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Pairing effects on spinodal decomposition of asymmetric nuclear matter

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Abstract

We present an analysis framed in the general context of two-component fermionic systems subjected to pairing correlations. The study is conducted for unstable asymmetric nuclear matter at low temperature, along the clusterization process driven by spinodal instabilities. It is shown that, especially around the transition temperature from the superfluid to the normal phase, pairing correlations may have non-negligible effects on the isotopic features of the clusterized low-density matter, which could be of interest also in the astrophysical context.

Introduction

The understanding of the properties of many-body interacting systems in terms of the interaction among their constituent particles is a very exciting challenge in several fields of physics.

The many-body problem is often approached by mean-field models, employing so-called effective interactions. Suitable extensions of these models allow to take explicitly into account effects of relevant interparticle correlations, such for example pairing correlations occurring under appropriate conditions in fermionic systems.

We focus in particular on the study of infinite nuclear matter; in such a system, it is well known that because of pairing nucleons can form paired states, analogous to the way with whom electrons pair in metals giving rise to a superconducting phase [1]. Pairing effects are widely investigated nowaday, according to the connection with astrophysical applications: neutron superfluidity in the crust and the inner part of neutron stars is in fact well known and has significant effects on cooling processes and glitch phenomena.

In the general context of many-body systems, another general feature is the possible occurence of different kinds of phase transitions. For nuclear matter at sub-saturation density and relatively low temperature ($T \leq 15$ MeV) in particular, liquid-gas phase transitions are expected to appear, driven by the unstable mean-field. Such a phenomenon is closely linked to the multifragmentation process experimentally observed in nuclear reactions. Taking into account the two-component structure of nuclear matter, an essential role in this mechanism pertains to the density behavior of the isovector part of the effective interaction and the corresponding term in the nuclear Equation of State, the symmetry energy, on which many investigations are concentrated. Along a phase separation process, the symmetry energy influences significantly the so-called isospin distillation mechanism, which leads to a different species concentration in the two phases, namely a more symmetric (with respect to the initial system) liquid phase and a more asymmetric gas phase.

The aim of our investigation is to evaluate the impact of pairing correlations, which are mostly active at low density, on mechanical (spinodal) instabilities of asymmetric nuclear matter.

1 Effective pairing interaction

Our study is undertaken in the framework of the Hartree-Fock-Bogolyubov (HFB) approach, which includes in a unified formalism the pairing and the mean-field effective interactions. We consider a local pairing interaction acting only between identical nucleons in a spin-singlet state, because pairing between protons and neutrons is strongly quenched in asymmetric matter [3] and we assume, as it has been often done, a strength interaction $v_{\pi q}$, where $q = p, n$ denotes neutrons or protons, respectively, which depends only on the density ρ_q of the considered species [4], so:

$$v_{\pi q}(\rho_n, \rho_p) \equiv v_\pi(\rho_q) = V_\pi \left[1 - \eta \left(\frac{2\rho_q}{\rho_0} \right)^\alpha \right], \tag{1}$$

where $\rho_0 = 0.16$ fm^{-3}. The other parameters in Eq.(1) are determined using pairing-gap function $\Delta(\rho_q)$, given by fitting the $^1S_0$ pairing gap in pure

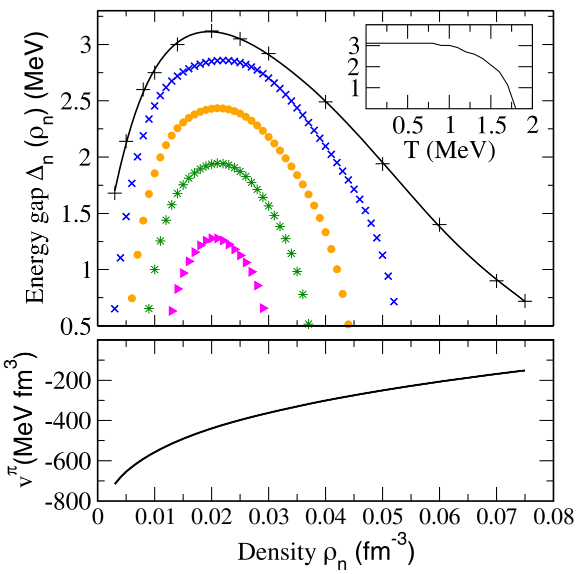

Figure 1: Top panel: The gap energy at zero temperature, as obtained in Brueckner calculations of pure neutron matter, as a function of the neutron density (black plus symbols). The figure also shows the gap energy obtained, solving eqs. (2) and (4), at several T values: 1 MeV (crosses), 1.3 MeV (dots), 1.5 MeV (stars), 1.65 MeV (triangles). The inset displays the gap energy as a function of the temperature, for the density $\rho_M = 0.02$ fm^{-3}. Bottom panel: The strength of the pairing interaction v_π as a function of the neutron matter density.

