Probing the nuclear symmetry energy with heavy-ion collisions

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Abstract

Heavy ion collisions (HIC) have been widely used to extract the parametrization of symmetry energy term of nuclear equation of state as a function of barionic density. HIC in fact are a unique tool in terrestrial laboratories to explore the symmetry energy around the saturation density ($\rho_0 = 0.16\, fm^{-3}$) from sub-saturation densities (Fermi energies) towards compressed nuclear matter ($\rho > 2 - 3\rho_0$) that can be reached at relativistic energies, as a function of different conditions of temperature, mass asymmetry and isospin. One of the main study at present is to reach a coherent description of EOS of asymmetric nuclear matter from heavy ion collisions of stable and exotic nuclei, nuclear structure studies and astrophysical observations. In this work an overview of the current status of the research is shortly reviewed together with new perspectives aimed to reduce the present experimental and theoretical uncertainties.

1 Introduction

The study of the Equation of State EOS($\rho, T, \beta$) of asymmetric nuclear matter (where $\beta = (\rho_n - \rho_p)/\rho$ is the isospin asymmetry as a function of the neutrons and protons densities respectively and $\rho = \rho_n + \rho_p$ is the barionic density) as a function of density, temperature and isospin asymmetry has been one of the most challenging goals of the almost last two decades from both experimental and theoretical sides [1–4]. This is not surprising because EOS for neutron rich matter provides fundamental understanding in many
Figure 1: Binding energy as a function of barionic density in a DBHF calculation for different values of the isospin asymmetry $\beta$. Adapted from Ref. [6].

aspects of nuclear physics and astrophysics phenomena. The nuclear EOS for asymmetric (infinite) matter is well described by the parabolic approximation $E(\rho, \beta) = E_0(\rho, 0) + S(\rho)\beta^2$ [5], where $E_0$ is the energy for nucleon of the symmetric matter ($\beta = 0$) and $S(\rho)$ is the symmetry energy describing, as a function of density, the change in nuclear energy (EOS) associated with neutron-proton asymmetry. Fig. 1 shows by a DBHF (Dirac-Brückner-Hartree-Fock) calculation [6], the EOS as function of the barionic density for different values of the isospin asymmetry. The symmetry energy can be also defined as the difference in $E/A$ between the two extreme cases, $\beta = 1$, neutron matter and $\beta = 0$, symmetric matter.

The behavior of $S(\rho)$ is the key ingredient in many aspects: i) nuclear structure as in masses determination [7], neutron skin-radii in neutron rich nuclei [8], nuclear collective motions (giant or pygmy dipole resonances) [9]; ii) nuclear dynamics and reaction mechanisms in heavy ions collisions from low to relativistic energies (10-2000 MeV/A) [10]; iii) the physics of compact stars where, for example, the symmetry energy determines the pressure of a neutron star matter around $\rho_0$ [11]. Nevertheless, the symmetry energy is so far relatively unconstrained, in particular at supra-saturation densities. Difficulties are originated by several reasons: i) symmetry term effects are generally small with respect to isoscalar forces; $N/Z$ asymmetries of entrance channel that can be obtained in laboratory experiments are forcibly limited (also taking into account the use of expected radioactive beams). ii) The symmetry energy is generally extracted by indirect way by using observables that transport codes rely to the symmetry energy. iii) At supra saturation densities the effect of three body forces (TBF) become impor-
tant in microscopic calculations of the energy functional [12] by enhancing the stiffness of symmetry energy at increasing densities. However, despite impressive progress in this field [13] extrapolations at supra saturation densities are not yet straightforward. iv) There is a lack of experimental data at supra-saturation densities.

Heavy ion collisions offer a way to explore the isospin degree of freedom in laboratory controlled conditions because they give access to different densities that are reached during the dynamics of the collision. From a narrow range around the saturation densities down toward sub-saturation densities (from 0.1\(\rho_0\) in the Fermi energy domain, up to 2-3 times \(\rho_0\) in the relativistic energy domain). In the next section some aspects related to sensitivity of observables to symmetry energy in heavy ion collisions will be reported considering as main pathway the different barionic densities of nuclear matter that can be explored in heavy ion collisions.

2 Symmetry energy constraints from HIC

Nuclear structure studies provide information on symmetry energy only around normal or at subnormal saturation (\(\approx 2/3\rho_0\)) [14] density, for example, by fitting nuclear masses of finite nuclei or from the analysis of neutron-skins radii of neutron rich nuclei. The compression and expansions phases of nuclear matter in heavy ion collisions potentially provide to pin down the symmetry energy from sub-saturation to supra-saturation densities.

