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Microscopic Calculation of Astrophysical S-factor and Branching Ratio for the ^{3}H(α, γ)^{7}Li Reaction

Alexander S. Solovyev[1,a], Sergey Yu. Igashov[1] and Yury M. Tchuvil'sky[2]

[1] *All-Russia Research Institute of Automatics (VNIIA), Moscow, 127055 Russia*
[2] *Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, Moscow, 119991 Russia*

Abstract. In the present work the radiative capture reaction ^{3}H(α, γ)^{7}Li has been investigated. The astrophysical *S*-factor and the branching ratio of the reaction have been calculated within a microscopic approach – the algebraic version of the resonating-group model. The lowest compatible with the Pauli exclusion principle wave functions of the translation-invariant oscillator shell model are adopted as the internal wave functions of the colliding clusters. The modified Hasegawa–Nagata NN-potential was employed in the calculations. The results are in good agreement with the experimental data.

1 Introduction

Knowledge of low-energy behavior of the fusion reactions cross sections (astrophysical *S*-factors) is important for the modern nuclear astrophysics [1–4]. Low-energy experimental measurements of these cross sections are rather difficult and not too reliable because values of the cross sections are strongly suppressed by the Coulomb barrier. For this reason direct theoretical calculations and extrapolations based on these calculations turn out to be the only way to obtain the cross sections and the astrophysical *S*-factors at the energies typical for the astrophysics. Microscopic approaches to the problem improve significantly the reliability of results.

The fusion (radiative capture) reaction ^{3}H(α, γ)^{7}Li plays an important role for big bang nucleosynthesis and the primordial lithium problem [5–9]. The interest in this reaction arose long ago and is retained by now. The results of the first measurement of the reaction cross section at the energies of the relative motion of colliding nuclei in center-of-mass system $E_{\mathrm{c.m.}} = 205\div565$ keV were published in Ref. [10]. The measured values turned out to be many times over than the estimates based on work [11], where the direct capture mechanism was not considered. The discrepancy stimulated the following investigations. As a result, another six experiments [12–17] were performed at energies $E_{\mathrm{c.m.}} = 150\div785$ keV, $300\div858$ keV, $79\div464$ keV, $80\div980$ keV, $50\div1200$ keV, and $100\div500$ keV respectively after the publication of the first experimental data [10]. In these works the values of the astrophysical *S*-factor are several times more than the values presented in Ref. [10]. As a whole, the data [12–17] are consistent at the energies $E_{\mathrm{c.m.}} > 250$ keV in the case that systematic errors are taken into consideration. However, at lower energies $E_{\mathrm{c.m.}} < 250$ keV the results [15, 17] and at energies $E_{\mathrm{c.m.}} < 150$ keV the data [14] show pronounced energy dependence of the astrophysical *S*-factor and are considerably greater than the data [16]. Thus, there is some scatter between the experimental data on the astrophysical *S*-factor for the ^{3}H(α, γ)^{7}Li reaction. Furthermore, none of these experiments covers the extremely low energies.

The first attempts to calculate the cross sections of the fusion reaction ^{3}H(α, γ)^{7}Li were undertaken in the works [18–20] based on the direct capture model. Later on calculations based on this model were made in Ref. [21]. The calculations of the astrophysical *S*-factor within the potential cluster model were performed in Refs. [22, 23]. But all these calculations are made in the framework of two-body model. Semi-microscopic calculation using the microscopic potential cluster model is presented in Ref. [24]. More advanced microscopic calculations based on the resonating-group model (RGM) [25] were carried out in Refs. [26–32]. Along with the traditional approaches to the consideration of the ^{3}H(α, γ)^{7}Li reaction some efforts were also applied to combine methods of such a type with the so-called *ab initio* approaches [33, 34]. In such comprehensive approaches calculations of asymptotic normalization coefficients for the wave functions (WFs) of the bound states of the ^{7}Li nucleus based on either the variational Monte Carlo method [35, 36] or on the no-core shell model [37] are combined with the traditional potential cluster model for the continuous states. However, these combined approaches are not capable to describe successfully both the magnitude and the energy dependence of the astrophysical *S*-factor at the same time. The results of *ab initio* calculations of the astrophysical *S*-factors for two mirror ^{3}H(α, γ)^{7}Li and ^{3}He(α, γ)^{7}Be reactions were presented in Ref. [38], where

[a] Corresponding author: alexander.solovyev@mail.ru

the states of discrete and continuous spectra were described uniformly using the fermionic molecular dynamics and realistic effective potentials. The astrophysical S-factor of the $^3He(\alpha,\gamma)^7Be$ reaction turns out to be in a reasonable agreement with experimental data while the astrophysical S-factor of the mirror $^3H(\alpha,\gamma)^7Li$ reaction is overestimated in Ref. [38].

