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# Closure of the single fluid magnetohydrodynamic equations in presence of electron cyclotron current drive

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**Abstract.** In the presence of electron cyclotron current drive (ECCD), the Ohm's law of single fluid magnetohydrodynamics (MHD) is modified as $\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta(\mathbf{J} - \mathbf{J}_{\mathrm{ECCD}})$. This paper presents a new closure relation for the EC driven current density appearing in this modified Ohm's law. The new relation faithfully represents the nonlocal character of the EC driven current and its main origin in the Fisch-Boozer effect. The closure relation is validated on both an analytical solution of an approximated Fokker-Planck equation as well as on full bounce-averaged, quasi-linear Fokker-Planck code simulations of ECCD inside rotating magnetic islands.

## 1 Introduction

Fluid models like single fluid magnetohydrodynamics (MHD) are commonly used to study macroscopic plasma dynamics. They are obtained by taking moments of the governing kinetic equations. This results in a hierarchy of equations in which the evolution of each of the moments depends on higher order moments. At some point this hierarchy is truncated by introducing closure relations that model these higher order moments in terms of the lower order moments. On the timescale of the macroscopic plasma evolution, the effect of electron cyclotron heating and current drive is described in the kinetic equation by quasi-linear diffusion of the electron distribution in velocity space. As shown in a recent paper by Hegna and Callen [1], the EC quasi-linear diffusion enters the single fluid MHD equations as a power source term in the energy balance equation, and as a parallel force term in Ohm's law. In addition it affects the closure for the resistivity through its effect on the electron-ion friction.

Because EC driven quasi-linear diffusion is dominantly in the direction of perpendicular momentum [2], the quasi-linear, parallel force contribution in Ohm's law is negligible. The effect of ECCD on Ohm's law must entirely be described in the closure relation for the resistivity, i.e. the electron-ion friction. Assuming a linear response of the plasma to the different driving forces, one may write $\mathbf{J} = \mathbf{J}_\Omega + \mathbf{J}_{\mathrm{ECCD}}$ where $\mathbf{J}_\Omega$ is the current density from the parallel electric field and $\mathbf{J}_{\mathrm{ECCD}}$ is the EC driven current density. This implies a modification of Ohm's law as commonly used:

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta(\mathbf{J} - \mathbf{J}_{\mathrm{ECCD}}) \tag{1}$$

with the usual (neoclassical) Spitzer resistivity $\eta$. The latter is unmodified by ECCD, but a separate closure relation still is needed to describe $\mathbf{J}_{\mathrm{ECCD}}$. A model for the closure of $\mathbf{J}_{\mathrm{ECCD}}$ has been proposed by Giruzzi et al. in [3].

In this contribution a new model for the closure of $\mathbf{J}_{\mathrm{ECCD}}$ is derived through approximations of the governing kinetic equation. The resulting closure relation faithfully represents the nonlocal character of the driven current and its origin in the Fisch-Boozer mechanism [4] and contains the closure relation of Giruzzi et al. as one of its limits. The model is validated against exact solutions of the approximate kinetic equation and compared to solutions of the full bounce-averaged, quasi-linear Fokker-Planck equation.

## 2 A new closure relation for the EC driven current density

To derive a model for the electron cyclotron driven current density we start from the gyrophase-averaged Boltzmann equation taking into account only collisions and the effect of the waves:

$$\frac{\partial f_e}{\partial t} = C(f_e) + Q_{\mathrm{EC}}(f_e) - v_\parallel \nabla_\parallel f_e, \tag{2}$$

where $f_e(v_\parallel, v_\perp)$ is the gyrophase averaged electron velocity distribution as a function of parallel and perpendicular velocities, respectively, $C(f_e)$ represents the effect of collisions, $Q_{\mathrm{EC}}(f_e)$ the averaged effect of the electron cyclotron waves, and the final term describes the convection along magnetic field lines of localized features in the electron distribution function with their parallel velocity.