neutron matter obtained with Brueckner calculations employing a potential Argonne v_{14} [5]. The corresponding values are plotted in the top panel of fig. 1 (black plus sign) as a function of the neutron density ρ_n. The maximum value of the gap ($\Delta \approx 3$ MeV) is reached at the density $\rho_M = 0.02$ fm^{-3}. Known Δ, we can then invert the gap equation:

$$-v_\pi(\rho_q) \int \frac{d\mathbf{p}}{(2\pi\hbar)^3} \frac{1}{2\xi} \left[1 - 2F^q(\mathbf{p})\right] = 1 \tag{2}$$

and by fitting pairing strength $v_\pi(\rho_q)$ on the functional dependence expressed by eq. (1), we get the parameters V_π, η and α. The corresponding behavior is shown in the bottom panel of fig. 1. In the previous equation,

$$F^q(\mathbf{p}) = \frac{1}{2}\left[1 - \frac{\xi}{E_\Delta} \tanh\left(\frac{E_\Delta}{2T}\right)\right] \tag{3}$$

represents the particle occupation number while E_Δ is defined as $E_\Delta = \sqrt{\xi^2 + \Delta^2}$, where $\xi = \left(\frac{\mathbf{p}^2}{2m} - \mu_q^*\right)$ with m the nucleon mass and μ_q^* the reduced chemical potential which can be obtained by fixing the particle number density:

$$\int \frac{d\mathbf{p}}{(2\pi\hbar)^3} \left[1 - \frac{\xi}{E_\Delta} \tanh\left(\frac{E_\Delta}{2T}\right)\right] = \rho_q. \tag{4}$$

Once the strength of the pairing interaction is fixed, eqs. (2) and (4) allow one to evaluate the pairing gap at finite temperature T; results are shown in fig. 1. The critical temperature $T_c(\rho_q)$ of the transition to the superfluid/superconducting phase (where the energy gap Δ starts to appear) is equal to $T_c = 1.8$ MeV at ρ_M [see inset in fig. 1 (top panel)]. The results discussed above are extended to the pp case, by assuming that the pairing strength is the same as in the nn case, just depending only on the density of the considered species.

2 Nuclear energy density functional

Taking into account the effective pairing interaction introduced and by adopting a simplified Skyrme-like effective interaction for the mean-field [6], the nuclear energy density functional can be written as it follows:

$$\rho\frac{E}{A} = \sum_q \left[2\int \frac{d\mathbf{p}}{h^3}F^q\frac{\mathbf{p}^2}{2m} + v_\pi\frac{|\tilde{\rho}_q|^2}{4}\right] + \rho\left[\frac{\mathcal{A}}{2}\left(\frac{\rho}{\rho_0}\right) + \frac{\mathcal{B}}{\sigma+1}\left(\frac{\rho}{\rho_0}\right)^\sigma + \frac{\mathcal{C}_{\rm sym}}{2}\mathcal{I}^2\right], \tag{5}$$

where $\rho = \rho_n + \rho_p$, $I = \frac{(\rho_n - \rho_p)}{\rho}$ is the asymmetry parameter and $\tilde{\rho}_q = 2\Delta_q/v_\pi$ denotes the so-called anomalous density. For the isovector part of the nuclear interaction we consider two representative parameterizations for the local density (ρ) dependence: one with a linearly increasing behaviour with density (asy-stiff), $\mathcal{C}_{\rm sym}(\rho) = 112.5\ \rho$ (MeV), and one with a kind of saturation above normal density (asy-soft), $\mathcal{C}_{\rm sym}(\rho) = \rho\,(241 - 819\ \rho)$ (MeV) [2, 6].