Fig. 2 (left panel) shows as significant prediction the time evolution of the density in a box around midrapidity region, obtained with a Stochastic Mean Field simulation [2] at 6 fm impact parameter in the reaction \(^{124}\text{Sn} + ^{64}\text{Ni}\) at 35 A.MeV. The plot shows a plateau close to \(\rho/\rho_0 = 1/3\) indicating the formation of a dynamical neck-like structure at sub-normal density in the midrapidity region in a short time scale [15]. In the right panel, a transport model calculation (IBUU04) [16] shows the evolution of barionic density as a function of collision time and incident beam energy for high-energy heavy ion central collisions. The density can span from 1.5 to 3.5 times with respect to the saturation density depending on the incident energy in the early phase of compression of the nuclear matter. The density dependence of \(S(\rho)\) can be expanded in a Taylor series with respect to the density: \(S(\rho) = S_0 + L_3 \left(\frac{\rho - \rho_0}{\rho_0}\right) + K_5 \left(\frac{\rho - \rho_0}{\rho_0}\right)^2 + \ldots\) where \(L\) and \(K\) are the slope and curvature parameters at \(\rho_0\) and \(S_0\) is the value of symmetry energy at \(\rho_0\). The slope of the symmetry energy at \(\rho_0\) is given by \(L = 3\rho_0 \left(\frac{dE_{\text{sym}}(\rho)}{d\rho}\right)_{\rho = \rho_0} \approx \frac{3f_0}{\rho_0}\) where
Figure 2: (left panel) SMF simulation showing the time evolution of the density at midrapidity in a semi-peripheral reaction at Fermi energies. (right panel) Calculation [16] of barionic density in the central collision (b=1 fm) $^{132}$Sn + $^{124}$Sn from 200 to 2000 A.MeV.

$P_0$ is the pressure of neutron matter at $\rho_0$. A value of the slope $L$ greater than 90-100 MeV refers to a “stiff” symmetry energy, while a small value of the slope $L$ ($L \approx 50$ MeV) is called “soft”. Under this latter assumption the potential part of symmetry energy vary slowly around saturation density. A theoretical linear correlation between $L$ and the neutron skin thickness of $^{208}$Pb has been found using different mean-field model simulations [8,17]. Indeed also in neutron stars the quantities $L$, $P_0$ and $R$ (the radius of a neutron star) are expected to be correlated [18]. Thus, putting stringent constraints to the density dependence of symmetry energy is not only a key requirement for dynamical models of heavy ion collisions but also for both nuclear structure studies of stable or exotic nuclei and for realistic astrophysical predictions on the EOS of compact stars.

At sub-saturation densities, in the Fermi energy domain, different experimental observables have been used to constraint the density dependence of the symmetry energy: isospin diffusion and equilibration [19, 20], neutron to proton ratio [21], transverse collective flow of light charged particles [22], ratio of fragments yields, isospin fractionation and isoscaling
in multi-fragmentation [23], heavy residue production in semi-central collisions [24], isospin migration of neutrons and protons between the low density “neck” region and denser matter in the proximity [25], cluster formation at very low densities [26]. Generally, in these experiments, projectiles and targets with the largest possible isospin asymmetries are studied, by using stable neutron-rich, neutron-poor isotopes or radioactive beams [27].

It is not the aim of this work to discuss in detail the relevant observables introduced above. Instead, we will evidence some aspects that can be a promising items for the next future.

i) **Clusterization at low density**: light ion clustering at very low density ($\rho/\rho_0 \leq 0.2$) have been recently studied [26] by using isospin asymmetric heavy ion collisions. The expansion and cooling of intermediate source in semi-central collisions produce light clusters whose temperature, density (determined by a coalescence technique [28]), and symmetry energy (determined indirectly by isoscaling analysis) are derived for the reconstructed source. At these low densities values, calculations based on quantum-statistical approach predict the formation of light clusters at densities ($\leq 1/10\rho_0$) and a finite value of symmetry energy and temperature. This gain in symmetry energy at low density due to the quantum cluster correlations seems to be in contrast with predictions within relativistic mean field approaches that indicates a symmetry energy linearly decreasing and vanishing at zero density. Recently it has been suggested to complement the coalescence approach with density determination (model dependent) by using particle-particle correlation techniques giving the size of emission source, thus exploiting the availability of next generations correlators (like FARCOS) and existing ones like MUST2 coupled to $4\pi$ detector like CHIMERA [29]. Indeed a connection is supposed between nuclear matter at low densities (similar temperature, densities and cluster correlations) and the neutrino sphere formed in core-collapse supernovae [17,30] thus opening new perspectives for these studies in terrestrial laboratories.