In the quasi-linear regime, electron cyclotron waves result in diffusion in velocity space localized at the electron cyclotron resonance or its harmonics [2]. Under typical conditions of ECCD in tokamaks, the electron cyclotron



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waves carry little or no parallel momentum, and the quasi-linear diffusion is dominantly in the direction of the perpendicular velocity. Using a non-relativistic approximation for the fundamental harmonic ordinary mode, the EC quasi-linear diffusion can be written as

$$Q_{\mathrm{EC}}(f_e) = \frac{\partial}{v_\perp \partial v_\perp} v_\perp D_{\mathrm{EC}} \delta(v_\parallel - v_{\parallel,\mathrm{res}}) \frac{\partial}{\partial v_\perp} f_e, \quad (3)$$

where $D_{\mathrm{EC}}$ is the amplitude of the EC quasi-linear diffusion coefficient, which is proportional to the wave power, and $v_{\parallel,\mathrm{res}} = (\omega - \Omega_{\mathrm{ce}})/k_\parallel$ is the parallel velocity of resonant electrons for a wave frequency $\omega$, parallel wave vector $k_\parallel$ and electron cyclotron frequency Ωce.

Next, we linearize the Boltzmann equation around the local Maxwellian $f_{\mathrm{M}}$ with temperature $T_e$, such that the EC quasi-linear diffusion term becomes $Q_{\mathrm{EC}}(f_e) = Q_{\mathrm{EC}}(f_{\mathrm{M}}$. The latter then becomes

$$Q_{\mathrm{EC}}(f_{\mathrm{M}}) = D_{\mathrm{EC}} \delta(v_\parallel - v_{\parallel,\mathrm{res}}) \left( \frac{v_\perp^2}{2 v_t^4} - 1/v_t^2 \right) \exp\left( -\frac{v_\parallel^2 + v_\perp^2}{2 v_t^2} \right), \quad (4)$$

where $v_t = \sqrt{kT_e/m_e}$ is the thermal velocity. We thus find that the waves drive a perturbation, which is characterized by an e bulge of electrons at supra thermal perpendicular velocities and a hole at sub thermal perpendicular velocities. This wave driven perturbation carries no current. A net current only arises as a consequence of the subsequent effect of collisions [4]. To proceed analytically with the calculation of this so-called Fisch-Boozer current, we approximate the collisions by a Krook-type collision operator

$$C(f_e) = -\nu(v)(f_e - f_M) \quad (5)$$

with a velocity dependent collision frequency $\nu(v) = \nu_t (v_t/v)^3$. The momentum loss that is implied by this operator represents the momentum transfer from electrons to ions. In the homogeneous case the solution then is

$$f_e(v_\parallel, v_\perp; t) - f_M = Q_{\mathrm{EC}}(f_M) \frac{1}{\nu(v)} (1 - e^{-\nu(v) t}). \quad (6)$$

In most experiments, however, the wave power deposition is extremely localized along a field line. When we restrict the ECCD power deposition to a finite interval $0 \leq x \leq L_{\mathrm{ECCD}}$ along a field line and take $t = 0$ as the time the power is switched on, the solution to (2) becomes

$$\delta f_e = \begin{cases} \frac{Q_{\mathrm{EC}}(f_M)}{\nu(v)} (1 - e^{-\nu(v) \min(x/v_{\parallel,\mathrm{res}}, t)}) & \\ & \text{for } 0 \leq x \leq L_{\mathrm{ECCD}} \\ \frac{Q_{\mathrm{EC}}(f_M)}{\nu(v)} (1 - e^{-\nu(v) \min(L_{\mathrm{ECCD}}, t v_{\parallel,\mathrm{res}} - (x - L_{\mathrm{ECCD}}))/v_{\parallel,\mathrm{res}}}) \times & \\ e^{-\nu(v)(x - L_{\mathrm{ECCD}})/v_{\parallel,\mathrm{res}}} & \text{for } L_{\mathrm{ECCD}} < x < L_{\mathrm{ECCD}} + t v_{\parallel,\mathrm{res}} \\ 0 & \text{for } x < 0 \text{ or } x \geq L_{\mathrm{ECCD}} + t v_{\parallel,\mathrm{res}} \end{cases} \quad (7)$$

A straightforward calculation of the first moment of this perturbation of the distribution function, i.e.

$$J_{\mathrm{ECCD}}(x, t) = -e \int d^3 v \, v_\parallel \delta f_e, \quad (8)$$

now provides the spatial and temporal evolution of the EC driven current density $J_{\mathrm{ECCD}}$ along a magnetic field line.

