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Quasi One-Dimensional Model of Natural Draft Wet-Cooling Tower Flow, Heat and Mass Transfer

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Abstract. The article deals with the development of CFD (Computational Fluid Dynamics) model of natural draft wet-cooling tower flow, heat and mass transfer. The moist air flow is described by the system of conservation laws along with additional equations. Moist air is assumed to be homogeneous mixture of dry air and water vapour. Liquid phase in the fill zone is described by the system of ordinary differential equations. Boundary value problem for the system of conservation laws is discretized in space using Kurganov-Tadmor central scheme and in time using strong stability preserving Runge-Kutta scheme. Initial value problems in the fill zone is solved by using standard fourth order Runge-Kutta scheme. The interaction between liquid water and moist air is done by source terms in governing equations.

1 Introduction

The principle of evaporative cooling is connected with releasing of latent heat during water evaporation. Releasing of latent heat is leading to an increase in moist air temperature which is connected with decrease of moist air density. Water evaporation is directly connected with an increase of moist air humidity. The density of warmed moist air is lower unlike the density of surrounding moist air. This density difference together with sufficient cooling tower height is creating natural draft. Typical natural draft cooling tower can be larger then 100 m in height and diameter.

The enthalpy of superheated water vapour at low vapour pressures is very close to enthalpy of saturated vapour h''_v corresponding to given temperature [1]

$$h_v = h''_v(T) = h'_w(T) + l(T), \tag{1}$$

h'_w is enthalpy of saturated water liquid and $l(T)$ is latent heat of vaporisation at temperature T. The difference in water liquid enthalpy $h_w = h'_w$ and water vapour enthalpy $h_v = h''_v$ clearly illustrate evaporative water cooling. Necessary condition for evaporative cooling is existence of mass transfer between water and moist air which is connected with difference in saturated specific humidity at water temperature $x''(t_w)$ and moist air specific humidity x

$$(x''(t_w) - x)) > 0. \tag{2}$$

This idea is based on assumption about existence of thin film of saturated moist air at water temperature on the interface of water and moist air.

Various models of natural draft cooling tower exist. Classical approach is described in reference [2], where simple algebraic draft equation is used to calculate moist air mass flow rate. An example of more complex approach based on commercial CFD code is the work [3]. As an example of other code can be mentioned e.g. work [4]. Current work can be understand as continuation of previous work of author, see e.g. [5], [6]. This work is leading to relatively complex CFD model of natural draft cooling tower flow, heat and mass transfer.

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2 Governing equations

The governing equations are describing the flow of homogeneous mixture of dry air and water vapour as in the work [5]. System of governing equations for quasi one-dimensional flow is

$$\frac{\partial(\mathbf{W}A)}{\partial t} + \frac{\partial(\mathbf{F}A)}{\partial z} = \mathbf{Q}, \tag{3}$$

where vector of conservative properties $\mathbf{W}$ and vector of fluxes $\mathbf{F}$ are

$$\mathbf{W} = \begin{bmatrix} \rho \\ \rho v \\ \rho w_v \\ \rho e \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} \rho v \\ \rho v^2 + p \\ \rho v w_v \\ v(\rho e + p) \end{bmatrix}. \tag{4}$$

Vector of sources is

$$\mathbf{Q} = \begin{bmatrix} A(z)\sigma_v(z) \\ p\frac{\mathrm{d}A}{\mathrm{d}z} - A(z)(\rho g + \zeta\rho\frac{v^2}{2}) \\ A(z)\sigma_v(z) \\ A(z)(\sigma_{qmodel}(z) + \rho g v) \end{bmatrix}. \tag{5}$$

First equation is overall continuity equation, second equation is momentum equation, third equation is water vapour continuity equation and the last equation is equation of energy. We define ad hoc internal energy of unsaturated humid air as

$$u = (w_a c_{v_a} + w_v c_{v_v})T = c_v T. \tag{6}$$

Water vapour mass fraction w_v is

$$w_v = \frac{x}{1+x} \tag{7}$$

and dry air mass fraction is

$$w_a = 1 - w_v. \quad (8)$$

The total energy of unsaturated moist air is expressed in the form

$$\rho e = \rho \left(c_v T + \frac{v_k v_k}{2} \right). \quad (9)$$

Pressure can be calculated using equation of state as

$$p = \frac{\rho R T \left(1 + \frac{M_a}{M_v} x\right)}{M_a (1 + x)} = \rho R T \left(\frac{M_v - w_v (M_v - M_a)}{M_a M_v} \right), \quad (10)$$

where M_v is molecular weight of water vapour and M_a is molecular weight of dry air.

