadronization based on these, but this will require a significant body of new experimental measurements.

In the upcoming years in hadronization studies, we should also begin considering in more detail hadronization in a nuclear environment. Baryon enhancement with respect to meson production has been explained in central heavy ion collisions with "recombination" models starting from a thermalized quark-gluon plasma. However, baryon enhancement is also observed in peripheral heavy ion collisions as well as in deuteron-gold collisions [66], implying an additional mechanism or mechanisms for hadronization in cold nuclear matter as well.

## 4 The cyclical process of proliferation and then synthesis of ideas and observations

There has been a proliferation of new ideas and observations over last 15–20 years in QCD, and I believe we are now in a rewarding period of synthesis of many of these ideas. This is certainly not the first such cycle in QCD and hadronic physics, with the many new observed particles in the 1950s and '60s leading up to the discovery of QCD itself. As we go through the process of synthesizing our various ideas and observations, we uncover increasing connections among what have historically been disparate subfields within QCD.

## 5 Final remarks

I believe that we are now in the early years of quantitative basic research in QCD, and that an Electron-Ion Collider is the facility to bring the era of quantitative QCD to maturity. However, we must keep in mind that measurements in complementary collision systems are extremely important, for there are things that can only be learned by comparison of different systems involving color in the initial state, final state, or both. After launching into the current era through an expansion of different ideas and observations along different directions, we are now taking steps to interweave these various new ideas and identify links among them. We are thus taking small, initial steps toward the ultimate dream of "grand unification" of QCD across different scales, from partons to neutron stars.

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