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# Scattering of ECRF waves by edge density fluctuations and blobs

Abhay K. Ram[1,a] and Kyriakos Hizanidis[2,b]

[1] *Plasma Science and Fusion Center, Massachusetts Institute of Technology, Cambridge, MA 02139. USA.*
[2] *National Technical University of Athens (part of HELLAS), School of Electrical and Computer Engineering, Association EURATOM-Hellenic Republic, GR-15773, Greece.*

**Abstract.** The scattering of electron cyclotron waves by density blobs embedded in the edge region of a fusion plasma is studied using a full-wave model. The full-wave theory is a generalization of the usual approach of geometric optics ray scattering by blobs. While the latter allows for only refraction of waves, the former, more general formulation, includes refraction, reflection, and diffraction of waves. Furthermore, the geometric optics, ray tracing, model is limited to blob densities that are slightly different from the background plasma density. Observations in tokamak experiments show that the fluctuating density differs from the background plasma density by 20% or more. Thus, the geometric optics model is not a physically realistic model of scattering of electron cyclotron waves by plasma blobs. The differences between the ray tracing approach and the full-wave approach to scattering are illustrated in this paper.

## 1 Introduction

The edge region of tokamak plasmas is replete with density fluctuations and turbulent structures such as blobs [1–3]. Radio frequency (RF) waves, commonly used for heating and for current profile control, have to propagate from the excitation structures to the core of the plasma through this active region. The fluctuations modify the propagation properties of the waves through reflection, refraction, and diffraction. We have been studying the scattering of RF waves by blobs using a full-wave theory [4, 5]. The theoretical approach is similar to that for Mie scattering of electromagnetic waves by dielectric particles [6]. The Faraday-Ampere system of equations are solved separately for wave fields inside the blob, and for wave fields in the surrounding plasma in which the blob is embedded. In the edge region, separating the blob from the background plasma, the electromagnetic boundary conditions, which follow from Maxwell's equations, have to be satisfied. These boundary conditions necessarily require the simultaneous excitation of all wave modes that can exist in the plasma. In the case of a cold plasma, the dispersion relation allows for only two independent wave modes which can be excited for any applied wave frequency. Thus, even though a specifically chosen wave mode is coupled into the plasma from an antenna structure, fluctuations will channel some of that power to the other plasma wave mode. For example, in the electron cyclotron range of frequencies, if an ordinary wave is coupled to the plasma from an external source, the fluctuations and the blobs will not only scatter the ordinary wave, but also couple some of the power to the extraordinary wave.

Previous studies on the scattering of RF waves by blobs have been based on the geometric optics ray tracing approximation [7, 8]. These studies are of limited applicability as effects like reflection and diffraction are ignored. Consequently, in these models, the density inside the blob has to be within a few percent of the background density. Experimental observations point to a much larger range of density fluctuations [9], so that the full-wave model is the most appropriate paradigm for wave scattering off blobs.

In the full-wave analytical model, we assume that the blobs are either spherical or cylindrical in shape. The axis of the cylindrical blobs is aligned along the magnetic field line. The plasma, both inside and outside the blobs, is assumed to be cold and homogeneous with arbitrary densities in either region; thus, we are not limited to small density fluctuations. The anisotropy induced by the magnetic field is such that the propagation characteristics and the polarization of the RF waves depend on the polar angle with respect to the direction of the magnetic field. In the full-wave theory, the vector Helmholtz equation is solved for the electromagnetic fields inside the blob, and for the scattered fields outside the blob. The incident wave induces charge oscillations inside the blob leading to waves propagating in all directions inside the blob. The geometry of the blob manifests itself through boundary conditions at the surface of the blob. This, in turn, induces scattered electromagnetic fields. A combination of the incident wave and the scattered fields has to match with the electromagnetic fields inside the blob so as to properly satisfy the boundary conditions.

