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System size dependence of the log-periodic oscillations of transverse momentum spectra ⋆

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Abstract. Recently the inclusive transverse momentum distributions of primary charged particles were measured for different centralities in $Pb + Pb$ collisions. A strong suppression of the nuclear modification factor in central collisions around $p_T \sim 6 - 7$ GeV/c was seen. As a possible explanation, the hydrodynamic description of the collision process was tentatively proposed. However, such effect, (albeit much weaker) also exists in the ratio of data/fits, both in nuclear $Pb + Pb$ collisions, and in the elementary $p + p$ data in the same range of transverse momenta for which such an explanation is doubtful. As shown recently, in this case, assuming that this effect is genuine, it can be attributed to a specific modification of a quasi-power like formula usually used to describe such p_T data, namely the Tsallis distribution. Following examples from other branches of physics, one simply has to allow for the power index becoming a complex number. This results in specific log-periodic oscillations dressing the usual power-like distribution, which can fit the $p + p$ data. In this presentation we demonstrate that this method can also describe $Pb + Pb$ data for different centralities. We compare it also with a two component statistical model with two Tsallis distributions recently proposed showing that data at still larger p_T will be sufficient to discriminate between these two approaches.

1 Introduction

Recently the inclusive transverse momentum distributions of primary charged particles were measured for different centralities in $Pb + Pb$ collisions at $\sqrt{s_{NN}} = 2.76$ TeV [1, 2]. Data were presented in terms of a nuclear modification factor which exhibits strong suppression in the case of central collisions for p_T around $6 - 7$ GeV/c [1, 2]. For more peripheral collisions this suppression becomes weaker but never vanishes. Several theoretical models based on a hydrodynamic description of the medium were proposed to study and explain the effect [1, 2]. Because, for some time already, it became popular to fit the different kinds of transverse momentum spectra measured in multiparticle production processes using a statistical approach based on a nonextensive quasi-power Tsallis formula (cf. Eq. (2) below) [3–8], such an approach was also used. However, it turned out that one needs at least a two component statistical model with two Tsallis distributions [9, 10] (in these papers these two components were identified with, respectively, *soft* and *hard* dynamics of the underlining production process) [1].

In the mean time it was realized that, when looking at the ratio *data/fit* for data on the p_T distributions from $p + p$ collisions, taken in the same range of transverse momenta and at the same energies as data from $Pb + Pb$ collisions [12–14] (and which, as shown in [15], can be adequately fitted by a single Tsallis formula), one then observes strong modulation in $Pb + Pb$ data and a similar (albeit much weaker) modulation also in $p + p$ data. However, for $p+p$ data any explanation based on hydrodynamic models is doubtful. Therefore the natural question one can pose is whether a single nonextensive Tsallis formula so successful in fitting p_T data [3–8, 15], can also describe these results. As shown in [16, 17] the answer is positive, here we extend the analysis presented there for the $p + p$ collisions to analysis of dependence of the shape of the p_T spectra of charged hadrons produced in $Pb + Pb$ collisions and its dependence on the size of a colliding system.

Transverse momentum distributions measured in $p + p$ interactions exhibit for large p_T roughly a power-like behavior, whereas they become purely exponential for small p_T. For a long time already, for different reasons, it was found reasonable to use instead of two different formulas for these two parts of phase space (reflecting, as it is believed, different dynamics operating there), some single interpolating formula [18]. The most know at present is its version known as *QCD inspired Hagedorn form* [19] (with parameters: m and T)

$$h(p_T) = C \cdot \left(1 + \frac{p_T}{m \cdot T}\right)^{-m}. \quad (1)$$

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[1] Similar idea of multicomponent approach, but based on a revised two- or multi-Boltzmann distribution instead of Tsallis distribution, was presented also in [11] (albeit it was applied there only to a rather limited range of transverse momenta).