We thus obtain the following picture for the process of ECCD: (1) The EC waves create a perturbation in velocity space localized at the resonant parallel velocity, which exists of a velocity space hole at small perpendicular velocities $v_\perp < \sqrt{2} v_t$ and a bulge at high perpendicular velocities $v_\perp > \sqrt{2} v_t$ with a zero net momentum; (2) This perturbation is convected along the field line out of the EC deposition region with the parallel velocity of the resonant electrons. As the perturbation is convected, the velocity space hole at low velocities is filled in more quickly by collisions than the bulge at high velocities is eroded, because of the velocity dependence of the collision frequency. The result is a net current which (3) subsequently decays at the slower collision rate of the high velocity electrons in the bulge.

In a final approximation, we represent the EC wave driven hole and bulge by two delta functions at perpendicular velocities $v_1$ and $v_2$ with associated collision rates $\nu_i = \nu_t/(v_{\parallel,\mathrm{res}}^2 + v_i^2)^{3/2}$, $i = 1, 2$:

$$\delta f_e \approx \delta(v_\parallel - v_{\parallel,\mathrm{res}}) \times \sum_{i=1,2} \frac{c_i}{v_i} \delta(v_\perp - v_i) \quad (9)$$

where the amplitudes $c_i$ are driven by the EC waves and decay due to collisions according to

$$\frac{\partial c_i}{\partial t} = \begin{cases} S_i - \nu_i c_i & 0 \leq x \leq L_{\mathrm{ECCD}} \\ -\nu_i c_i & x > L_{\mathrm{ECCD}}. \end{cases} \quad (10)$$

The conservation of particles in the EC diffusion leads to a relation of the sources $S_i$ as $S_1 = -S_2$. This may be looked upon as the balance between two counter driven current density perturbations, one associated with $v_1$ in the counter direction and the other associated with $v_2$ in the co-direction which are represented by the equations

$$\frac{\partial J_1}{\partial t} = -S_{\mathrm{ECCD}} - \nu_1 J_1 + v_{\parallel,\mathrm{res}} \nabla_\parallel J_1 \quad (11)$$

and

$$\frac{\partial J_2}{\partial t} = +S_{\mathrm{ECCD}} - \nu_2 J_2 + v_{\parallel,\mathrm{res}} \nabla_\parallel J_2 \quad (12)$$

where $S_{\mathrm{ECCD}}$ is non zero only between $0 < x < L_{\mathrm{ECCD}}$. The EC driven current density is now defined as the sum of these two,

$$J_{\mathrm{ECCD}} \equiv J_1 + J_2, \quad (13)$$

Together this set of equations (11) − (13) represents our new closure relation. In the limit of $\nu_1 \to \infty$, $J_1$ becomes identical zero, and our model reduces in principle to the one proposed by Giruzzi et al. [3]. However, the latter model does not include a convective transport term but relies on a high parallel diffusivity for the equilibration of the driven current density along a field line, while in addition also a finite perpendicular diffusivity is included. The source term $S_{\mathrm{ECCD}}$ is related to the EC current drive efficiency which is defined as the ratio of the total driven current over the absorbed power $\eta_{\mathrm{ECCD}} \equiv I_{\mathrm{ECCD}}/P_{\mathrm{EC}}$ as obtained, for example, from a bounce-averaged, quasi-linear Fokker-Planck calculation:

$$S_{\mathrm{ECCD}} = \eta_{\mathrm{ECCD}} p_{\mathrm{EC}} \frac{2\pi R \nu_2}{1 - \frac{\nu_2}{\nu_1}} \quad (14)$$

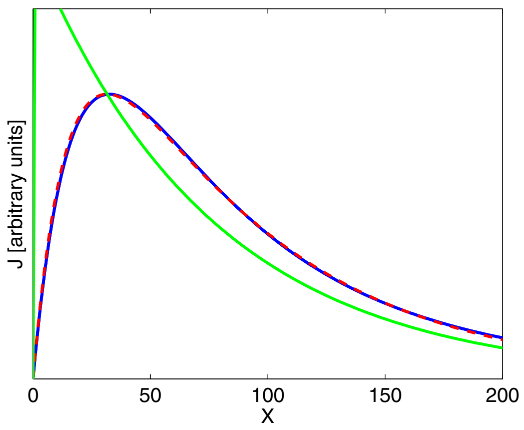


**Figure 1.** The EC driven current density along a field line. The blue curve represents the analytical solution 7 of the approximated Fokker-Planck equation. The parameters are given in the text. The red dashed curve represents the fit with the new closure equation for $J_{\text{ECCD}}$ (11) - (13) with $v_1 = 0$ and $v_2 = 2.7$. The green curve represents the limit of $\nu_1 \to \infty$ corresponding to the model of Giruzzi et al. [3].

where $p_{\text{EC}}$ is the local (non-flux-surface-averaged) EC absorbed power density.