3 Fill heat and mass transfer

The model of heat and mass transfer derived in this section is more advanced than previously used model [5] and it is easier to combine it with governing equations of moist air flow.

3.1 Derivation of governing equations

Infinitesimal contact area can be expressed using vertical coordinate z as

$$\mathrm{d}A = a_q A_{fr} \mathrm{d}z, \quad (11)$$

where a_q is transfer area per unit volume and A_{fr} is cross sectional area of the fill. Mass transfer can be expressed by using mass transfer coefficient α_m, the difference of saturated specific humidity at water temperature $x''(t_w)$ and moist air specific humidity x and contact area dA as

$$\mathrm{d}\dot{m}_w = \alpha_m a_q A_{fr} (x''(t_w) - x). \quad (12)$$

Change in water mass flow rate with height z is then

$$\frac{\mathrm{d}\dot{m}_w}{\mathrm{d}z} = \alpha_m a_q A_{fr} (x''(t_w) - x). \quad (13)$$

Energy balance between water and moist air can be expressed as

$$\dot{m}_a \mathrm{d}h_{1+x} = \dot{m}_w \mathrm{d}h_w + h_w \mathrm{d}\dot{m}_w = \alpha (t_w - t_a) \mathrm{d}A + h_v(t_w) \mathrm{d}\dot{m}_w, \quad (14)$$

where h_{1+x} is moist air enthalpy, h_w is enthalpy of water, α is heat transfer coefficient, t_w is water temperature, t_a is moist air temperature and $h_v(t_w)$ is enthalpy of water vapour at water temperature t_w. Previous equation can be rewritten as

$$\dot{m}_w \mathrm{d}h_w = \alpha (t_w - t_a) \mathrm{d}A + l(t_w) \mathrm{d}\dot{m}_w, \quad (15)$$

with latent heat of vaporisation $l(t_w)$. The change in water temperature can be using previous equation and using equation for change in water mass flow rate expressed as

$$\frac{\mathrm{d}t_w}{\mathrm{d}z} = \frac{a_q A_{fr}}{\dot{m}_w c_w} \left[\alpha (t_w - t_a) + \alpha_m (x''(t_w) - x) l(t_w) \right]. \quad (16)$$

Further modifications of governing equations are based on the definition of Lewis factor [2]

$$Le_f = \frac{\alpha}{\alpha_m c_p}, \quad (17)$$

where

$$c_p = \frac{1}{1+x} c_{p_a} + \frac{x}{1+x} c_{p_v}. \quad (18)$$

Change in water temperature can be expressed using the definition of Lewis factor as

$$\frac{\mathrm{d}t_w}{\mathrm{d}z} = \frac{\alpha_m a_q A_{fr}}{\dot{m}_w c_w} \left[Le_f c_p (t_w - t_a) + (x''(t_w) - x) l(t_w) \right]. \quad (19)$$

Merkel number [2] can be in general expressed as

$$Me = \int_0^A \frac{\alpha_m}{\dot{m}_w} \mathrm{d}A = \int_0^{h_{fill}} \frac{\alpha_m a_q A_{fr}}{\dot{m}_w} \mathrm{d}z, \quad (20)$$

where h_{fill} is height of the fill. Averaged value of non dimensional mass transfer coefficient is

$$\frac{\alpha_m a_q A_{fr}}{\dot{m}_w} = Me / h_{fill}. \quad (21)$$

Governing equations for water mass flow rate can be simplified using known value of Merkel number Me as

$$\frac{\mathrm{d}\dot{m}_w}{\mathrm{d}z} = Me / h_{fill} \, \dot{m}_w (x''(t_w) - x), \quad (22)$$

$$\frac{\mathrm{d}t_w}{\mathrm{d}z} = \frac{Me / h_{fill}}{c_w} \left[Le_f c_p (t_w - t_a) + (x''(t_w) - x) l(t_w) \right]. \quad (23)$$

3.2 Calculation of source terms

Because the general mass balance between water and water vapour in infinitesimal control volume of height dz is

$$\mathrm{d}\dot{m}_w = \dot{m}_a \mathrm{d}x, \quad (24)$$

we can express mass source as

$$\sigma_v = \frac{\dot{m}_a}{A(z)} \frac{\mathrm{d}x}{\mathrm{d}z} = \frac{\alpha_m a_q A_{fr}}{A(z)} \left(x''(t_w) - x \right), \quad (25)$$

and after manipulation

$$\sigma_v = \frac{Me / h_{fill}}{A(z)} \dot{m}_w \left(x''(t_w) - x \right). \quad (26)$$