The other is a Tsallis formula [20] (with two parameters: q and T, C is a normalization constant)

$$f(p_T) = C \cdot \left[1 - (1-q)\frac{p_T}{T}\right]^{1/(1-q)}. \tag{2}$$

For our purposes, both formulas are equivalent for $m = 1/(q-1)$ (and we shall use them interchangeably.) They both represent the simplest way of describing the whole observed range of measured p_T distributions (and they both replace p_0, a momentum customarily used to separate *soft* and *hard* parts of the momentum phase space, by the parameter T, which can be interpreted either as a scale parameter for the hard component or a kind of temperature for the soft one). The best examples are the recent successful fits [15] to very large p_T data measured by the LHC experiments CMS [12], ATLAS [14] and ALICE [13] for $p+p$ collisions. However, albeit these fits look pretty good, closer inspection shows that the ratio of *data/fit* is not flat but shows some kind of specific log-periodic oscillations as a function of p_T [16, 17] (cf. also [21, 23]).

At this point one should remember that in other branches of science, whenever one encounters pure power-like distributions, one finds that in many cases these distributions are *decorated* by specific log-periodic oscillations, i.e., they are multiplied by some *dressing factor* R, which is customarily taken in the form of [22]:

$$R(p_T) = a + b\cos\left[c \cdot \ln(p_T + d) + f\right]. \tag{3}$$

(here presented as function of p_T, which is our variable).

2 Derivation of the dressing factor R for Tsallis distributions

Log-periodic oscillations are usually regarded as an indication of the presence of some hierarchical, multiscale, fine-structure, most probably of some kind of (multi) fractal origin. In what concerns oscillations apparently seen in [12–14] data, which is our case, it was assumed in [16, 17] that they are not an experimental artifact but, rather, that they are caused by some genuine dynamical effect and, as such, they should be studied carefully. The rationale was that these oscillations are seen by all experiments, at all energies at which data were taken, and show almost identical patterns. Now, in addition, they are changing with size of the colliding objects, as seen in the $Pb + Pb$ data. As in other places, where such oscillations were investigated for simple power-like distributions [22], we further assumed that to account for them the original Tsallis formula (either $h(p_T)$) from Eq. (1) or $f(p_T)$ from Eq. (2)) has to be multiplied by a log-oscillating function R, as given by Eq. (3), with the parameters connected to the original parameters of the respective form of Tsallis distribution used. For completeness of the presentation we shall repeat shortly its derivation (cf. [16, 17] for details).

Start from the simple pure power law distribution,

$$O(x) = C \cdot x^{-m} \tag{4}$$

This function is scale invariant, i.e.,

$$O(\lambda x) = \mu O(x) \tag{5}$$

with $m = -\ln\mu/\ln\lambda$. However, because $1 = \exp(\imath 2\pi k)$,

$$\mu\lambda^m = 1 = \exp(\imath 2\pi k), \quad k = 0, 1, \ldots, \tag{6}$$

i.e., it means that, in general, the index m can become complex,

$$m = -\frac{\ln\mu}{\ln\lambda} + \imath\frac{2\pi k}{\ln\lambda}. \tag{7}$$

Such form of the power index results in R as given by Eq. (3) when one keeps only $k = 0, 1$ terms [22].

However, Tsallis distribution is not a pure power-law but rather a quasi-power distribution, it contains a scale T for the variable considered and it has also a constant term, as in Eqs. (1) or (2). One must therefore find a variable in which our Tsallis distribution will show scaling property. It turns out that the evolution of the differential $df(p_T)/dp_T$ of a Tsallis distribution $f(p_T)$ with power index n performed for finite difference, $\delta p_T = \alpha(nT + p_T)$, replacing differential dp_T (here α is a kind of scaling factor, for $\alpha \to 0$ oscillations vanish) and using variable

$$x = 1 + \frac{p_T}{nT} \tag{8}$$

results in the desired scale invariant relation, which in our case takes the form of (cf. [16, 17] for details of derivation):

$$g[(1+\alpha)x] = (1 - \alpha n)\, g(x), \tag{9}$$

This means that one can write Eq. (1) in the form:

$$g(x) = x^{-m_k}, \tag{10}$$

with

$$m_k = -\frac{\ln(1-\alpha n)}{\ln(1+\alpha)} + \imath k\frac{2\pi}{\ln(1+\alpha)}, \tag{11}$$

or, more generally, as the sum

$$\begin{aligned} g(x) &= \sum_{k=0}^{\infty} w_k \cdot Re\left(x^{-m_k}\right) = \\ &= x^{-Re(m_k)} \sum_{k=0}^{\infty} w_k \cdot \cos\left[Im(m_k)\ln x\right]. \end{aligned} \tag{12}$$

