A unified homographic law for fusion excitation functions above the barrier

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Abstract. We report on a systematics of fusion cross section data at energies above the reaction threshold to those of disappearance of fusion process. By an appropriate scaling of both cross sections and energy, a fusion excitation function common to all the data points is established. A universal description of the fusion excitation function relying on basic nuclear concepts is proposed and its dependence on the reaction cross section used for the cross section normalization is discussed.

1 Introduction

The evolution of the fusion reaction mechanism above the barrier to the Fermi-energy region is closely related to the existing competition between (nuclear) mean field and two-body dissipation. Thus, the study of both the complete (CF) and incomplete (IF) fusion excitation functions may be a proper way to understand and to constrain the ingredients entering in theoretical models, such as Sky3D [1], TDHF3D [2] or DYWAN [3], used to describe heavy-ion reactions at that energy range. Examples of burning issues are the choice of effective interactions [4] and the modeling of nucleon-nucleon collisions.

In two recent papers [5, 6] we have published a systematic study of both CF and IF fusion cross sections $\sigma_{\text{fus}}$ for incident energies $E_{\text{lab/nucleon}} = E_{\text{in}}$ higher than about 3–4A MeV. From the works published during the past 40 years we have collected 382 CF and CF+IF $\sigma_{\text{fus}}$ data points belonging to 81 reaction systems with a vast variety of projectile–target pairs. By properly reducing $\sigma_{\text{fus}}$ values with the (total) reaction cross section $\sigma_{\text{reac}}$ and by applying an original energy scaling we have demonstrated that, irrespectively of the details of a given reaction system, the fusion excitation function follows a simple universal functional law. On that way we have been able to identify those data points for which, the most likely, the reported $\sigma_{\text{fus}}$ values suffer from a non-fusion contribution [5, 6] and those for which other reaction mechanism has been erroneously identified as fusion [7]. Discarding these data one ends with 76 systems and 316 CF+IF $\sigma_{\text{fus}}$ data points plus 57 CF data points. Besides on experimental uncertainties of $\sigma_{\text{fus}}$, the exact parameter values of the universal functional law are somewhat dependent on the values of $\sigma_{\text{reac}}$ used for the $\sigma_{\text{fus}}$ normalization. In this contribution we focus on the data normalization problem and the impact of the implemented $\sigma_{\text{reac}}$.

2 Scaling of fusion cross section

Figure 1 displays the 316 CF+IF kept $\sigma_{\text{fus}}$ data points as a function of $E_{\text{in}}$. Clearly, most of these non-scaled raw $\sigma_{\text{fus}}$ data points gather in a narrow domain of the $\sigma_{\text{fus}}$ vs $E_{\text{in}}$ plane although lighter systems (blue, cyan, and green symbols) gather along an arclike structure decreasing with $E_{\text{in}}$ while heavier ones (pink, red, and orange symbols) follow a line which sharply rises with $E_{\text{in}}$.

The fact that systems of low and high mass do not fall together is natural due to the known dependence of $\sigma_{\text{fus}}$ on the system size. In order to compare so many different systems it is not enough to scale only the cross sections. We have shown [5, 6] that energy has to be scaled too. In order to express on the same footing the mass asymmetric systems with those which are mass symmetric the system energy should be expressed in units of the so-called system available energy that is nothing but the center-of-mass energy per nucleon

$$E_{\text{avail}} = \frac{E_{\text{cm}}}{A_{\text{tot}}} = \frac{E_{\text{lab}}}{A_{p}A_{t}} \left(\frac{A_{p}A_{t}}{A_{p} + A_{t}}\right)^{2},$$

where $A_{\text{tot}} = A_{t} + A_{p}$ is the system mass and $A_{p}$ and $A_{t}$ are projectile and target mass, respectively.