Heat source is

$$\sigma_q = \frac{\dot{m}_a}{A(z)} \frac{\mathrm{d}h_{1+x}}{\mathrm{d}z}. \quad (27)$$

Sensible heat source can be expressed as

$$\sigma_{qs} = \frac{\dot{m}_a}{A(z)} \frac{\mathrm{d}h_{1+x} - l(t_a) \mathrm{d}x}{\mathrm{d}z}. \quad (28)$$

After substitution is sensible heat source

$$\sigma_{qs} = \frac{1}{A(z)} \left[\alpha a_q A_{fr} (t_w - t_a) + \frac{\mathrm{d}\dot{m}_w}{\mathrm{d}z} \left(l(t_w) - l(t_a) \right) \right]. \quad (29)$$

Finally we get

$$\sigma_{qs} = \frac{\alpha a_q A_{fr}}{A(z)} (t_w - t_a) + \sigma_v \left[l(t_w) - l(t_a) \right]. \quad (30)$$

After application of Lewis factor Le and Merkel number Me we have

$$\sigma_{qs} = Le_f \frac{Me / h_{fill}}{A(z)} \dot{m}_w c_p (t_w - t_a) + \sigma_v \left[l(t_w) - l(t_a) \right]. \quad (31)$$

Because internal energy in the model of moist air flow in section 2 is calculated from $0K$ we have to modify heat source as

$$\sigma_{qmodel} = \sigma_{qs} + 273.15 c_{p_v} \sigma_v. \quad (32)$$

4 Numerical solution

The numerical method used in the previous work [5] has severe problem close to discontinuities connected with existence of unphysical oscillations. The more sophisticated numerical method described in this section is chosen to overcome this problem.

4.1 Numerical method

Numerical solution of moist air flow uses cell-centered Finite Volume Method in conservative form. Semi-discrete scheme of Kurganov & Tadmor [7] is chosen. System of governing equations (3) can be rewritten as

$$\frac{\mathrm{d}\mathbf{W}_i}{\mathrm{d}t} + \frac{1}{\mu_i}\left[\mathbf{F}^*_{i+1/2}A_{i+1/2} - \mathbf{F}^*_{i-1/2}A_{i-1/2}\right] = \frac{\Delta z_i}{\mu_i}\mathbf{Q}_i, \quad (33)$$

where

$$\mu_i = \Delta z \frac{A_{i+1/2} + A_{i-1/2}}{2}. \quad (34)$$

Numerical fluxes are of Rusanov kind [7] and for control volume face $k = i + 1/2,\ i - 1/2$ we have

$$\mathbf{F}^*_k = \frac{1}{2}\left[\mathbf{F}(\mathbf{W}^R_k) + \mathbf{F}(\mathbf{W}^L_k)\right] - \frac{a_k}{2}\left[\mathbf{W}^R_k - \mathbf{W}^L_k\right], \quad (35)$$

where local propagation speed $a_{i\pm1/2}$ is maximum value of wave speed over cells i, i+1 and i−1. Values on left side and on right side of cell edge are based on linear reconstruction

$$\mathbf{W}(z) = \mathbf{W}_i + \mathbf{s}_i(z - z_i),\ for\ \ z_{i-1} < z < z_{i+1}. \quad (36)$$

The slopes $\mathbf{s}_i$ are calculated using generalized minmod limiter

$$\mathbf{s}_i = \mathrm{minmod}(\theta\mathbf{s}^-, \mathbf{s}^C_i, \theta\mathbf{s}^+_i),\ \ 1 \le \theta \le 2, \quad (37)$$

where θ determines the value of chosen slope. Value of θ higher then 1 is leading to lower artificial viscosity. It has been shown that scalar second order semi-discrete central scheme satisfies total variation diminishing (TVD) property for θ up to 2 [7]. Slopes are calculated as

$$\mathbf{s}^-_\mathbf{i} = \frac{\mathbf{W}_i - \mathbf{W}_{i-1}}{z_i - z_{i-1}}, \quad (38)$$

$$\mathbf{s}^\mathbf{C}_\mathbf{i} = \frac{\mathbf{W}_{i+1} - \mathbf{W}_{i-1}}{z_{i+1} - z_{i-1}}, \quad (39)$$

$$\mathbf{s}^+_\mathbf{i} = \frac{\mathbf{W}_{i+1} - \mathbf{W}_i}{z_{i+1} - z_i}. \quad (40)$$