Comparison of the theoretical calculations of the astrophysical S-factor for the $^3H(\alpha,\gamma)^7Li$ reaction demonstrates qualitative and quantitative discrepancies between them like the experimental data. So, the questions concerning the magnitude and the energy dependence of the astrophysical S-factor for the $^3H(\alpha,\gamma)^7Li$ reaction at low energies remain open. In the present work, we apply the single-channel algebraic version of the RGM (AVRGM) [39, 40] to calculate the astrophysical S-factor and the branching ratio for the discussed reaction at low energies. It should be noted that the AVRGM was utilized earlier in Ref. [41] for the study of this reaction at zero-energy value. However, the energy dependences of the astrophysical S-factor and the branching ratio, which is also important as a test of the model reliability, were not studied in Ref. [41]. The AVRGM is consistent and effective microscopic nuclear model to investigate few-nucleon systems and nuclear reactions at low energies in the case that few channels are open. The details of the calculations procedure may be found in Refs. [42, 43].

2 The radiative capture cross section

E1-transitions from continuous spectrum states of the $\alpha + t$ system to the ground ($J_f = 3/2$, $\pi_f = -1$) and the first excited ($J_f = 1/2$, $\pi_f = -1$) states of the ^7Li nucleus make a dominant contribution to the total cross section of the radiative capture $^3H(\alpha,\gamma)^7Li$ (see, for example, Refs. [28, 35]). Three partial E1-transitions from the continuous spectrum states of the $\alpha + t$ system with quantum numbers (J_i, l_i) equal to (1/2, 0), (3/2, 2), and (5/2, 2) allowed by the selection rules populate the ground state of the ^7Li nucleus. For the partial E1-transitions to the first excited state of the ^7Li nucleus values of the quantum numbers (J_i, l_i) are (1/2, 0) and (3/2, 2). Here J and l are the angular and the orbital momenta, π is the parity, the indices i, f denote the initial and final states respectively. Channel spin s is equal to 1/2.

Thus, at low energies the total cross section of the $^3H(\alpha,\gamma)^7Li$ reaction is the sum of five partial cross sections characterized by the quantum numbers which are given above. In the long-wavelength limit the partial radiative capture cross sections are determined by the reduced matrix elements of the electric dipole operator $M^{E}_{1\mu}$ [44–46]:

$$\sigma_{i\to f}(E_{\rm c.m.}) = \frac{8\pi}{9\hbar(2l_i+1)}\left(\frac{E_\gamma}{\hbar c}\right)^3 \left|\left\langle J_f^{\pi_f}\left\| M_1^{\rm E}\right\| J_i^{\pi_i} l_i s\right\rangle\right|^2, \quad (1)$$

where E_γ is the energy of emitted photon. The smallness of the Coulomb interaction compare to nuclear interaction is taken into account in Exp. (1). The initial WF is assumed to be a partial wave of a unit-flux scattering WF.

The astrophysical S-factor is defined as follows [47]

$$S(E_{\rm c.m.}) = E_{\rm c.m.}\exp\left(\sqrt{E_{\rm G}/E_{\rm c.m.}}\right)\sigma(E_{\rm c.m.}), \quad (2)$$

where $E_{\rm G}$ is the Gamow energy. The cross section σ is characterized by the strong energy dependence at the sub-barrier energies while the astrophysical S-factor is a smooth function of the energy for non-resonant reactions.

3 Brief overview of the AVRGM

In the single-channel RGM [25] the total WF of the seven-nucleon system in $\alpha + t$ fragmentation is sought in the form of antisymmetrized product of the internal WFs of the clusters φ_α, φ_t and the WF of the relative motion f which depends on the Jacobi vector $\mathbf{q}$. Schematically, the total WF is written as

$$\Psi = \mathrm{A}\left\{\varphi_\alpha \varphi_t f(\mathbf{q})\right\}. \quad (3)$$

