Effect of breakup coupling on fusion for $^{6,7}$Li+$^{24}$Mg systems

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Abstract. To study the effect of breakup coupling on fusion we have derived fusion cross sections in the framework of continuum discretised coupled channels (CDCC) method using the coupled channels code FRESCO for the systems $^{6,7}$Li+$^{24}$Mg. The CDCC predicted fusion cross sections for the $^7$Li+$^{24}$Mg system agree well with the experimental fusion data whereas for the $^6$Li+$^{24}$Mg system the agreement is reasonable at below barrier energies. However, within the limits of the present work no definite conclusion could be obtained from the quality of agreement at above barrier energies for the $^6$Li+$^{24}$Mg system.

1 Introduction

Over the past years extensive efforts have been devoted on the studies of fusion reactions induced by weakly bound nuclei that have small separation energies which subsequently lead to a large probability of breaking up when approaching the field of other nuclei [1]. This subject has been centred mainly on studying the effect of breakup process on the fusion cross sections. Although vast works have been carried out both experimentally and theoretically in this field, still the subject is far from being fully understood. There is a special interest in this field due to the recent increasing availability of radioactive beams of very weakly bound nuclei. This has primarily been motivated by the fact that many reactions of astrophysical interest are induced by weakly bound nuclei. Also if the large size of the radioactive nuclei leads to a remarkable enhancement of fusion cross sections, super-heavy nuclei could be more easily produced. However the radioactive beams are produced with very low intensity still now, therefore precise measurement of fusion cross sections is still difficult, though some results have started coming in [2-5]. On the other hand, it is very convenient to produce fusion reactions induced by high intensity stable beams that are weakly bound nuclei. Therefore the study of fusion reactions induced by these stable weakly bound nuclei may test theoretical models and also may be useful when compared with similar studies of reactions involving unstable halo nuclei. There are three suitable stable weakly bound nuclei for this kind of study. These are $^6$Li, $^7$Li and $^9$Be that have different cluster structures and breakup threshold energies in the range 1.47 MeV to 2.47 MeV. A large number of fusion cross sections measurements have been carried out with these nuclei on various targets at energies around the Coulomb barriers. It is to be noted that for light and medium mass systems, such as $^{6,7}$Li+$^{12,13}$C, $^{6,7}$Li+$^{16}$O, $^{6,7}$Li+$^{27}$Al, $^{9}$Be+$^{27}$Al, $^{6,7}$Li+$^{62}$Zn, $^{9}$Be+$^{62}$Zn [6,7], the main evaporation channels include charged particles, so the residues produced from the complete fusion (CF) process (in which whole of the weakly bound nucleus fuses with whole of the target nucleus) and the residues produced from the incomplete fusion (ICF) process (in which a part of the weakly bound projectile fuses with whole of the target and the rest escapes with the corresponding beam velocity) are similar. Therefore for such systems experimentally it is very difficult to separate CF and ICF events and so only their total fusion (TF) have been measured [8]. The TF cross sections when compared with the fusion cross sections of strongly bound system show no effect of breakup on TF. Whereas for heavy systems, like $^{6,7}$Li+$^{208}$Bi, $^{9}$Be+$^{208}$Pb [9], experimentally it is possible to separate CF and ICF events since the compound nuclei evaporate mainly by neutrons. The CF cross sections for such systems when compared with the predictions of single-barrier penetration model show CF suppression of ~ 30% at above barrier energies. So from literature, it is found that the study of the effect of breakup on fusion has mainly involved by comparing the measured fusion excitation functions to either realistic theoretical predictions which do not include couplings to the breakup channels or to the measured fusion excitation functions of strongly bound nuclei for which breakup is expected to be weak [1]. In this context by considering the couplings to the breakup channels only using the continuum discretized coupled channels (CDCC) method, calculations of fusion cross sections were reported for the systems $^{6,7}$Li+$^{16}$O [10], $^{6,7}$Li+$^{59}$Co and $^{6,7}$Li+$^{208}$Bi [11]. For the $^{6,7}$Li+$^{16}$O systems, the CDCC predicted fusion
cross sections were found to agree well with the measured fusion cross sections obtained by the γ-ray method and it was conjectured that no significant effect of breakup on fusion is observed. Whereas for the $^6,7\text{Li}+^{28}\text{Co}$ and $^6,7\text{Li}+^{209}\text{Bi}$ systems, it was found that the breakup enhances the total fusion at energies just around the barrier, but it hardly affects the total fusion at energies well above the barrier. Here we report the CDCC calculations of the total fusion cross sections for the $^6,7\text{Li}+^{24}\text{Mg}$ systems for which the measured total fusion excitation functions obtained from the characteristic γ-ray method have already been reported [12].