[a] e-mail: abhay@mit.edu
[b] e-mail: kyriakos@central.ntua.gr

From experimental observations it is found that the speed of propagation of the blobs is of the order of $5 \times 10^3$ m s$^{-1}$ [9]. This is slower than the group velocity of the incoming EC waves by several orders of magnitude. The frequency of temporal variation of the fluctuations is less than 1 MHz. So that the typical EC wave frequencies are several orders of magnitude higher than the fluctuation frequency. Consequently, in our model, we assume that the blobs are stationary and that the density inside the blob is time invariant. Observations show that the scale lengths of the blobs typically range from a fraction of a centimeter to several centimeters [9].

## 2 Description of the plasma

### 2.1 Spherical coordinate system

In the spherical coordinate system, with the origin being at the center of the blob, the position vector $\mathbf{r}$ is given by $(r, \theta, \phi)$, where $r$ is the radial distance, $\theta \in [0, \pi]$ is the polar angle, and $\phi \in [0, 2\pi]$ is the azimuthal angle. The propagation vector $\mathbf{k}$ is represented in its own spherical coordinate system by $(k, \theta', \phi')$. The ambient magnetic field, in which the plasma and the blob are immersed, is taken to be uniform and along the $z$−direction of the Cartesian coordinate system. The electromagnetic RF fields have a time dependence of the form $\exp(-i\omega t)$, where $\omega$ is the angular frequency of the RF fields.

In the coordinate system of the propagation vector $\mathbf{k}$, the plasma permittivity tensor is obtained by an appropriate transformation of the Cartesian permittivity tensor [10]. This transformed tensor is

$$\overleftrightarrow{\mathbf{K}} = \begin{pmatrix} K_\perp \sin^2\theta' + K_\parallel \cos^2\theta' & \left(K_\perp - K_\parallel\right)\sin\theta'\cos\theta' & -iK_\times \sin\theta' \\ \left(K_\perp - K_\parallel\right)\sin\theta'\cos\theta' & K_\perp \cos^2\theta' + K_\parallel \sin^2\theta' & -iK_\times \cos\theta' \\ iK_\times \sin\theta' & iK_\times \cos\theta' & K_\perp \end{pmatrix}. \tag{1}$$

The elements of $\overleftrightarrow{\mathbf{K}}$ are given by

$$\begin{aligned} K_\perp &= 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} - \sum_\iota \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2}, \\ K_\times &= -\frac{\omega_{ce}}{\omega}\frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} + \sum_\iota \frac{\omega_{ci}}{\omega}\frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2}, \\ K_\parallel &= 1 - \frac{\omega_{pe}^2}{\omega^2} - \sum_\iota \frac{\omega_{pi}^2}{\omega^2}, \end{aligned} \tag{2}$$

where $\omega_{pe}$ ($\omega_{pi}$) and $\omega_{ce}$ ($\omega_{ci}$) are the angular electron (ion) plasma frequency and cyclotron frequency, respectively, and the index $\iota$ represents all the ion species in the plasma. The permittivity tensor of the background plasma and of the blob are expressed in terms of their respective densities and ion compositions.

### 2.2 Cylindrical coordinate system

For a cylindrical blob, we ignore the effects of the ends of the cylinder, and assume that the axis of the blob is aligned along the magnetic field. Using the conventional description of the cylindrical coordinates, with the origin located at the center of the blob, the propagation vector is $(k_\perp, m, k_\parallel)$, where $k_\perp$ is the component of $\mathbf{k}$ along the radial direction, $m$ is the azimuthal mode number, and $k_\parallel$ is the component along the axial direction. The plasma permittivity can be obtained from (1) by setting $\theta' = 0$.

## 3 Electromagnetic fields inside and outside the blob

The spatial dependence of the electric fields in a plasma is obtained from the Faraday-Ampere equation

$$\nabla \times \nabla \times \mathbf{E}(\mathbf{r}) = \frac{\omega^2}{c^2} \overleftrightarrow{\mathbf{K}}.\, \mathbf{E}(\mathbf{r}), \tag{3}$$

where $c$ is the speed of light. We solve for $\mathbf{E}(\mathbf{r})$ by using the Fourier transform

$$\mathbf{E}(\mathbf{r}) = \int d^3k\, \mathbf{E}(\mathbf{k})\; e^{i\mathbf{k}.\,\mathbf{r}}. \tag{4}$$