In the AVRGM [39, 40] in contrast to the conventional RGM the WF of relative motion is sought in the form of expansion over the basis of the oscillator functions. As a result, the total WF is represented as a linear combination of the basis WFs of the AVRGM which are labeled by the following quantum numbers: total angular momentum J, its projection M, parity π, orbital momentum l, channel spin s, and number of oscillator quanta ν. Unknown expansion coefficients $C_{J^\pi Mls\nu}$ of the total WF satisfy the infinite set of linear uniform algebraic equations [39, 48]

$$\sum_{\nu=\nu_0}^{\infty}\left(\left\langle J^\pi Mls\tilde{\nu}\right|H\left|J^\pi Mls\nu\right\rangle - E\,\delta_{\tilde{\nu}\nu}\right)C_{J^\pi Mls\nu} = 0, \quad \tilde{\nu} = \nu_0, \nu_0+2, \ldots, \quad (4)$$

where ν_0 is the minimum number of oscillator quanta allowed by the Pauli exclusion principle, H and E are the Hamiltonian and the total energy of the system respectively. The details of calculations of the matrix elements of the Hamiltonian are given, for example, in Refs. [42, 43]. The modified Hasegawa–Nagata NN-potential [49] is used in our study. This potential provides a good description of electromagnetic properties of light nuclei [50] and electromagnetic processes with them [28].

For the bound states the expansion over the oscillator basis is usually truncated for a sufficiently large $\nu = \nu_{\max}$. Thus, the infinite set of Eqs. (4) is reduced to the finite one (ν, $\tilde{\nu} \le \nu_{\max}$). The $\nu_{\max}$ value is chosen so that the further increasing of $\nu_{\max}$ does not influence on the energy of the corresponding bound state. We chose $\nu_{\max} \sim 200$ in the calculations.

Neglect of the terms with large value of oscillator quantum number violates the asymptotic behavior of the WF of the continuous spectrum. Therefore, the next step of the AVRGM approach is the replacement of the expansion coefficients by their asymptotic values [40, 51–53] starting from sufficiently large $\nu = \nu_{\rm as}$. Thus, the set of Eqs. (4) can be reduced to a finite one of the non-uniform linear algebraic equations [39, 48]:

$$\sum_{\nu=\nu_0}^{\nu_{\rm as}-2}\left(\left\langle J^\pi Mls\tilde{\nu}\right|H\left|J^\pi Mls\nu\right\rangle - E\delta_{\tilde{\nu}\nu}\right)C_{J^\pi Mls\nu} = F_{J^\pi Mls\tilde{\nu}}, \quad \tilde{\nu} = \nu_0, \nu_0+2, \ldots, \nu_{\rm as}, \quad (5)$$

