EPJ Web of Conferences **93**, 01046 (2015)
DOI: 10.1051/epjconf/20159301046

The effect of the nuclear Coulomb field on atomic ionization at positron-electron annihilation in β^+ - decay

Sergey Fedotkin

Institute for Nuclear Research, 03680, Prospect Nauki 47, Kiev, Ukraine

Abstract. We consider the process of the annihilation of a positron emitted at β^+ - decay and a K-electron of the daughter atom. A part of energy during this process is passed to another K- electron and it leaves the atom. The influence of the Coulomb field on the positron and the ejected electron is considered. It was calculated the probability of this process for an atom with arbitrary Z is calculated. For the nucleus Ti the effect of the Coulomb field essentially increases the probability of the considered process.

1 Introduction

At annihilation positrons and atomic electrons may happen some radiationless processes besides of usual single-quantum annihilation. In this case occurs the nuclear excitation or ionization of the atom [1]. These processes may happen also at the annihilation of a positron and an electron of the daughter atom created in β^+-decay. The nuclear excitation at the annihilation of positrons was studied experimentally and theoretically in [2,3] for β^+-decay of ^{45}Ti and the following excitation of the nucleus ^{45}Sc.

In this paper we consider the process of annihilation of positrons emitted in β^+- decay and K-electrons of daughter atoms. Part of the energy during this process is passed to another K-electron. As a result this electron leaves the atom. The parameter $\xi = Z\alpha c/\upsilon$ for this process can be much bigger than 1 at very small velocity of the ejected electron υ and the Born approximation becomes in this case incorrect (here Z is the nuclear charge, α is the fine structure constant, c is the speed of light). Because of this, in contrast to paper [4], the wave function of continuum spectra in Coulomb field for the electron, which is ejected from atom, and the Green function for electron in a Coulomb field are used here.

2 Calculation of probability of process.

Let us consider the most probable case when the positron annihilates with a K-electron and this energy is passed to another K-electron. The Feynman diagram corresponding to this process is shown in Figure 1. Here I and F are initial and final nuclear states, e_{K1} and e_{K2} are K-shell atomic electrons; ω- energy transmitted as a result of annihilation of K_1 - electron and positron to another K-shell electron K_2; e_ε - electron escaping from atom with energy ε and momentum $\mathbf{p}_\varepsilon$; ν is neutrino with momentum $\mathbf{p}_\nu$.

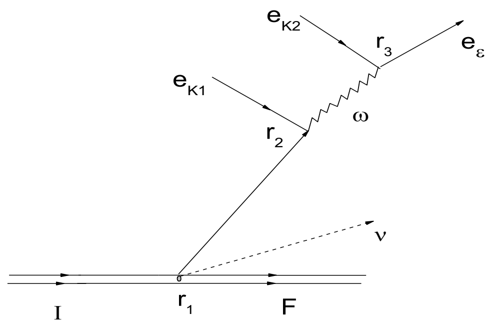

Figure 1. Feynman diagram corresponding to the process of positron- K1 -electron annihilation and ejection from an atom of another K2 - electron.

The general expression for the probability of this process after integration over final states of electron and neutrino has the following view

$$W_{\beta^+K,1S} = 2\pi \sum_{s_e, s_\nu} \int \frac{d\mathbf{p}_\nu d\mathbf{p}_\varepsilon}{(2\pi)^6} \left|U_{fi}\right|^2 \delta(E_0 - \varepsilon - p_\nu)$$

Here $E_0 = E_I - E_F + 2E_K$. In the calculation of the amplitude of probability of this process U_{fi} for the atomic electron hydrogen like wave functions are used. The electron which leaves the atom is described by the wave function of continuum spectra in Coulomb field.

[a] Corresponding author: sfedotkin@inbox.ru

For the Green function the Green function for an electron in a Coulomb field in term of Whittaker functions $W_{\eta_0,1/2}(r)$ [5] is used. After the calculation of the amplitude U_{fi} according to the Feynman diagram in Figure1 the probability $W_{\beta^+K,1s}$ in a case of allowed Fermi transitions may be written in the following view (here and hereinafter is used system of the units $\hbar$=c=1):

$$W_{\beta^+K,1s} = \frac{4\alpha^2 (Zm\alpha)^6}{\pi^3} G_V^2 \left| M_{FI}^{\beta^+} \right|^2 J_{\beta^+K,1s} \quad , \qquad (1)$$

where m is the electron mass, $M_{FI}^{\beta^+}$ is the nuclear matrix element, corresponding to nuclear transition from state I to F, G_V is a constant of the weak interaction. $J_{\beta^+K,1s}$ is the following integral over the electron energy

$$J_{\beta^+K,1s} = \int_m^{E_0} d\varepsilon (E_0-\varepsilon)^2 \sqrt{\varepsilon^2 - m^2}\, [(\varepsilon+m)(\varepsilon-3m)^2 + 3(\varepsilon-m)^3] \left| I_{23}(\varepsilon) \right|^2$$