One accounts for the proportionality of cross section to the size of a reaction system by the renormalizing $\sigma_{\text{fus}}$ by (total) reaction cross section $\sigma_{\text{reac}}$ at the same $E_{\text{in}}$. Accurate measurement of the total $\sigma_{\text{reac}}$ is rather hard so that these data are scarce and it would be inappropriate to apply them to the ensemble of our $\sigma_{\text{fus}}$ data. Therefore, one commonly resorts to phenomenological approaches to calculate $\sigma_{\text{reac}}$, a solution which suffers for its own uncertainties and ambiguities. Thus, the uncertainty arising from the use of a particular parameterization of $\sigma_{\text{reac}}$ has to be investigated. Among a number of phenomenological parameterizations we have investigated five of them [8–12] including the pioneer one due to Bass [8] and the most recent one by Tripathi [12]. In order to infer reliability of
each of the models in figure 2 we compare them with 134 experimental $\sigma_{\text{reac}}$ measured for 46 systems. All these approaches rely on the strong absorption picture of nuclear processes and differ among themselves in the way the basic relation

$$\sigma(E) = \pi R^2 (1 - \frac{V}{E})$$

is parameterized. In Eq. (2) the cross section depends on the inverse of the (center-of-mass) energy $E$ while the radius $R$ and the potential depth $V$ may be, in a first approximation, considered constant for a given system. Various models differ in the treatment of $R$ and $V$ by introducing or not a certain dependence on energy and/or on some other system property, usually $A_{\text{tot}}$ or $A_{\text{sys}}$, to the one or both of them.

From figure 2 one may conclude that the Bass approach overpredicts the measured cross sections at all $E_{\text{avail}}$, that the one of Kox underpredicts them at low energies ($E_{\text{avail}} \ll 2 \text{ MeV/nucleon}$) while that of Gupta at higher energies ($E_{\text{avail}} \gtrsim 2 \text{ MeV/nucleon}$). These conclusions are plausible from the histograms obtained by projecting those ratios on the ordinate of each of panels in figure 2.

Figures 3 and 4 show the $\sigma_{\text{fs}}$ data of figure 1 scaled in both cross section and energy. In figure 3 shown are normalizations with $\sigma_{\text{reac}}$ of Bass, Gupta, and Kox, respectively and in figure 4 those with $\sigma_{\text{reac}}$ of Shen and that of Tripathi as well as the one labeled Mixed obtained by a heuristic approach. By investigating the projection histogram on the ordinate axis of the ratios between experimental and model reaction cross sections we heuristically define the most appropriate combination of model predictions. So, the Mixed $\sigma_{\text{reac}}$ is taken as that of Tripathi [12] for $E_{\text{avail}} < 2 \text{ MeV/nucleon}$ and $A_{\text{tot}} < 86$, as that of Gupta [9] for $E_{\text{avail}} < 2 \text{ MeV/nucleon}$ and $A_{\text{tot}} \geq 86$, while for $E_{\text{avail}} \geq 2 \text{ MeV/nucleon}$ mixed $\sigma_{\text{reac}}$ is the average of the Shen [11] and Tripathi [12] predictions.

It is interesting to note that out of the 316 data points in the case of Gupta and that of Kox 2 and 41 points, respectively violate the physically allowed range for the normalized cross section values $\sigma_{\text{fs}}/\sigma_{\text{reac}} = \sigma_{\text{red}}$, namely between 0 and 1. All these points are due to $\sigma_{\text{red}}$ overflow that occurs at low energies.

Assuming the applicability of the strong absorption concept in the first approximation both $\sigma_{\text{fs}}$ and $\sigma_{\text{reac}}$ may be expressed by the same functional form given by Eq. (2). Consequently, for the reduced cross section one gets a simple homographic law

$$\sigma_{\text{red}}(E) = \frac{\sigma_{\text{fs}}(E)}{\sigma_{\text{reac}}(E)} = a + \frac{b}{c + E},$$

where $a$, $b$, and $c$ are free parameters and $E$ is taken as $E_{\text{avail}}$ [5, 6]. The $\sigma_{\text{red}}$ data points of each panel of fig-
The best fit curves overlap. The fit results of di-
malization cases disagree by at most 3 to 4% of
the red plotted as a function of \( E_{\text{avail}} \) that may be
seen from the dashed black curve that is drawn in
the best fit result gives for the energy of the
disappearance of the CF+1F fusion process the same value:
\( E_{\text{avail}} = 12.98 \pm 0.06 \text{ MeV/nucleon} \). Also, at low energy all
the best fit curves overlap.