Since we do not know *a priori* the details of the dynamics of processes under consideration (i.e. we do not know the weights w_k), in what follows we use (as before) only the two first terms, $k = 0$ and $k = 1$, getting dressed Tsallis distribution

$$g(p_T) = \left(1 + \frac{p_T}{nT}\right)^{m_0} \cdot R(p_T) \tag{13}$$

with dressing factor which now has the following form:

$$R(p_T) \simeq \left\{w_0 + w_1\cos\left[\frac{2\pi}{\ln(1+\alpha)}\ln\left(1 + \frac{p_T}{nT}\right)\right]\right\}. \tag{14}$$

The parameters in general modulating factor given by Eq. (3) are now identified as follows:

$$\begin{aligned} &a = w_0, \quad b = w_1, \quad c = 2\pi/\ln(1+\alpha), \\ &d = nT, \quad f = -c \cdot \ln(nT) \end{aligned} \tag{15}$$

Notice that this *dressing procedure* introduces three new parameters: scaling factor α and weights w_0 and w_1.

3 Transverse momentum distributions in Pb+Pb collisions

Recently, the inclusive transverse momentum distributions of primary charged particles are measured for different centralities in $Pb+Pb$ collisions at $\sqrt{s_{NN}} = 2.76$ TeV [1, 2], see Fig. 1. The data, presented in terms of nuclear modification factor, show strong suppression in central collisions for p_T around $6-7$ GeV/c [1, 2]. As shown in Fig. 1, these data can be fitted using a Tsallis distribution in the form of Eq. (1) with parameters as listed in Table 1. In Fig. 2 we show *data/fit* ratios, which exhibit rather dramatic log-oscillatory structure, increasing for most central collisions. As shown there it can be fitted, for all centralities, by using a dressing factor R as defined in Eq. (3), with parameters listed in Table 2.

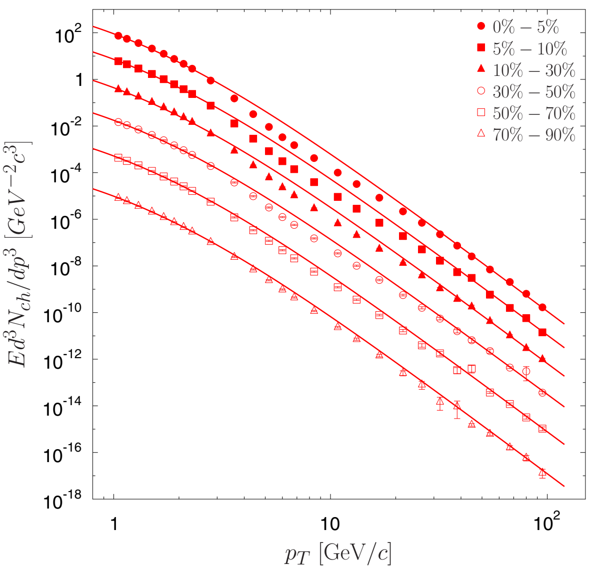

Figure 1. Transverse momentum distributions of particles produced in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV [13] fitted by Eq. (1). For better readability, results for different centralities are scaled by 10^{-i}, $i = 0, 1, 2, \cdots, 5$ from the most central to the most peripheral collisions.

Table 1. Parameters used in Eq. (1) to fit the spectra presented in Figs. 1-3

centrality [%]	C	m	T
0 – 5	11000	7.0	0.145
5 – 10	8750	6.95	0.145
10 – 30	5300	6.95	0.145
30 – 50	2100	6.9	0.145
50 – 70	625	6.95	0.145
70 – 90	119	7.0	0.145
$p+p$	22.65	7.1	0.145

From Fig. 2 one can deduce that the amplitude of the oscillating term in Eq. (3) reaches its maximum for the most central collisions, and smoothly decreases when going to more peripheral interactions. As seen in Fig. 3 there is also reasonable agreement between results from $p+p$ and most peripheral $Pb+Pb$ collisions. However, closer look at parameters reveal that parameters c and f differs substantially in both cases (cf. Table 2). This can be attributed to changes in the scaling parameter α in Eq. (15).