Substituting this form into (3) yields

$$\int d^3k\, \overleftrightarrow{\mathbf{D}}.\, \mathbf{E}(\mathbf{k})\; e^{i\mathbf{k}.\,\mathbf{r}} = 0, \tag{5}$$

where

$$\overleftrightarrow{\mathbf{D}}(\mathbf{k}, \omega) = \frac{c^2}{\omega^2}\left(\mathbf{kk} - k^2 \overleftrightarrow{\mathbf{I}}\right) + \overleftrightarrow{\mathbf{K}} \tag{6}$$

is the dielectric tensor. Here, $\mathbf{kk}$ is a dyadic and $\overleftrightarrow{\mathbf{I}}$ is the unit tensor. In general, the above equation is satisfied if

$$\overleftrightarrow{\mathbf{D}}(\mathbf{k}, \omega) . \mathbf{E}(\mathbf{k}) = 0. \tag{7}$$

For a non-trivial solution to the Fourier transformed electric field, we require that

$$\det\left(\overleftrightarrow{\mathbf{D}}(\mathbf{k}, \omega)\right) = 0, \tag{8}$$

where det denotes the determinant of the tensor. From (8), we obtain the dispersion relation for the waves in a plasma. In spherical coordinates, the dispersion relation relates the angle of propagation $\theta'$ to the magnitude of $\mathbf{k}$

$$\tan^2\theta' = \frac{-K_\parallel (n_k^2 - K_R)(n_k^2 - K_L)}{(n_k^2 - K_\parallel)(K_\perp n_k^2 - K_R K_L)}, \tag{9}$$

where $ck/\omega = n_k$ is the index of refraction, $K_R = K_\perp + K_\times$, and $K_L = K_\perp - K_\times$.

In cylindrical coordinates, the dispersion relation is

$$\begin{aligned} K_\perp n_{k\perp}^4 &- \left[\left(K_\perp - n_{k\parallel}^2\right)(K_\perp + K_\parallel)\right] n_{k\perp}^2 \\ &+ \left[\left(K_\perp - n_{k\parallel}^2\right)^2 - K_\times^2\right] K_\parallel = 0, \end{aligned} \tag{10}$$

where $c\mathbf{k}/\omega = \mathbf{n}_k$. The dispersion relation is independent of the azimuthal mode numbers $m$.

It is easy to show that, in spherical coordinates, for a given angle of propagation $\theta'$ in (9), $n_k^2$ is positive [4, 5].

Similarly, in cylindrical coordinates, for a real $n_{k\|}$ in (10), $n_{k\perp}^2$ is real. Thus, $n_k$ in spherical coordinates, or $n_{k\perp}$ in cylindrical coordinates, is either purely real or purely imaginary. Of the four roots for $n_k$ or $n_{k\perp}$, only two are independent. The other two roots are negatives of the independent roots and can be obtained, in spherical coordinates, by appropriate rotations in $\theta'$ and $\phi'$.

In spherical coordinates, along the two principal directions of propagation $\theta' = 0$ and $\theta' = \pi/2$, the respective dispersion relations for the waves are

$$n_k^2 = K_R, \;\; n_k^2 = K_L, \;\; K_\| = 0, \qquad (11)$$

$$n_k^2 = K_\|, \;\; n_k^2 = \frac{K_R K_L}{K_\perp}. \qquad (12)$$

In Eq. (11), the last expression gives plasma oscillations. In Eq. (12), the second expression leads to upper hybrid and lower hybrid resonances when $K_\perp = 0$. For an arbitrary $\theta'$, we will classify the two roots of $n_k^2$ obtained from Eq. (9) according to their limiting values when $\theta' \to \pi/2$. The root leading to the first dispersion relation in Eq. (12) will be referred to as the ordinary wave or O wave, and the root leading to the second dispersion relation as the extraordinary wave or X wave. The same can be easily done for the cylindrical coordinate system.

## 4 Results and discussion

The consequences emanating from a full-wave physics model, which includes a diversity of scattering phenomena, is best illustrated using a single cylindrical blob as a scatterer. Then, it is easy to compare the full-wave model with the geometrical optics, ray tracing, model that has been the basis of previous studies [7, 8].