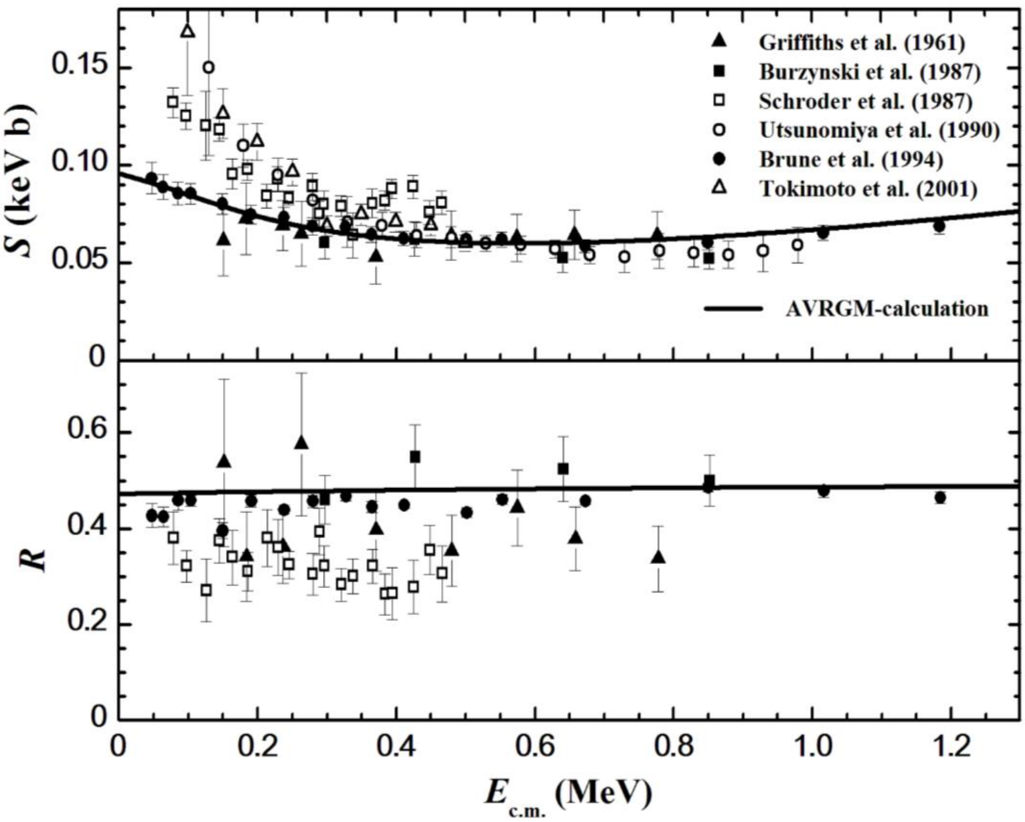

Figure 1. Calculated astrophysical S-factor and branching ratio R for the ^{3}H(α, γ)^{7}Li reaction, as well as related experimental data [12–17].

where

$$F_{J^\pi Mls\tilde{\nu}} = -\sum_{\nu=\nu_{as}}^{\nu_{max}} \left\langle J^\pi Mls\tilde{\nu} \middle| H \middle| J^\pi Mls\nu \right\rangle C^{(as)}_{J^\pi Mls\nu}. \quad (6)$$

The solution to the set of equations (5) results in determination of the values of the expansion coefficients at $\nu < \nu_{as}$ and the nuclear phase shift, which appears in the expression for the asymptotic expansion coefficients $C^{(as)}_{J^\pi Mls\nu}$ with $\nu_{as} \le \nu \le \nu_{max}$.

4 Results and discussion

Explicit expressions for the matrix elements of the electric dipole operator for the $\alpha + t$ system in the oscillator representation and a method of their calculation are given in Ref. [54]. These matrix elements and values of the expansion coefficients $C_{J^\pi Mls\nu}$ are necessary for calculation of the cross section (1) and the astrophysical S-factor (2) for the ^{3}H(α, γ)^{7}Li reaction in the framework of the AVRGM.

In the present study two adjustable parameters – the oscillator radius r_0 and the intensity of central Majorana forces g_c were tuned to reproduce the energies of the ground states of the ^{4}He, ^{3}H, and ^{7}Li nuclei, as well as the energy of the first excited state of the ^{7}Li nucleus [55, 56] together with the experimental data on the astrophysical S-factor and the branching ratio

$$R(E_{c.m.}) = \sigma_1(E_{c.m.}) / \sigma_0(E_{c.m.}) \quad (7)$$

of the ^{3}H(α, γ)^{7}Li reaction in the best way. Here σ_0 and σ_1 denote the capture cross sections to the ground and the first excited states of the ^{7}Li nucleus respectively. The calculation of the branching ratio and its comparison with the experimental data is an additional test of reliability of the chosen model. It should be noted that the data [16] cover largest energy range and have relatively small errors in comparison with other experimental data. That is why the fit is performed using these data. Final values of the parameters are r_0 = 1.386 fm and g_c = 1.021.

Our calculation of the astrophysical S-factor and the branching ratio for the ^{3}H(α, γ)^{7}Li reaction within the AVRGM is shown in Fig. 1. As it is seen from Fig. 1, our theoretical results are in a very good agreement with the experimental data from Refs. [12, 13, 16].

The calculation of the energies for the ^{4}He, ^{3}H, and ^{7}Li nuclei and also their experimentally measured values from Refs. [55, 56] are presented in Table 1.

Table 1. Calculated and experimentally measured values of energies for the ^{4}He, ^{3}H, and ^{7}Li nuclei.

Energy (MeV)	Experiment	Calculation
$E(^4$He)	–28.296	−28.296
$E(^3$H)	–8.482	−6.467
$E(^7$Li)	−39.244	–36.002
$E(^7$Li*)	−38.766	–35.920

Table 1 demonstrates a reasonable agreement between the theoretical and experimental values of the energies.

5 Conclusion

In the present paper the astrophysical *S*-factor and the branching ratio for the fusion reaction ^{3}H(α, γ)^{7}Li at low energies are calculated in the framework of a microscopic approach – the AVRGM. A comparison of the obtained results with the experimental ones demonstrates a good agreement. The possible ways for the development of the study are: the extension of the model by including the other cluster configurations – multi-channel approach, the refinement of the internal cluster WFs, involving in the fitting procedure other types of data including the data concerning the mirror ^{3}He(α, γ)^{7}Be reaction.

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