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Disentangling the nuclear shape coexistence in even-even Hg isotopes using the interacting boson model

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Abstract. We intend to provide a consistent description of the even-even Hg isotopes, $^{172-200}$Hg, using the interacting boson model including configuration mixing. We pay special attention to the description of the shape of the nuclei and to its connection with the shape coexistence phenomenon.

1 Introduction

Shape coexistence has been observed in many mass regions throughout the nuclear chart and turns out to be realized in more nuclei than anticipated a few decades ago [1].

Recently, a lot of new results have become available for the even-even Po, Hg and Pt nuclei, for which experimental information was highly needed. In this mass region, the intruder bands are easily singled out for the Pb and Hg nuclei and the excitation energies display the characteristic parabolic pattern with minimal excitation energy around the $N = 104$ neutron mid-shell nucleus. In the case of Hg there is an intense experimental activity for the light isotopes near the mid-shell region, which is providing a very complete set of excitation energies, E2 transition rates, isotopic shifts, α-hindrance factors, etc. These new results are painting a new landscape where the inclusion of intruder states is a key ingredient to understand the physics of this mass region. In particular, in a recent COULEX experiment for $^{184-188}$Hg [2], very detailed information on the shape of these nuclei has been obtained.

In a set of previous articles we studied the Pt [3–5] and the Hg [6] nuclei extensively with the Interacting Boson Model (IBM) [7], incorporating proton 2p–2h excitations (IBM-CM) [8]. The conclusion of these studies was that configuration mixing in the Pt nuclei is somehow "concealed", while in the case of Hg its main effect does not appear for the ground state but for the first two 2^+ states.

The IBM-CM allows the simultaneous treatment and mixing of several boson configurations which correspond to different particle–hole shell-model excitations [8]. Hence, the model space corresponds to a $[N]\oplus[N+2]$ boson space. The boson number N is obtained as the sum of the number of active protons (counting both proton particles and holes) and the number of valence neutrons, divided by two. Thus, the Hamiltonian for two-configuration mixing is written as

$$\hat{H} = \hat{P}^{\dagger}_{N}\hat{H}^{N}_{\rm ecqf}\hat{P}_{N} + \hat{P}^{\dagger}_{N+2}\left(\hat{H}^{N+2}_{\rm ecqf} + \Delta^{N+2}\right)\hat{P}_{N+2} + \hat{V}^{N,N+2}_{\rm mix}\,, \tag{1}$$

where $\hat{P}_N$ and $\hat{P}_{N+2}$ are projection operators onto the $[N]$ and the $[N+2]$ boson spaces, respectively, $\hat{V}^{N,N+2}_{\rm mix}$ describes the mixing between the $[N]$ and the $[N+2]$ boson subspaces, $\hat{H}^{i}_{\rm ecqf}$ is the extended consistent-Q Hamiltonian (ECQF) with $i = N, N{+}2$ (see [7]), and Δ^{N+2} can be associated with the energy needed to excite two particles across the $Z = 82$ shell gap.

Within this formalism we have performed a fit to the excitation energies and $B(E2)$ transition rates of $^{172-200}$Hg in order to fix the parameters for the IBM-CM Hamiltonian. The results from the fitting procedure are summarized in Table 3 of Ref. [6] and they will be used in the calculations shown in the present contribution.

Our aim is to describe the shape evolution of Hg isotopes, paying special attention to the mid-shell nuclei, $^{180-188}$Hg. To do so, we present results i) from the IBM-CM coherent state formalism, ii) for the quadrupole deformation parameter β extracted from the experimental $B(E2)$ values, and, iii) on quadrupole shape invariants.

2 Shape evolution from different approaches

The IBM-CM calculations, carried out in Ref. [6], display a strongly evolving character of the wave function in the $[N]$ and $[N+2]$ space along the Hg isotope chain. The lightest and the heaviest Hg isotopes show a rather pure $[N]$ composition for the lowest lying states, while the isotopes near the mid-shell, $^{180-186}$Hg, present a mixed character (see Fig. 12 of Ref.[6]), especially for the lowest two 2^+ states. These changes in the wave function are expected to modulate the deformation of the nucleus.