Consider a cylindrical blob whose axis is aligned along the ambient magnetic field; the plasma being homogeneous in that direction. Let us assume that the incident EC plane wave, either the ordinary O wave or the extraordinary X wave, is propagating along the $x$-direction. In the ray tracing approach, the plane wave, represented by a single ray, corresponds to the initial condition $k_\| = 0$. The $k_\perp$ is then determined from the appropriate dispersion relation. Since the incident ray is normal to the surface of the blob, the refracted ray inside the blob also has $k_\| = 0$; the corresponding $k_\perp$ being determined from the plasma dielectric inside the blob. As the ray exits the farther end of the blob, it is refracted for the second time, while maintaining $k_\| = 0$. After the second refraction, the ray is back in the background plasm in which it originated. Consequently, the $k_\perp$ of the final refracted ray is the same as that of the incident ray, and the refractive process through the blob does not lead to any changes in the spectrum of the incident ray. The polarization of the wave and the power flow are conserved as well; in cold plasma there is no dissipation. In other words, the ray passes through the cylindrical blob as if it does not exist. This refractive scattering process is independent of whether the incoming wave is an O wave or an X wave; i.e., it is independent of the polarization of the wave.

The full-wave scattering model leads to completely different results. In displaying the results, we will primarily look at the time-averaged Poynting vector

$$\langle \mathbf{S} \rangle \;=\; \frac{1}{2\mu_0} \mathrm{Re}\,(\mathbf{E} \times \mathbf{B}^*)\,, \qquad (13)$$

where < ... > denotes the time-average, $\mu_0$ is the free-space permeability, and $\mathbf{B}^*$ is the complex conjugate of $\mathbf{B}$ – the magnetic induction vector given by Faraday's equation

$$\mathbf{B} \;=\; \frac{1}{i\omega}\, \nabla \times \mathbf{E}. \qquad (14)$$

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Figure 1a and 1b display the contours of constant $S_N = |\mathrm{Real}\,\langle \mathbf{S} \rangle|/|\mathrm{Real}\,\langle \mathbf{S}_0 \rangle|$ as a function of $x$ and $y$, for an incident O wave and X wave, respectively. $\langle \mathbf{S}_0 \rangle$ is $\langle \mathbf{S} \rangle$ evaluated for the incoming wave. The plasma and blob parameters are as indicated in the figure caption. It is clear that the scattering process is dependent on the polarization of the incident wave; there are distinct differences in the structure of the wave spectrum transmitted into the core of the plasma. The differences are further amplified by the results shown in Figs. 2a and 2b. In these figures, we plot the Fourier spectrum of $(S_N)_x$, denoted by $(S_N)_k$, evaluated in the shadow of the blob. The normalized magnitude of the Poynting flux, $S_N$, is calculated for points outside the blob with $x > 0$ and $y = 0$. The Fourier spectrum of this data is plotted in Figs. 2a and 2b. $k_0$ is the incoming $\mathbf{k}$ along the $x$-direction. If refraction was the sole scattering process for both incoming waves, the spectra would be located exactly at $k_x - k_0 = 0$ with the height of the peak being unity. The differences with the results from pure refraction are quite obvious. The blob affects the X wave differently from the O wave. This is clearly indicated by the difference in the spread of the spectrum of the transmitted wave, and its amplitude. These feature cannot be reproduced by using the ray tracing formalism. Consequently, the full-wave theory is a more complete description of the scattering of EC waves.