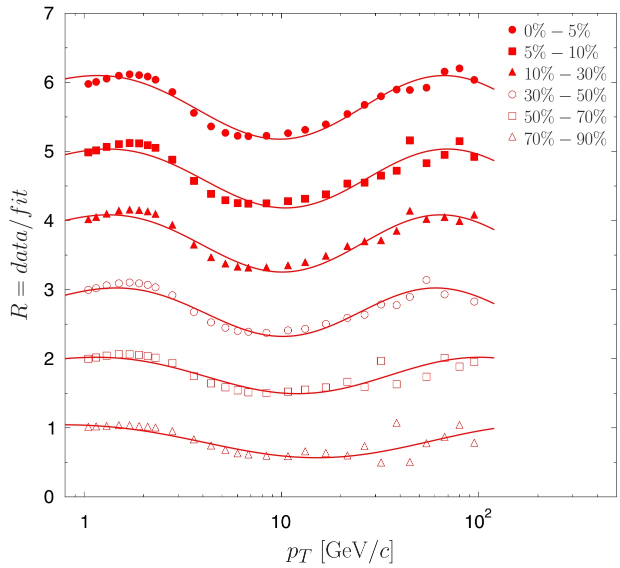

Figure 2. Data/fit ratio from the Fig. 1 fitted by Eq. (3). For better readability, results for different centralities are shifted by $i = 0, 1, 2, \cdots, 5$ from the most peripheral to the most central collisions.

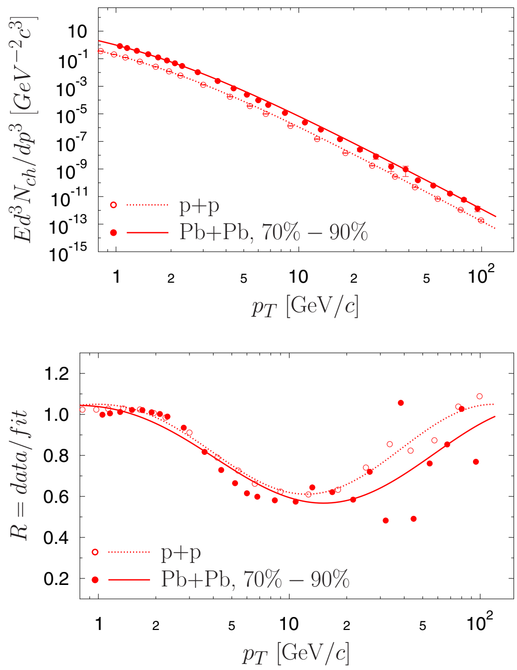

Figure 3. Comparison of results for p_T distributions in $p+p$ collisions with the most peripheral $Pb+Pb$ ones. Data taken from [13].

Table 2. Parameters used in Eq. (3) to fit the data/fit ratios presented in Figs. 2-3

centrality [%]	a	b	c	d	f
0 – 5	0.638	0.461	1.664	0.368	−0.719
5 – 10	0.609	0.426	1.690	0.368	−0.889
10 – 30	0.668	0.414	1.725	0.368	−0.910
30 – 50	0.673	0.351	1.789	0.368	−1.072
50 – 70	0.757	0.263	1.494	0.368	−0.622
70 – 90	0.806	0.238	1.199	0.368	−0.143
$p + p$	0.830	0.219	1.407	0.368	−0.428

3.1 Centrality dependence of parameters

When analyzing results from nuclear collisions it is customarily to look especially for their sensitivity on the number of nucleons participating in collision (i.e., participants, N_{part}) and on the number of binary nucleon-nucleon collisions, N_{coll}. On Fig. 4 the ratio of oscillating to constant term, b/a in Eq. (3), is presented as a function of N_{part}. One observes a smooth increase of this ratio when going to more central collisions. On Fig. 5 the same ratio is