The mean-field potential U_q can be derived from the potential part of the energy density functional (eq. (5)) $U_q = \left[\frac{\partial(\rho E_{\rm pot}/A)}{\partial\rho_q}\right]_{\tilde{\rho}_q}$ [7]. We notice that, because of the density dependence of the pairing strength, U_q gets a contribution also from the pairing energy density $U_q^\pi = \Delta^2/v_\pi^2\ \partial v_\pi/\partial\rho_q$.

2.1 Spinodal instability in asymmetric matter

We now turn to an examination of the possible occurence of mechanical (spinodal) instabilities in asymmetric nuclear matter at a fixed temperature T. The spinodal region is defined as the ensemble of unstable points for which the free-energy surface is concave in at least one direction. This is determined by the curvature matrix

$$C = \begin{pmatrix} a & c/2 \\ c/2 & b \end{pmatrix}, \tag{6}$$

where $a = \partial\mu_p/\partial\rho_p$, $b = \partial\mu_n/\partial\rho_n$, $c = 2\partial\mu_p/\partial\rho_n$ and $\mu_q = \mu_q^* + U_q$ denotes the chemical potential. The lower eigenvalue gives the minimal free-energy

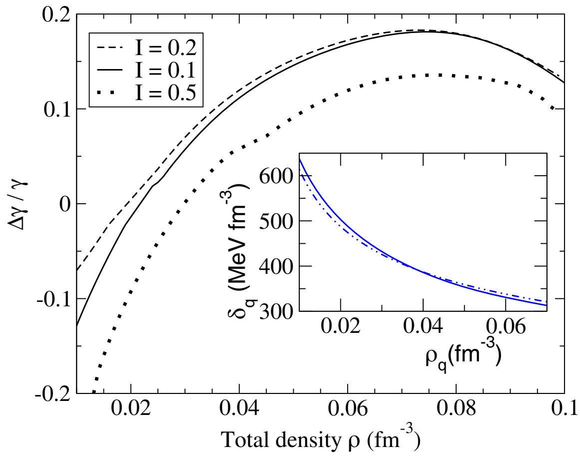

Figure 2: The percentage variation at zero temperature of the quantity γ (see text) as a function of the matter density, for three values of the asymmetry I. The inset shows the density behavior of the quantity δ_q (see text), in normal (dashed line) and superfluid (full line) matter, again at zero temperature.

curvature; if the latter is negative, the associated eigenvector gives the direction of phase separation. In asymmetric nuclear matter ($\rho_n \neq \rho_p$) at low density, instabilities correspond to isoscalar-like density oscillations: the two species move in phase but with different amplitudes, according to the eigenvector components $(\delta\rho_p, \delta\rho_n)$. In particular, defining the angle α as $\tan\alpha = \delta\rho_n/\delta\rho_p$, from the diagonalization of the the matrix C one obtains:

$$\tan 2\alpha = \frac{c}{a-b}. \tag{7}$$

It is generally observed that the asymmetry of the instability direction, $\delta I = (\delta\rho_n - \delta\rho_p)/(\delta\rho_n + \delta\rho_p)$ is smaller than the system initial asymmetry, leading to the formation of more symmetric nuclear clusters. This is the so-called isospin distillation mechanism, that is mainly ruled by the effect of the symmetry potential, which enhances the neutron-proton attraction.

3 Pairing effect on spinodal instability

Here our aim is to investigate how pairing correlations may affect these features. To undertake this analysis we need to evaluate the expression of the elements of the matrix C in presence of pairing correlations. We notice that the term c gets no contribution from the pairing interaction. One relevant quantity, to evaluate the width of the isospin distillation effect and which could be affected by pairing interaction, is instead the difference $\gamma = a - b$ of eq. (7). Thus we consider its percentage variation in superfluid nuclear matter, with respect to normal nuclear matter, at different global

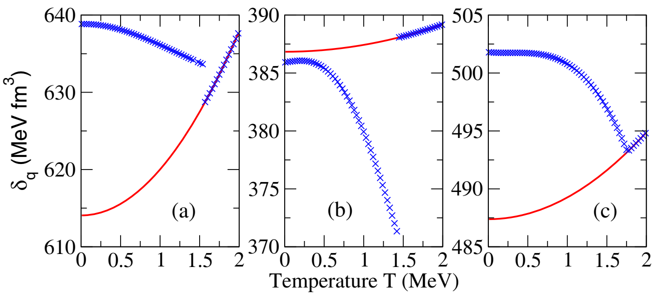

Figure 3: The quantity δ_q is represented as a function of the temperature, for three ρ_q density values: 0.5 ρ_M (a), 2 ρ_M (b) and ρ_M (c). Crosses indicate calculations including pairing, whereas full lines are for normal nuclear matter.

densities and asymmetries and zero temperature. Results are displayed in fig. 2, in the case of the asy-stiff symmetry potential.