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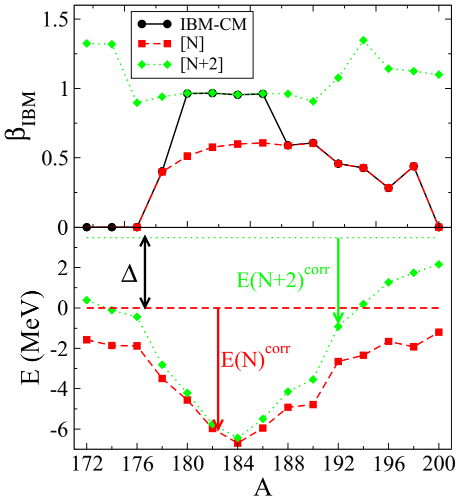

Figure 1. Upper part: value of β_{IBM} from the IBM-CM mean-field calculation. Lower part: absolute energy of the lowest regular and lowest intruder states for $^{172-200}$Hg. The arrows correspond to the correlation energies in the N and N+2 subspaces.

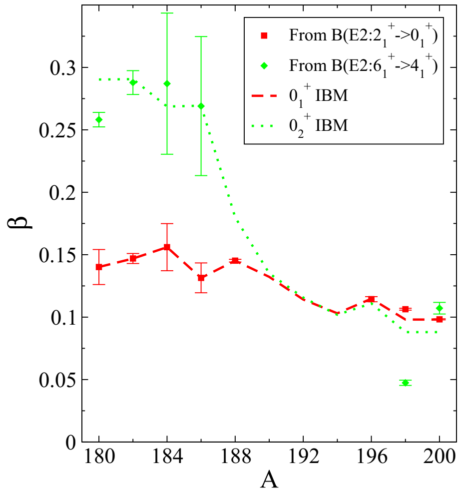

Figure 2. Value of β extracted from $B(E2)$ values.

A first approach to disentangle the shape evolution in Hg isotopes results from the geometric interpretation of the IBM, which is obtained within the intrinsic state formalism, proposed by Ginocchio *et al.* [9]. In this approach one derives the equilibrium value of the shape variables β_{IBM} (deformation) and γ_{IBM} (degree of triaxiality) by means of a variational procedure. These variables can be connected with the standard Hill-Wheeler variables [5, 9], in particular, β_{IBM} is directly connected with the deformation parameter of the collective model, β, while $\gamma_{IBM} = \gamma$. To study the geometry of the IBM-CM, Frank *et al.* [10] proposed a new method which takes into account the existence of two families of states.

We have calculated the energy surface for the Hg chain of isotopes and the corresponding equilibrium value of the deformation parameter β_{IBM}. In the upper part of Fig. 1 we present the value of β_{IBM} for the unperturbed configurations (omitting the mixing term in the Hamiltonian), as the dashed line with squares, for the regular states, while using a dotted line with diamonds for the intruder states. It is clear that the unperturbed configurations result in a very different value of β_{IBM} (around 0.5 for the regular state and 1.0 for the intruder one) and that the full IBM-CM varies between the regular value at the beginning and the end of the shell, and the intruder one at the mid-shell region.

This can be partially understood inspecting the bottom part of Fig. 1, where the unperturbed energies for the lowest two 0^+ states are shown. One observes very close-lying unperturbed configurations near mid-shell, however, contrary to the situation in the Pt nuclei, the intruder state never becomes the ground-state. Note that the full IBM-CM energy, though not shown, very closely follows the regular unperturbed configuration in Fig. 1 (lower part). Combining the upper (deformation) and the lower part (energies), this looks like a *conundrum*. As we shall explain later, this results from the fact that energies obtained from mean-field calculations do not include the full dynamics, which is particularly important in the case of two close-lying configurations, as is the case here. The equilibrium value of γ for the whole chain has also been calculated, giving rise to an oblate shape at the beginning and at the end of the shell, while prolate around the mid-shell region, *i.e.*, $^{180-186}$Hg.