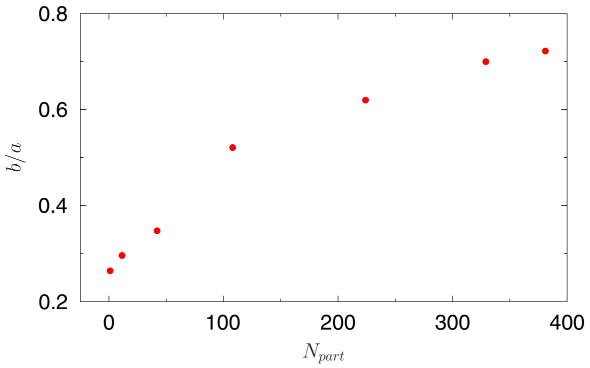

Figure 4. Ratio of the values of b/a parameters obtained from the fits presented on Fig. 2 as a function of number of nucleons participating in collision, N_{part}.

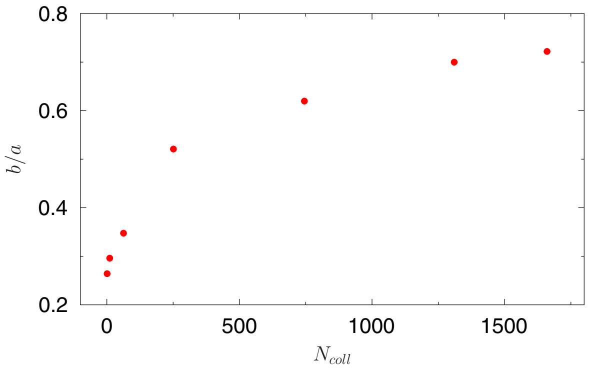

Figure 5. Ratio of the values of b/a parameters obtained from the fits presented on Fig. 2 as a function of number of collisions, N_{coll}.

plotted as a functions of N_{coll}. Again, there is a smooth

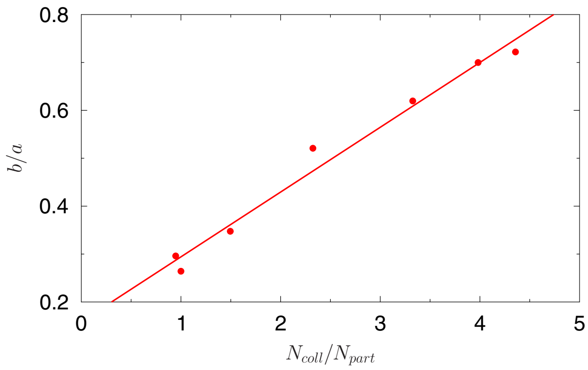

Figure 6. Ratio of the values of b/a parameters obtained from the fits presented on Fig. 2 as a function of number of collisions per participant nucleon, N_{coll}/N_{part}.

increase of the b/a ratio with increasing number of collisions. Interestingly, when plotting the ratio b/a as a function of the number of collisions per the number of participants, N_{coll}/N_{part}, one finds a visible linear increase, see Fig. 6. Such behavior suggests that the influence of oscillatory part increases with increasing percentage of binary collisions (usually attributed to hard scattering, possibly to particles produced from jets).

3.2 Two component Tsallis fit

As mentioned already before, recently a different method has been proposed to describe a structure clearly visible in the transverse momentum spectra obtained in $Pb + Pb$ collisions [9, 10] (see also [11]). Recognizing that a single Tsallis fit is not able to fully reproduce the observed structure, it was argued that one should resort to two power-laws, i.e., to two Tsallis distributions. According to these authors, they could be attributed to two possible mechanisms of particle production, also mentioned before, soft and hard, each with different sensitivity to N_{part}. The increased (as in our case) number of parameters allows them to obtain very good fits, cf., Fig. 7, where we present an example of the use of this method for the most central, $c = 0 - 5\%$, Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV using the following double-Tsallis formula:

$$h_2(p_T) = C_1 \cdot \left(1 + \frac{p_T}{m_1 T_1}\right)^{-m_1} + C_2 \cdot \left(1 + \frac{p_T}{m_2 T_2}\right)^{-m_2}, \quad (16)$$