The obtained best fit result is
displayed by the red full curve for each \( \sigma_{\text{red}} \). Interestingly
enough, independently of the \( \sigma_{\text{red}} \) chosen for the
normalization procedure, the best fit curve gives for the energy of
disappearance of the CF+1F fusion process the same value:
\( E_{\text{avail}} = 12.98 \pm 0.06 \text{ MeV/nucleon} \). Also, at low energy all
the best fit curves overlap. The fit results of different nor-
malization cases disagree by at most 3 to 4% of \( \sigma_{\text{red}} \) and
this occurs at intermediate values of \( E_{\text{avail}} \). That may be
easily inferred by the dashed black curve that is drawn in
each panel of figures 3 and 4 and which represents the best
fit result in the case of the heuristic (Mixed) \( \sigma_{\text{reac}} \). Al-
though the best fit result does not depend strongly on the
details of the \( \sigma_{\text{reac}} \) parameterization this subject is open
for further investigation. However, the homographic func-
tional law (3) seems to be established without any doubt.

3 Complete fusion cross section

Only twelve experiments have explicitly been designed to
measure both complete and incomplete fusion components

\[ \begin{align*}
\text{Normalized fusion cross sections} & \quad \sigma_{\text{red}} \text{ plotted as a function of } E_{\text{avail}}. \quad \\
\text{The } \sigma_{\text{reac}} \text{ used for normalization is due to } & \quad \text{a) Bass [8], b) Gupta [9], and } \\
\text{c) Kox [10], respectively. The full red curve in each panel is due to a fit with the } & \quad \text{homographic function (3), whereas the } \\
\text{dashed black curve is due to the same kind of fit of panel c) in figure 4.}
\end{align*} \]

Figure 3.

\[ \begin{align*}
\text{Normalized fusion cross sections} & \quad \sigma_{\text{red}} \text{ plotted as a function of } E_{\text{avail}}. \text{ The } \sigma_{\text{reac}} \text{ used for normalization is due to a) Shen} \\
\text{[11], b) Tripathi [12], and c) the Mixed one (see text for details), respectively. The full red curve in each panel is due to a fit with the } \\
\text{homographic function (3), whereas the dashed black curve is due to the fit of panel c).}
\end{align*} \]

Figure 4.
4 Discussion and conclusions

In the main panel of figure 5 is repeated panel c) of figure 4, namely $\sigma_{\text{red}}^{\text{CF}}$ for the CF+IF data points obtained with the heuristic $\sigma_{\text{rec}}^{\text{CF+IF}}$ displayed as a function of $E_{\text{avail}}$. The orange background drawn around the full red best fit curve is due to the uncertainties of the fit parameters. Much smaller errors of the fit parameters are in accordance with the already discussed stability of the CF+IF $\sigma_{\text{red}}^{\text{CF}}$ data. In the main panel is repeated the full violet best fit curve to the CF data. Making difference of both allows to infer the main properties of the incomplete fusion excitation function: IF process opens around $E_{\text{avail}} \approx 1.5$ MeV/nucleon, reaches maximal value at the energy of CF disappearance ($E_{\text{avail}} \approx 6$ MeV/nucleon), and vanishes at $E_{\text{avail}} \approx 13$ MeV/nucleon. An inset in the main inset displays how the best-fit CF excitation function decreases relatively to the CF+IF one, an observable which has been investigated a long time ago by Morgenstern et al. [14].

To summarize, the scrutiny of the existing fusion cross sections well above the reaction barrier allowed us to establish a universal dependence of these data on energy. The established homographic functional description of both the complete and the complete plus incomplete fusion excitation functions is rather stable and the inferred global features of these excitation functions quite weakly depend on the details of the data normalization. Nevertheless, the normalization may be improved if additional high quality measurements on both fusion and reaction cross section would be available.

References