For both the reactions $^6\text{Li}+^{24}\text{Mg}$ and $^7\text{Li}+^{24}\text{Mg}$, the measured total fusion cross sections were compared with the optical model calculations and are shown in Fig. 1. It was observed that the measured cross sections are nearly equal to the total reaction cross sections at lower energies and their difference increases with the increase of bombarding energy [12]. The measured cross sections were also compared with the one-dimensional barrier penetration model (1D BPM) using the code CCFULL in the no coupling limit. The results of 1D BPM calculations do not agree with the measured fusion cross sections at sub-barrier energies. The theoretical cross sections remain practically unaltered on inclusion of coupling to the excited states of $^{24}\text{Mg}$. It was found that the CCFULL calculations agree with the measured fusion cross sections at higher bombarding energies, and the low energy data appears to indicate an enhancement of fusion cross sections [12]. So in continuation of this work, it will be interesting if the effect of breakup coupling is considered in the calculations.

2 CDCC calculations & results

In the continuum discretised coupled channels (CDCC) formalism [13,14] the effect of breakup is considered by the cluster-folding model for $^6\text{Li}$ and $^7\text{Li}$ using the coupled reaction channels code FRESCO [15]. The method used is similar to that described by Keeley et al [10].

In this formalism, the breakup process of a weakly bound nucleus is interpreted in terms of inelastic excitation of the nucleus into the continuum energy eigen-states above its breakup threshold. In this method, the breakup continuum states are described in terms of a finite number of discrete states called “bins”, which are suitably constructed from the original continuum states. Each bin is treated as an excited state and coupling among these discretised continuum states i.e. “bins” are treated exactly in a coupled channels approach.

![Figure 1](image1.png)

**Figure 1.** Total fusion cross sections as a function of centre-of-mass energy for the $^6,7\text{Li}+^{24}\text{Mg}$ system. The dash-dot-dot line shows reaction cross sections from optical model calculations. The dashed line represents the 1-D BPM calculations using code CCFULL in the no coupling limit. The dot-dot black line represents the coupled channels calculations considering target excitation using code CCFULL [12]. The solid line shows the fusion cross sections obtained by projectile breakup coupling in the CDCC method.

![Figure 2](image2.png)

**Figure 2.** Binning scheme for the breakup continuum in the $^6\text{Li}$ nucleus. The bin states are represented by boxes. For details see the text.

2.1 The $^6\text{Li}+^{24}\text{Mg}$ reaction

The model space of breakup continuum for $^6\text{Li}$ nucleus used in our calculation is very much similar to that used by G.R. Kelley et al [16]. In the calculations, $^6\text{Li}$ is considered as having a cluster structure of $\alpha+d$ for its bound and continuum states. The continuum above the breakup threshold at 1.47 MeV is discretised with respect to linear momentum $\hbar k$ of $\alpha$-d relative motion. The bins are taken to be equally spaced in wave number $k$, each of bin width $\Delta k = 0.25 \text{ fm}^{-1}$ in the range $0 \leq k \leq 0.75 \text{ fm}^{-1}$, corresponding to the $^6\text{Li}$ excitation energy of $1.47 \leq E_x \leq 10.27 \text{ MeV}$ with respect to $^6\text{Li}$ ground state energy. The contribution from higher excited states is expected to be negligible. Each momentum bin is treated as an excited state of $^6\text{Li}$ nucleus with excitation energy equal to mean energy of the bin and having spin $J$ and parity $\pi = (-1)^J$.

The angular momentum is related by $\vec{J} = \vec{L} + \vec{s}$ where
\( \tilde{s} \) is spin of the deuteron and \( \tilde{L} \) is the relative angular momentum of the \( \alpha-d \) cluster system. In the calculation \( L \) is limited to 0, 1 and 2. The contribution from higher \( L \) is negligible. The binning scheme is suitably modified in presence of the three \( L = 2 \) resonant states (3’, 2.19 MeV; 2’, 4.31 MeV and 1’, 5.65 MeV), in order to avoid double counting of these states. The three \( L = 2 \) resonance states are also treated as momentum bins of width taken from Ref [17]. The binning scheme for \(^7\)Li nucleus is shown in Fig. 2. The ground state of \(^7\)Li is taken to be the pure \( L = 0 \) state. The ground state and the \( s \)-wave continuum states are generated by a core-valence interaction potential of Woods-Saxon form with geometrical parameter \( R = 1.9 \) fm and diffuseness parameter \( a = 0.65 \) fm [18], the depth being adjusted to reproduced the binding energy 1.47 MeV. The \( p \)-wave and \( d \)-wave continuum states are generated with the same potential but with slightly increased depth. Since the \(^7\)Li nucleus is modelled as two clusters of \( \alpha \) and \( d \), the excitation of the nucleus \(^7\)Li is interpreted in terms of the interaction between each cluster and the target nucleus \(^24\)Mg. The cluster-folding model potentials for the interactions, \( \alpha-^{24}\)Mg and \( d-^{24}\)Mg are evaluated at 4/6 and 2/6 of the incident energy of \(^7\)Li beam, respectively. As no experimentally measured elastic scattering angular distribution data for \( \alpha+^{24}\)Mg and \( d+^{24}\)Mg reactions are available in the literature, we have used global optical model potential parameters in describing their interactions at the corresponding energies. The \( \alpha+^{24}\)Mg and \( d+^{24}\)Mg potentials are taken from references [19-20] respectively. Both Coulomb and nuclear couplings are included in the calculations. The coupling of ground state to continuum and continuum to continuum states are included in the calculations. The fusion cross sections thus obtained from the calculations are shown in Fig. 1 by the solid line.