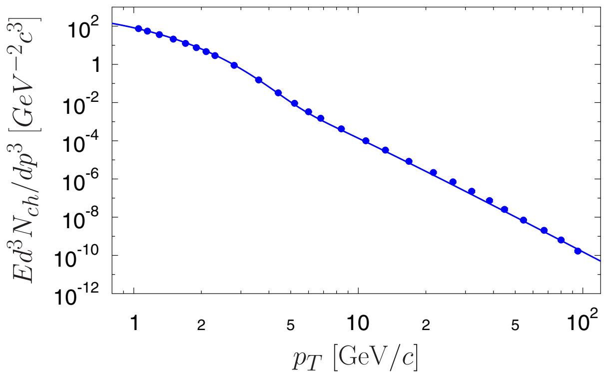

Figure 7. Transverse momentum distribution of particles produced in most central, $c = 0 - 5\%$, Pb+Pb collisions at $\sqrt{s_{NN}} =$ 2.76 TeV [13] fitted by Eq. (16) with parameters: $C_1 = 419.133$, $C_2 = 1100.0$, $T_1 = 0.158$, $T_2 = 0.369$, $m_1 = 6.172$, and $m_2 = 66.660$.

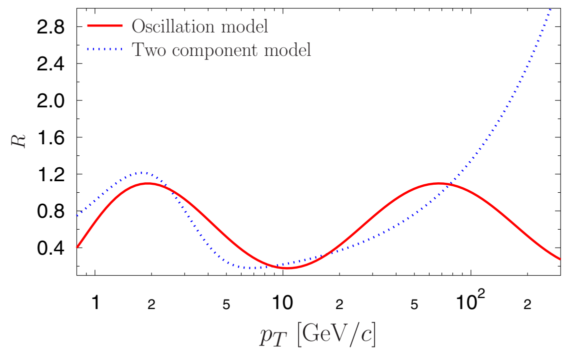

Figure 8. Ratios of two component fit to single Tsallis fit (dotted line), and Tsallis with oscillating term (Eq. (3)) for oscillation model (solid line) plotted for the most central, $c = 0-5\%$, Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV.

Table 3. Parameters used in Eq. (1) to fit the spectra presented

centrality [%]	C	m	T
0 – 5	11000	7.0	0.145
5 – 10	8750	6.95	0.145
10 – 30	5300	6.95	0.145
30 – 50	2100	6.9	0.145
50 – 70	625	6.95	0.145
70 – 90	119	7.0	0.145
$p + p$	22.65	7.1	0.145

The result presented on Fig. 7 is indeed encouraging. Regarding Eq. (16) as effectively the two first terms in a kind of expansion of our dressed Tsallis formula (13), it is not surprising that Eq. (16) vaguely follows our results, as can be seen in Fig. 8. It presents the comparison of the ratio of best fits obtaining from both formulas, respectively Eq. (16) and Tsallis formula with oscillating term, Eq. (3), to a single Tsallis fit, Eq. (1), plotted for the most central, $c = 0 - 5\%$, Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV.

However, both approaches diverge dramatically for large values of p_T. This is because the log-oscillating formula (14) contains term which can be positive or negative whereas the two-Tsallis one is always positive and therefore the ratio plotted in Fig. 7 must ultimately grow as a function of p_T. Should the data in that region be available, it would allow us to decide which model better describes the mechanism of particle production in relativistic heavy ion collisions.

4 Summary

Recently, the inclusive transverse momentum distributions of primary charged particles were measured for different centralities in $Pb + Pb$ collisions at $\sqrt{s_{NN}} = 2.76$ TeV. Data presented in terms of the nuclear modification factor show a strong suppression in central collisions for p_T around 6 – 7 GeV/c. The dependence of the shape of the p_T spectra on the size of a colliding system has been discussed using Tsallis distribution as a reference spectrum. It is remarkable that this kind of suppression is also observed in $p + p$ collisions where the mechanism of particle production is believed to be different. This suppression, visualized as possibly signal of log-oscillatory behavior known from other branches of physics, has been attributed there to imaginary part in the Tsallis power index. When analyzing $Pb + Pb$ data along the same lines, one gets that the amplitude of the corresponding (much stronger than in $p + p$ case) oscillations increases linearly as a function of number of collisions per participant nucleon, N_{coll}/N_{part}. We compared our results with recent proposition of using two-power laws Tsallis fits to describe such data and proposed way of experimental differentiating between these approaches.

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