ii) **Neutron/Proton ratio**: The symmetry energy contribution of the mean field potential at sub-saturation density is predicted to be repulsive for neutron and attractive for protons [2]. Thus the ratio (or double ratio) of neutron over protons energy spectra (in c.m. system) is sensitive to the stiffness of the symmetry energy. In fact, comparisons of double $p-n$ ratios to ImQMD calculations have quantitatively constrained the symmetry energy [19]. However, problems related to this observable are mainly, neutron detections efficiency and the momentum dependence of the symmetry potential (mass splitting) that mimic the same effects of the symmetry potential at high $p_t$ [31]. Recently, it has been shown that energy spectra of light
clusters follow an isoscaling law related to local chemical potentials [32]. This scaling property could permit to measure ratios of charged particles (like \(t/^{3}He\)) as an equivalent alternative and/or complementary observable to n/p ratios. Anyway cluster production mechanism simulation is yet an open problem in transport models. Indeed the use of 4π detectors (for event characterization) with new generation neutron detectors and the possibility to probe different observables in the same experiment could improve also the constraining power of the n/p ratio observable with respect to available simulations.

iii) *Isospin diffusion and isospin migration*: isospin transport phenomena are related to the presence of a dilute region that is expected to be formed in the mid-rapidity region formed in semi-peripheral towards semi-central HIC; due to the density gradient with respect to normal saturation density, a "migration" of neutrons towards the dilute region is predicted, driven by the slope of the symmetry energy (that is larger for the *asy-stiff* parametrization than for the *ast-soft* parametrization at sub-saturation density). Conversely, if projectile and target have a large initial difference in N/Z asymmetry

Figure 3: Density dependence of the symmetry energy. The region enclosed by a thick line at high density refers to old Au+Au FOPI data on elliptic flow. The hashed magenta region to new ASYEOS experiment preliminary data. The blue hashed region (HIC) is mainly from Sn+Sn isospin diffusion data (see text).
the transport phenomena in dilute matter drives the isospin equilibration through the neck (isospin diffusion). Isospin diffusion relies on the magnitude of the symmetry energy at sub-saturation density and is larger for the asy-soft parametrization, thus isospin equilibration is predicted to be reached rapidly in asy-soft than in asy-stiff behavior. The degree of isospin diffusion is generally rescaled as an isospin transport ratio depending by the observable $X$ [19,33] which is linear dependent on isospin asymmetry (for example $^7\text{Li}/^7\text{Be}$ isobars) and by using different combinations of neutron-rich and proton-rich projectiles and targets. Isospin enrichment in the low density neck region has been investigated looking at the $N/Z$ of fragments which are dynamically emitted at midrapidity [25,34]. Estimation of the slope parameter $L$ has been obtained by comparison with prediction of transport models (BUU or stochastic mean field (SMF), QMD, AMD, Constrained Molecular Dynamics, etc). Advances in isospin equilibration studies are related to proposals to use radioactive beams in order to enhance the sensitivity to the symmetry energy and reduce the present uncertainties; this aspect is also shared with neck fragmentation studies. However, experimental direct observations of the time dependent matter density, as it is predicted by transport codes during the dynamical process and if fragments are produced in the dilute phase of matter, still remain a real challenge [15]. Finally careful comparison of different transport codes is also important in extracting information on the density dependence on symmetry energy. As an example of such comparison, it is noticed that ImQMD code produces IMFs in the neck region with much less neutron enrichment than SMF model and this latter model favors stronger isospin equilibration effects [35]. Thus error bars in density dependence of extracted values of the symmetry energy (should) reflect at present both experimental and theoretical uncertainties.