One can see that it is possible to reach an effect of 20% for the variation of γ at total densities around 0.08 fm^{-3}. A similar effect, at zero temperature, is also reflected on the asymmetry of the instability direction δI.

Let us now move to consider nuclear matter at finite temperature T. Since the features of the unstable modes are determined by derivatives of proton and neutron chemical potentials, a deeper insight into the amplitude of the pairing effect can be obtained by looking directly at the quantity $\delta_q = \partial\mu_q^*/\partial\rho_q + \partial U_q^\pi/\partial\rho_q$.

The quantity δ_q is represented in fig. 3, as a function of T, for three values of the density ρ_q ($\rho_M/2$, ρ_M, $2\rho_M$). Calculations including pairing correlations (blue circles) are compared with the results of normal nuclear matter (red dashed lines). As a rather interesting effect, we clearly observe at the critical temperature the appearance of discontinuities, analogous to those discussed in literature [1] for the heat capacity, in the behavior of δ_q, which is connected to the matter compressibility. This jump however disappears at the density $\rho_q = \rho_M$ [fig. (1b)], where the energy gap Δ is maximum at all temperatures.

Finally, we turn to a discussion of the impact of pairing correlations directly on the asymmetry δI. Results are displayed in fig. 4, for nuclear matter at $\rho = 0.08$ fm^{-3}, three values of the asymmetry I and for the two parameterizations of the symmetry energy introduced above. We notice that the overall effect of the isospin distillation mechanism is rather important ($\delta I/I$ is lower than 1 in all cases) and it is larger in the asy-stiff case, in agreement with previous studies [2]. The asymmetry δI is more sensitive to the choice of the symmetry energy parameterization (black vs. red lines) than to the introduction of pairing correlations (full vs. dashed lines). Thus our results essentially confirm the leading role of the symmetry energy in the isospin distillation mechanism. However, new interesting effects appear

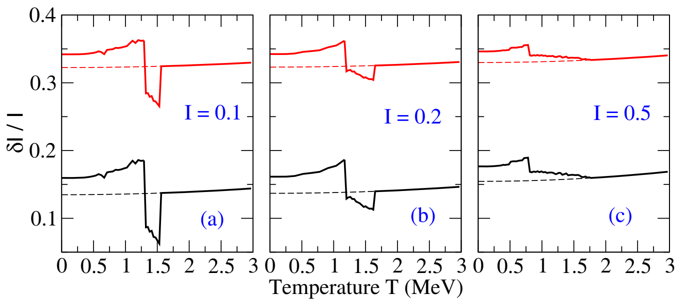

Figure 4: (Colour online) The asymmetry of the unstable oscillations as a function of the temperature, for nuclear matter at total density $\rho = 0.08$ fm^{-3}, three asymmetry values and for the asy-stiff (black) and the asy-soft (red) parameterizations.

at moderate temperatures. Owing to the trend followed by the chemical potential derivative (see fig. 3), the calculations including the pairing interaction exhibit two discontinuities, corresponding to neutron and proton critical temperatures, which may cause significant variations of δI. As observed in fig. 3, the δ_q discontinuity is more pronounced at densities around $2\rho_M \approx 0.04$ fm^{-3}. This explains why for the density $\rho = 0.08$ fm^{-3} the largest effect for δI is seen at small asymmetries [see fig. 3(a)]. Hence, under suitable density and temperature conditions, pairing correlations may lead to significant deviations of the asymmetry from its average and so may induce non-negligible variations in the isotopic content of the clusterized low-density matter. These results are so relevant in the general context of nuclear fragmentation and clustering mechanisms and could influence the properties of supernova matter and those of (proto-)neutron star crusts, where clustering phenomena take place at rather low temperature.

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