## 3 Model validation

In order to illustrate how the EC driven current evolves, we performed a number of calculations using the solution (7) to the approximate, linearized Boltzmann equation with the Krook type collision operator along a given magnetic field line. In these calculations, time is normalized to the thermal collision time and length to the distance traveled by a thermal electron in a collision time *i.e.*$v_t/nu_t$. For typical tokamak parameters the thermal electron velocity $v_t$ is of order $10^7$ m/s, the thermal collision frequency $nu_t$ is of order $10^4$ to $10^5$ Hz, and a typical width of the EC power deposition is of order 2 to 10 cm, which corresponds to an ECCD power deposition width in normalized units of order $10^{-3}$. We choose a typical resonant parallel velocity of $v_{\|,\text{res}} = 2v_t$. Figure 1 (blue curve) shows the EC driven current density, that is established after a sufficient number of collision times (> 100), as a function of the length along a field line crossing through the EC power deposition region. What this figure illustrates in a striking manner is that the EC driven current is generated while the perturbation of the distribution function flows out of the EC power deposition region, which is only a tiny region near $x = 0$ in this figure: the EC driven current is highly nonlocal.

A virtually exact match between the solution of the approximate Boltzmann equation and the proposed closure relation (11) - (13) for the EC driven current density is obtained with a proper choice of parameters for $v_1$ and $v_2$. This is illustrated for the case of Fig. 1 by the red dashed curve which is obtained by setting $v_1 = 0$ and $v_2 = 2.7$, which corresponds to $\nu_1 = 1/8$ and $\nu_2 = 1/38$, in equations (11). Similarly, the green curve shows the solution

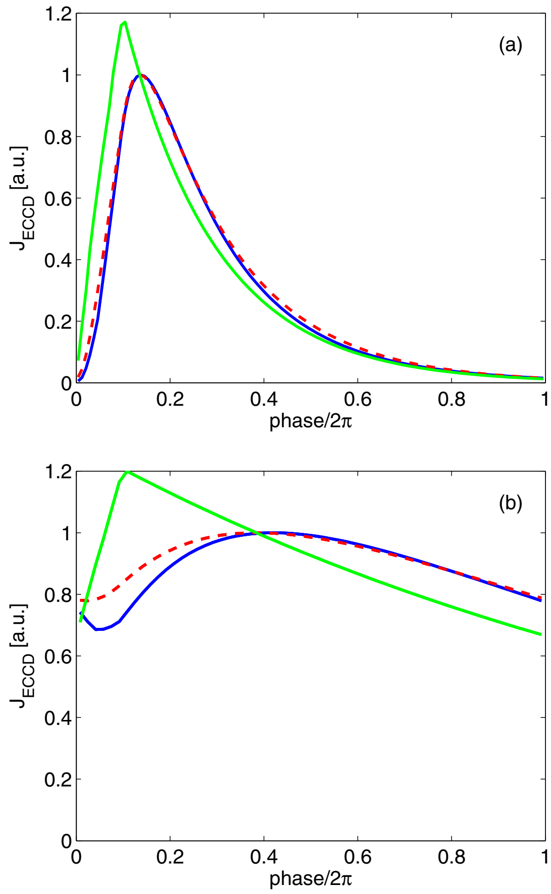


**Figure 2.** The EC driven current density near the O-point of a rotating magnetic island in ASDEX Upgrade: a) for an island rotation frequency of 3 kHz, b) for 23 kHz. The blue curve represents the un-approximated bounce-averaged quasi-linear Fokker-Planck simulation described in Ref. [6]. The red dashed curve represents a fit with the new closure equation for $J_{\text{ECCD}}$ (11) - (13) (parameters: $\nu_1 = 88.4$ kHz, $\nu_2 = 15.2$ kHz and $v_{\|,\text{res}} \to \infty$, i.e. flux surface averaged). The green curve represents the limit of $\nu_1 \to \infty$ corresponding to the model of Giruzzi et al. [3].

that is obtained in the limit of $\nu_1 \to \infty$ holding $\nu_2$ constant. The extreme localization of the ECCD power along the field line in this case results in a large over estimate of the local current density in the power deposition region. Note, however, that the total length of the field line displayed in Fig. 1 is of the order of $2 \times 10^5$ - $10^6$ m, which corresponds to $10^4$ - $10^5$ toroidal revolutions of the field line. This means that over this length the field line crosses the power deposition region many times with the sum of all these crossings adding up to a much more smooth profile on the flux surface spanned by the field line.