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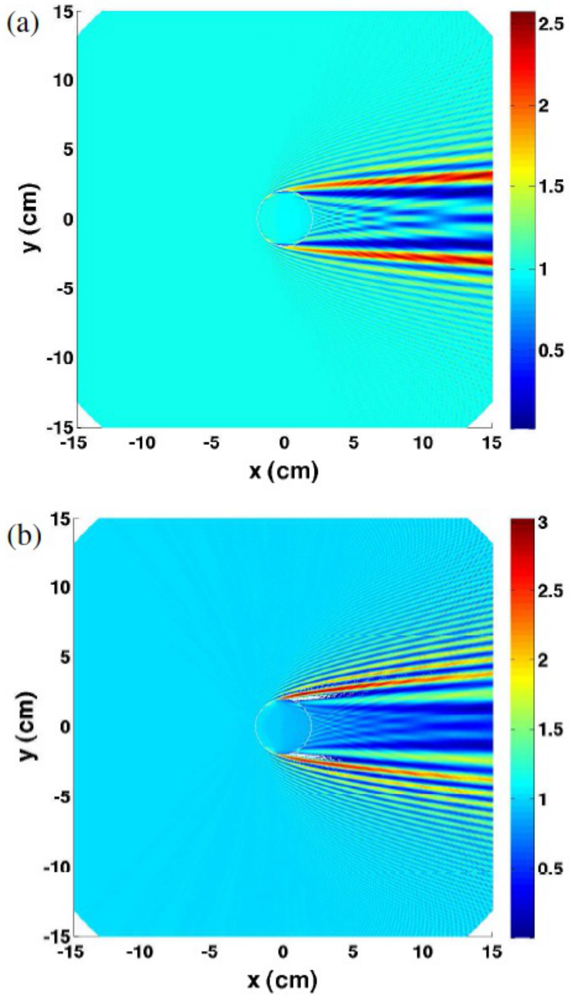


**Figure 1.** Contours of constant $|\mathrm{Re}\,\langle \mathbf{S} \rangle| / |\mathrm{Re}\,\langle \mathbf{S}_0 \rangle|$, for an incident (a) O wave and (b) X wave. The axis of the cylindrical blob is centered at $x = y = 0$, and aligned along the $z$-direction. The radius of the blob is 2 cm, and the plasma density inside the blob is $4 \times 10^{19}$ m$^{-3}$ – twice the density of the background plasma outside the blob. The plasma is composed of electrons and deuterons. The magnetic field is 4.13 T and the wave frequency is 170 GHz. The incoming plane wave is incident from the left hand side of the blob, with the initial $\mathbf{k}_0 = k_0\hat{\mathbf{x}}$ along the $x$-direction.

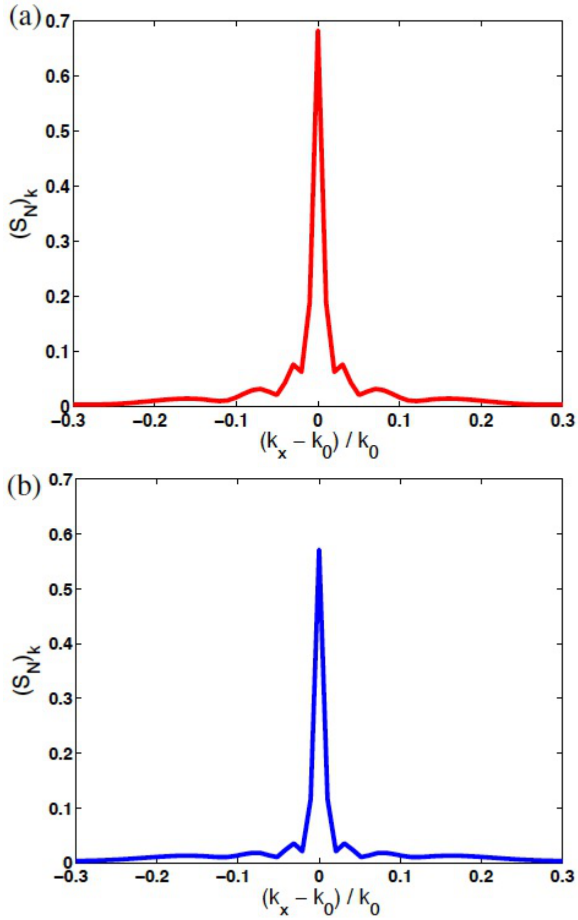


**Figure 2.** Spectra of the forward scattered waves, shown in Figs. 1a and 1b, for an incident (a) O wave, and (b) X wave. Here $k_x$ is the $x$-component of the wave vector $\mathbf{k}$.