### 2.2 The \(^7\)Li+\(^{24}\)Mg reaction

In the calculation, the nucleus \(^7\)Li is considered to have a cluster structure of \( \alpha+t \) for its bound and continuum states. The \( \alpha+t \) continuum above the breakup threshold at 2.47 MeV is discretised into a series of momentum bins with respect to the momentum \( \hbar k \) of the \( \alpha+t \) relative motion. The model space was similar to that as described in G.R. Kelley [16] and K. Rusek et al [21], \( k \) being limited to \( 0 \leq k \leq 0.75 \) fm\(^{-1} \) with \( \Delta k = 0.25 \) fm\(^{-1} \). This binning scheme is suitably modified in presence of resonant states (7/2, 4.63 MeV and 5/2, 6.68 MeV) in order to avoid double counting. The binning scheme for \(^7\)Li is shown in Fig. 3. The \( L = 0, 1 \) and 3 continuum states are included in the \(^7\)Li calculation. Coupling to \( L = 2 \) continuum is not considered in our calculations as its contribution to the total cross sections was much less than that due to coupling to the \( L = 0 \) continuum [16]. The \( L = 3 \) resonances (7/2, 4.63 MeV and 5/2, 6.68 MeV) in \(^7\)Li are also treated as momentum bins with widths taken from Ref. [17]. For the ground state, first excited state (1/2, 0.480 MeV) and non-resonant continuum bins, the \( \alpha-t \) binding potential of B. Buck and A.C. Merchant [22] is used. The cluster-folding model potentials for \( \alpha-^{24}\)Mg and \( t-^{24}\)Mg interactions are evaluated at 4/7 and 3/7 of the incident energy of \(^7\)Li beam. For the \( t+^{24}\)Mg interaction, as no experimental angular distribution data is available in the literature, we have used the global optical model potential of triton by Xiaohua Li et al [23]. Here also both Coulomb and nuclear coupling is considered. Also ground state to continuum and continuum to continuum states are included in the calculations. The fusion cross sections thus obtained are shown in Fig. 1 by the solid line.

For both the systems, the calculations were carried out for \(^6\)Li- bombarding energies from above-barrier to below-barrier-energies in steps of 1 or 2 MeV.

### 3 Discussion

We have obtained the TF in the CDCC formalism for the systems \(^6,7\)Li+\(^{24}\)Mg at energies around the respective Coulomb barriers. The CDCC predicted TF cross sections when compared with the measured TF cross sections as seen in Fig. 1, show reasonably good agreement for the \(^7\)Li+\(^{24}\)Mg reaction both for below-barrier and above-barrier energies. So for the \(^7\)Li+\(^{24}\)Mg system, there seems to be no effect of breakup on fusion similar to the systems \(^6\)Li+\(^{16}\)O as reported by N. Keeley et al [10]. For the \(^7\)Li+\(^{24}\)Mg reaction, reasonable agreement is observed at below-barrier energies. From the quality of agreement at above barrier energies for \(^7\)Li+\(^{24}\)Mg, no definite conclusion can be made at these energies.

It is to be noted that the nucleon transfer induced breakup mechanisms, are found to be an important processes for systems such as \(^6\)Li+\(^{208}\)Pb [24], \(^7\)Li+\(^{159}\)Tb [25]. The \( p \)-pickup triggered breakup for \(^7\)Li- induced and \( n \)-stripping triggered breakup for \(^7\)Li- induced reactions have been seen to be the prominent transfer induced processes [24,25]. Both these transfer processes have positive g.s. Q-values, thus favouring the transfer induced
breakup channels. For the $^6$Li+$^{24}$Mg systems, knowledge of the relative probabilities of such nucleon transfer triggered breakup processes requires a detailed experimental investigation. Such two-step processes if present may also influence the fusion process. Therefore, before making any definite conclusion about the influence of breakup coupling on the fusion process, the effect of these two step processes must also be investigated. However, the nucleon transfer induced breakup cannot be handled in the CDCC formalism.

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References