Generalized minmod limiter is defined as

$$\mathrm{minmod}(a,b,c) = \begin{cases} \min(a,b,c), & if\ a,b,c \ge 0, \\ \max(a,b,c), & if\ a,b,c \le 0, \\ 0, & \text{otherwise.} \end{cases} \quad (41)$$

Semi-discrete system (33) for every control volume can be written as

$$\frac{\mathrm{d}\mathbf{W}}{\mathrm{d}t} = L(\mathbf{W}). \quad (42)$$

Temporal integration of system (42) in control volume is based on four stage order three explicit strong stability preserving Runge-Kutta method with nonnegative coefficients according to [8]

$$\mathbf{W}^{(1)} = \mathbf{W}^n + \frac{1}{2}\Delta t L(\mathbf{W}^n), \quad (43)$$

$$\mathbf{W}^{(2)} = \mathbf{W}^{(1)} + \frac{1}{2}\Delta t L(\mathbf{W}^{(1)}), \quad (44)$$

$$\mathbf{W}^{(3)} = \frac{2}{3}\mathbf{W}^n + \frac{1}{3}\mathbf{W}^{(2)} + \frac{1}{6}\Delta t L(\mathbf{W}^{(2)}), \quad (45)$$

$$\mathbf{W}^{n+1} = \mathbf{W}^{(3)} + \frac{1}{2}\Delta t L(\mathbf{W}^{(3)}). \quad (46)$$

This scheme has CFL coefficient of 2, but in the work of Kurganov & Tadmor [7] it has been shown that combination of Runge-Kutta method with their method can lead to severe restriction of CFL.

Boundary conditions are applied similarly like in the work [5].

Ordinary differential equation (22) for water mass flow rate and (23) for water temperature are solved simultaneously with system of governing equations for moist air flow. Integration of equations (22) and (23) is based on standard four step fourth order Runge-Kutta method according to [9]. Simultaneous solution of moist air flow and water mass flow rate and temperature equations continues until steady state is reached.

5 Results

Rotational hyperboloid shaped natural draft cooling tower with

$$A(z) = \frac{\pi}{4}\left(0.006977z^2 - 1.2764z + 131.61\right)^2 \quad (47)$$

is selected [5]. Tower is 150 m high with fill zone placed at height of 11.5 m. Calculation was performed for fill height of 2 m. Water inlet mass flow rate is $\dot{m}_w = 17200$ kg/s. Inlet water temperature is $t_{wi} = 34.9$°C. Air inlet temperature is $t_{ai} = 22$°C and specific humidity at air inlet is $x_i = 7.622$ g/kg. The atmospheric pressure is $p = 98100$ Pa. Merkel number is assumed to be $Me = 0.815$, Lewis factor is $Le_f = 0.9$ and loss coefficient per meter of the fill zone is $\zeta = 12$ m^{-1}.

Numerical simulation uses CFL = 1.6 and $\theta = 2$ set on the basis of numerical experiments. Water temperature decreases in the fill zone to 26.41°C and its mass flow rate decreases to 16985 kg/s. There is a dry air mass flow rate of 16392.9 kg/s in natural draft cooling tower. The heat addition in the fill zone is connected with the decrease in density as shown in the figure 1 and with the temperature increase in the figure 2. The decrease in water temperature in fill zone is visible in the figure 3 where the temperature of moist air is increasing similarly like adiabatic saturation temperature, which is lower then air temperature. The mass transfer between flowing water and moist air is documented in the figure 4 with jump in the specific humidity in the fill zone. The distributions of adiabatic saturation temperature in the figure 2 and the distribution of saturated specific humidity in the figure 4 indicate unsaturated moist air inside cooling tower. Velocity distribution in the figure 5 is reflecting primarily the shape of the cooling tower. There is relatively small increase in velocity in the fill zone compensating decrease in the moist air density.

6 Conclusions

Model of natural draft wet cooling tower flow, heat and mass transfer is developed. The numerical method based