As an additional validation of the new closure relation, we compare its predictions to the results of a bounce-averaged, quasi-linear Fokker-Planck calculation. For this we use results obtained with the RELAX Fokker-Planck code which employs a full quasi-linear diffusion operator and a Maxwellian background collision operator with a correction term expressing the momentum conservation in electron-electron collisions [5]. The parameters used correspond to a typical ASDEX Upgrade experiment in which ECCD is applied for NTM suppression, and which has been modelled extensively in a recent paper by B. Ayten et al. [6]. In particular, we will compare with the temporal behavior of the current density as obtained by Ayten et al. near the O-point of a rotating magnetic island. These results are represented by the blue lines in Fig. 2 a and b, which refer to an island rotation frequency of 3 and 23 kHz, respectively. Since the Fokker-Planck code calculates a flux surface averaged driven current density under the assumption that the parallel transport is infinitely fast, we compare with the results of equations (11) - (13) under the same assumption. The parameters, $\nu_1$ and $\nu_2$ to be used are estimated by fitting the rise of the current density to steady state on an equilibrium flux surface in the middle of the power deposition profile. In particular, we obtain for the case described in Ref. [6] $\nu_1 = 88.4$ kHz and $\nu_2 = 15.2$ kHz. We then use these parameters to model the current density evolution near the O-point of a rotating magnetic island when the ECCD is well centered on the radius at which the island is localized. The results are shown in Fig. 2 a and b as the red dashed lines. Note that the rotation frequency for the first case is smaller than $\nu_2$, while in the second case it is larger. We observe that the new closure relation provides a good fit to all the results of the Fokker-Planck code without any further changes of the parameters. Notice that the Fokker-Planck calculations for the high rotation frequency of 23 kHz shows a small dip when the O-point crosses the power deposition region. This dip is the result of the trapping of resonant electrons and represents the Ohkawa-effect [7] which is not described by this simple model of the Fisch-Boozer current drive. Unlike the Fish-Boozer current, the Ohkawa-effect is an immediate response to the EC quasi-linear diffusion.

## 4 Conclusions

The main result of this paper is formed by equations (11) - (13), which provide a new closure relation for the EC driven current density that faithfully represents the non-local character of the driven current and its origin in the Fisch-Boozer mechanism [4]. In single fluid MHD modelling, this closure relation is to be used in combination with the usual modification of Ohm's law (1). In the model the EC driven current density is the sum of two contributions representing the EC driven quasi-linear modification of the electron distributing function: a hole at low perpendicular velocities and a bulge at high perpendicular velocities. The dominant transport is provided by parallel convection of these perturbations with the parallel resonant velocity. This parallel convection is the main transport mechanism that will result in almost constant (in space) driven current density over closed magnetic surfaces. Additional transport terms representing parallel diffusion and (anomalous) radial diffusion could be added to the model as well. In the limit that the collisionality of the electrons in the hole goes to infinity, i.e. neglecting the dynamics of the hole itself, the new model reduced to the closure as proposed by Giruzzi et al. [3].

The new model has been validated on both an analytical solution of the approximated Fokker-Planck equation discussed above. This validation in particular addressed the strongly non-local character of the evolution of the EC driven current density along a magnetic field line. In practice this will lead to almost constant (in space) driven current density over closed magnetic surfaces, an assumption that lies at the basis of the usual bounce averaged Fokker-Planck code modelling of ECCD. A further validation of the new model has been provided by comparison of flux surface averaged predictions of the model with full quasi-linear Fokker-Planck code simulations of ECCD inside rotating magnetic islands. The Fokker-Planck code results used in this comparison, were obtained with the RELAX code and are taken from a recent paper by B. Ayten et al. on the Fokker-Planck code modelling of ECCD for NTM suppression in ASDEX Upgrade [6].

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