Here

$$I_{23}(\varepsilon) = \int \frac{d\mathbf{q}}{q^2} I_3(\mathbf{p}_\varepsilon - \mathbf{q})\, I_2(\mathbf{q}) \quad , \qquad (2)$$

where

$$I_3(\mathbf{p}_\varepsilon - \mathbf{q}) = \int d\mathbf{r}_3\, e^{i\mathbf{r}_3(\mathbf{q}-\mathbf{p}_\varepsilon) - \eta r_3} F(i\xi, 1; i(p_\varepsilon r_3 + \vec{p}_\varepsilon \vec{r}_3)) \quad (3)$$

$$I_2(\mathbf{q}) = \Gamma(1-\eta_0) \int \frac{d\mathbf{r}_2}{r_2} e^{i\mathbf{r}_2\mathbf{q} - \eta r_2} W_{\eta_0,1/2}(-2ibr_2) \qquad (4)$$

Below there are the definitions of some quantities in equations (3) and (4) :

$\eta = Zm\alpha$, $\eta_0 = iZ\alpha(2E_K - \varepsilon)/b$, $b = \left[(2E_K - \varepsilon)^2 - m^2\right]^{1/2}$

The integrals $I_3(\mathbf{p}_\varepsilon - \mathbf{q})$ (3) and $I_3(\mathbf{p}_\varepsilon - \mathbf{q})$ (4) are calculated analytically and have the following form:

$$I_2(\mathbf{q}) = \frac{4\pi}{q^2 + (\eta - ib)^2} \left(1 + \frac{\eta_0}{(1-\eta_0)2q} \left[\Phi_+(q) - \Phi_-(q) \right] \right),$$

where

$$\Phi_\pm(q) = (b + i\eta \pm q)\, {}_2F_1(1,1;2-\eta_0; \frac{\eta + i(b \pm q)}{2ib})$$

and

$$I_3(\mathbf{k}) = -\frac{dJ(\mathbf{k})}{d\eta}, \quad J(\mathbf{k}) = 2\pi \left(\frac{k^2}{2} + \frac{\eta^2}{2} \right)^{i\xi - 1} \left(\frac{k^2}{2} + \frac{\eta^2}{2} + \mathbf{pk} - i\eta p \right)^{-i\xi}$$

We obtained the analytical but cumbersome expression for integral $I_{23}(\varepsilon)$ (2). The expression for the probability $W_{\beta^+K,1s}$ (1) is general and may be used for nuclei with arbitrary Z. Using eq. (2) for $I_{23}(\varepsilon)$ the probability $W_{\beta^+K,1s}$ (1) and the ratio of this probability to the probability W_{β^+} of usual β^+- decay were calculated for nucleus ${}^{45}_{22}Ti$. It was obtained the following result :

$$\frac{W_{\beta^+K,1s}}{W_{\beta^+}} \approx 1.2 \cdot 10^{-5} \qquad (5)$$

An analogous ratio was obtained for a case with free-particle expressions for the Green function and wave function of electrons ejected from atoms [4] :

$$\frac{W^0_{\beta^+K,1s}}{W^0_{\beta^+}} \approx 4 \cdot 10^{-6} \qquad (6)$$

The use of the wave function of continuum spectra in an Coulomb field for the electron in the final state instead of the plane wave approximation reduces the probability of this process in a case of β^+-decay of ^{45}Ti for several times. However, the use of the Green function for an electron in a Coulomb field gives the opposite effect and the probability of ionization of the atom increases. Consequently taking into account the Coulomb field is important for calculations of probabilities of β^+-decay as well as for more complicated processes of β^+-decay with following atomic ionization and positron annihilation. It is important also for calculations of ratios of these probabilities.

References

1. S. Shimizu., T. Mucoyama, Y. Nakayama, Phys.Rev. **173**, 405 (1968)
2. G.P. Borozenec, I.N. Vishnevskii, V.A. Zheltonozhskii, Sov. J. Nucl. Phys. **43**,14 (1983)
3. V.M. Kolomietz , O.G. Puninskii, S.N. Fedotkin, Izv. Akad. Scien.USSR, Phys. Ser. **52**, 12 (1988)
4. S.N. Fedotkin, Nucl. Phys. Atom. Energy. **13**, 223 (2012)
5. R.J. Glauber, P.C. Martin, Phys. Rev. **104**,158 (1956)