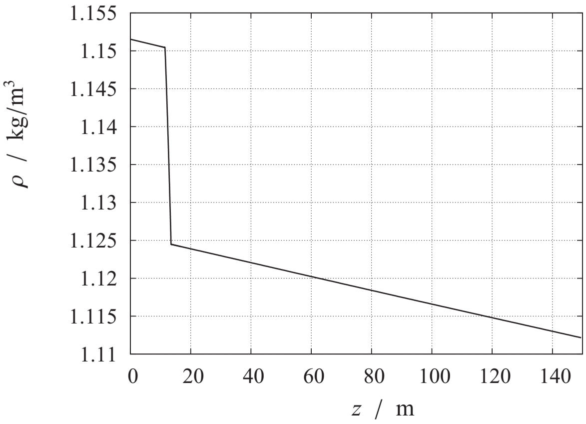

Fig. 1. Density of moist air distribution inside natural draft cooling tower

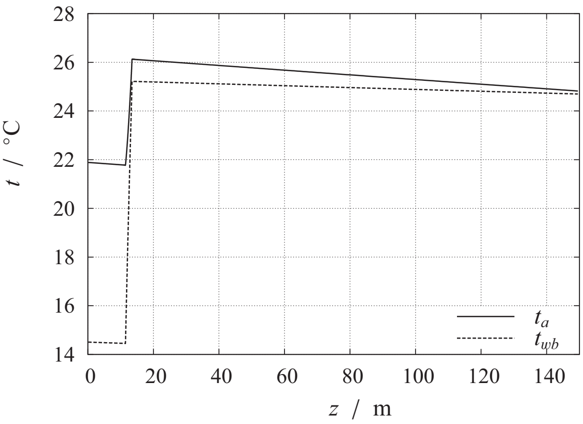

Fig. 2. Distribution of moist air temperature t_a and adiabatic saturation temperature t_{wb}

on semi-discrete Kurganov & Tadmor scheme is used. The key part of the model is simultaneous solution of moist air flow equations and water mass flow rate and temperature equations. The results of numerical solution of ODEs for water temperature and mass flow rate allows to calculate sources in governing equations for moist air flow. The limitations of present approach are mainly connected with the definition of moist air as mixture of dry air and water vapour which is not sufficient especially in the case of supersaturated air. Therefore, it is selected the case where supersaturation does not occur.

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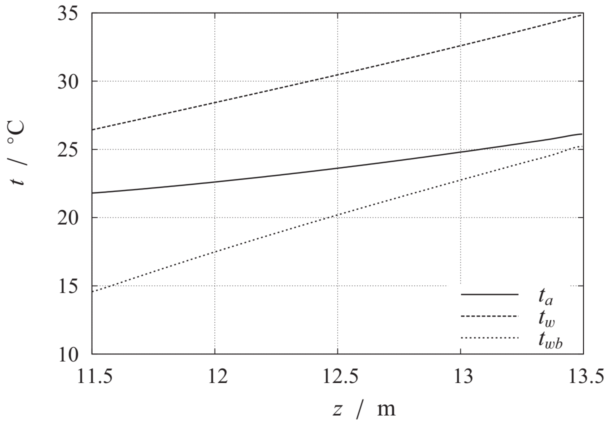

Fig. 3. Distribution of moist air temperature t_a, water temperature t_w and adiabatic saturation temperature t_{wb} in fill zone

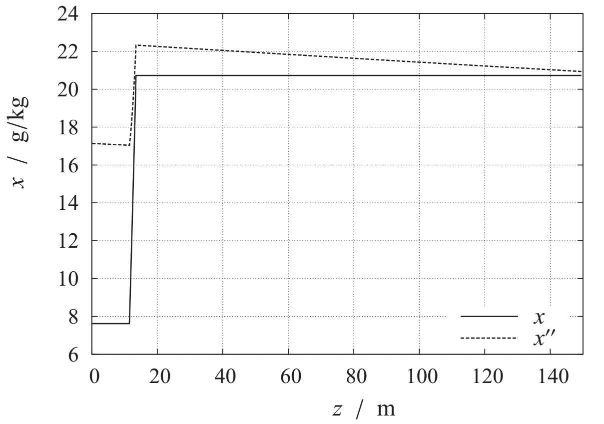

Fig. 4. Distribution of specific humidity x and saturation specific humidity at moist air temperature $x''(t_a)$

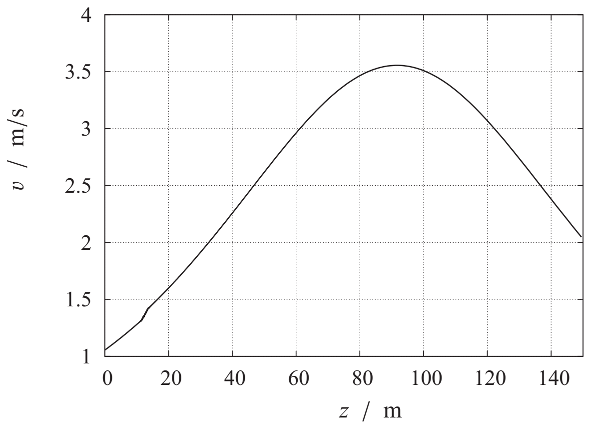

Fig. 5. Natural draft cooling tower velocity distribution

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