Fig. 3 shows a summary of density dependence of symmetry energy obtained from heavy ion reactions. Data at low densities (shown as blue shaded area) are in favor of values $30 \leq S_0 \leq 34$ MeV and $0.5 \leq \gamma \leq 1$ assuming that the symmetry energy is parameterized as a power low as a function of the $\gamma$ parameter: $S(\rho) = C_k \times (\rho/\rho_0)^{2/3} + C_p \times (\rho/\rho_0)^{\gamma}$ (1) where $C_k \approx 12.5$ MeV (as predicted by a free Fermi gas model) and the second term is the potential part of symmetry energy. These constraints were mainly extracted in Sn+Sn [3] collisions data on isospin diffusion. Constraints obtained till now by means of other observables by HIC generally also fall into these relatively large limits.

iv) Femtoscopy: particle-particle correlation functions at small relative angles are a promising probe because the particles emission sequence [36] as well the space-time properties of emitting source are sensitive to $S(\rho)$ [37].
But few experiments exist and the obtained results are still controversial. Requirements for such experiments are in fact a good event characterization, high angular and energy resolution correlator array, high statistics, possibility to detect also neutron channel for proton-neutron correlations, etc; conditions that are rarely simultaneously available in the same experimental setup till now. Thus, this is a challenge for the next future.

v) **Multifragmentation and isoscaling:** In multifragmentation events, measuring light particles and fragments yields formed in an equilibrated source at freeze-out at sub-saturation density allow, following statistical models, to measure the (free) symmetry energy, using methods such as isoscaling [23]. In this evaluation it is of crucial importance to deduce the primary partitions of hot fragments in the formed source at freeze-out from the detected secondary cold fragments. New advances in this aspect for experimentally reconstructed hot fragments have been recently presented [38]. Similar studies are also performed by INDRA-VAMOS collaboration.

Investigations at supra-saturation density (where few constraints exist till now), are expected to give non contradicting results when the full parametrization of EOS is applied to describe both astrophysical observa-
tions and heavy ion phenomenology. The largest set of data available around 1 A.GeV range, mainly on pions and light charged particles, is from FOPI collaboration [39]. The controversial interpretation with different transport models of $\pi^-/\pi^+$ yield ratio produced in Au+Au collisions from 0.4 to 1.5 A.GeV are well summarized in Ref. [40]. In Fig. 3 the region enclosed by red thick line shows the constraints obtained by comparison of neutron and proton elliptical flow ratio from Au+Au collision at 0.4 A.GeV from a recent re-analysis of FOPI-LAND data in comparison with predictions of the UrQMD model [41]. Results correspond to $S_0 = 34 MeV$ and $\gamma = 0.9 \pm 0.4$ values. The ASYEOS experiment at GSI [42] has as main goal to measure with high statistic and the most efficiently possible the neutrons and $Z=1$ elliptic flow and their ratio in $Au+Au$, $^{96}Ru+^{96}Ru$ and $^{96}Zr+^{96}Zr$ reactions at 0.4 A.GeV. Fig. 3 shows results of constraint on symmetry energy obtained by comparing neutron to Hydrogens elliptic flow ratio with UrQMD model predictions [43]. Preliminary results in the figure (dashed area) correspond to a value of $\gamma = 0.7 \pm 0.2$. A density range spanning between 1.2-2.0$\rho_0$ (at around 0.4 A.GeV) has been estimated by means of IQMD model [44] relative to proton elliptic flow observable. For this reason the area for flow measurements in Figs. 3-4 approximately covers this range. Finally in Fig. 4 the derivative of the symmetry energy, parameterized as in eq. (1), $L'(\rho) = 3\rho_0 \times dS/d\rho$, is plotted as a function of the barionic density for the same data of Fig. 3. Evidently at $\rho/\rho_0 = 1$ this plot gives estimations of the average value and error bar on the slope $L$ at saturation density. For example, $L = 83 \pm 26$ for FOPI-LAND data and $L = 70 \pm 13$ for ASYEOS data. New experiments planned at RIKEN with Samurai-TPC and future experiments with higher energies stable and radioactive beams of FAIR-GSI could open new perspectives in this field.

3 Summary

A brief review of experimental determination of symmetry energy by HIC has been presented in this paper. The availability in the next future of new facilities for radioactive beams, from low to relativistic energies and new experimental setups will improve the present both experimental and theoretical limitations by designing dedicated experiments in which many observables are measured and correlated together and by improving progressively transport model simulations. Advances are also expected by cooperative efforts going on among different fields (nuclear structure, heavy ion collisions, astronomical observations), adopting a common “language” and objectives.
related to the multi-facets aspects of symmetry energy phenomenology.

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References

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