In a second approach, we use a phenomenological interpretation to extract information on the quadrupole deformation β. The key point is to consider the geometrical view of the nucleus and to extract the β parameter for a given band, either regular or intruder, from a known $B(E2)$ value

$$\beta = \frac{\sqrt{B(E2 : I_i \rightarrow I_f)}}{\langle I_i K 2 0 | I_f 0\rangle \frac{3}{4\pi} Z e R_0^2}, \qquad (2)$$

where $R_0 = 1.2A^{1/3}$ fm and $< ....|.. >$ is the Clebsch-Gordan coefficient. Around the mid-shell region we assume that 0_1^+ and 2_1^+ states belong to the regular band while the 6_1^+ and 4_1^+ states belong to the intruder band. In Fig. 2, we depict the value of β using this method. One can easily single out the presence of two configurations with very different deformation; the lower configuration, which corresponds to the regular state, is less deformed than the higher one and can be identified with an intruder state. This is an empirical evidence about the nature of the ground state, which, close to mid-shell, always corresponds to a less deformed and regular configuration, in agreement with the lower panel of Fig. 1, confirming the failure of the IBM mean-field calculation in providing a correct value of β_{IBM}. Therefore, the ground state is always regular, while the 0_2^+ band corresponds to an intruder configuration near mid-shell, although the β value drops dramatically from $A = 188$ and onwards, denoting that the character of this band is no longer intruder, but changes in character.

In a third approach, we construct quadrupole invariants to extract information about the nuclear deformation in a model independent way [11, 12]. Even though the shape of the nucleus is not an experimental observable, it is still possible to extract from the data direct information

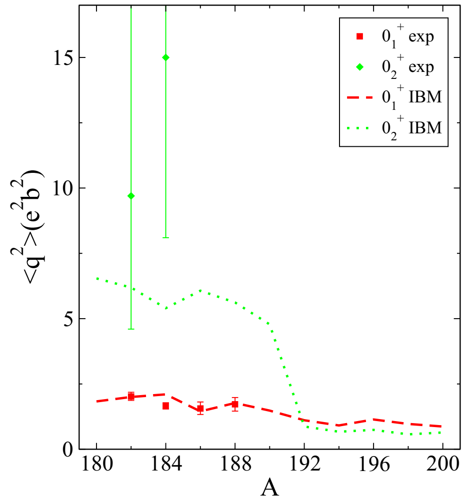

Figure 3. Comparison of the experimental and theoretical q^2 shape invariant given in e^2b^2 units for the first two 0^+ states.

about various moments characterizing the nuclear shape corresponding with a given eigenstate. Using Coulomb excitation, it is possible to extract the most important diagonal and non-diagonal quadrupole and octupole matrix elements, including their relative signs and, in a model independent way, extract information about nuclear deformation as shown by Kumar [11] (see also [12]).

From the theoretical point of view the nuclear shape can be calculated using quadrupole shape invariants. In particular q^2 corresponds to,

$$q^2 = \sqrt{5}\langle 0^+ \| [\hat{Q} \times \hat{Q}]^{(0)} | 0^+ \rangle = \sum_r \langle 0^+ \| \hat{Q} \| 2_r^+ \rangle \langle 2_r^+ \| \hat{Q} \| 0^+ \rangle, \tag{3}$$

where q denotes the nuclear intrinsic quadrupole moment and the sum is running over the complete basis of intermediate states with $L = 2$.

A comparison with the experimental values can be carried out whenever a large enough set of reduced E2 matrix elements can be extracted from, *e.g.*, Coulomb excitation experiments. Such a comparison constitutes a very stringent test for the theoretical model and, at the same time, provides a clear picture of the nuclear shape.

In Fig. 3 we compare the IBM-CM results with recent Coulomb excitation experiments of $^{182-188}$Hg at REX-ISOLDE and Miniball, allowing to extract a useful set of reduced E2 matrix elements [2]. It turns out that our IBM-CM calculations indeed give rise to values of q^2 that differ by a factor of ≈ 3 between the 0_1^+ and 0_2^+ states around the mid-shell region. The general picture provided through this approach corresponds to a ground state slightly deformed with regular character, while the 0_2^+ state presents a rapid evolution being intruder around mid-shell region, but turning into a regular character from ^{190}Hg and onwards.

3 Conclusions

We have studied the shape evolution of a chain of Hg isotopes, $^{172-200}$Hg, using the IBM-CM intrinsic state formalism. We also obtained values of β extracted from reduced E2 transition rates, and, finally we considered quadrupole shape invariants. The mean-field approach describes very well the presence of two different structures with very different deformations, but, is not able to describe correctly the evolution of the ground state deformation. The second approach and the quadrupole shape invariants indicate compelling evidence that the ground state is always slightly deformed while the structure of the 0_2^+ state changes dramatically along the chain of Hg isotopes.

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