Neutrinoless double beta nuclear matrix elements around mass 80 in the nuclear shell-model

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Abstract. The observation of the neutrinoless double-beta decay can determine whether the neutrino is a Majorana particle or not. For theoretical nuclear physics it is particularly important to estimate three types of matrix elements, namely Fermi (F), Gamow-Teller (GT), and tensor (T) matrix elements. In this paper, we carry out shell-model calculations and also pair-truncated shell-model calculations to check the model dependence in the case of mass $A=82$ nuclei.

1 Introduction

The double beta decay is a second order process of the weak interaction which increases the atomic number of a nucleus by two. There are two modes of double-beta decays. The $2\nu$ mode ($2\nu\beta\beta$) is expected within the Standard Model and is characterized by the additional emission of two anti-neutrinos. Up to now $2\nu\beta\beta$ decay half-lives have been measured in ten cases in experiment. In contrast, the $0\nu$ mode can only take place if the neutrino is a Majorana particle. This demands an extension of the Standard Model of electroweak interactions because it violates the lepton number conservation. The $0\nu\beta\beta$ mode ($0\nu\beta\beta$) is one of the best probes for physics beyond the Standard Model. Many theoretical methods have been applied so far to evaluate the nuclear matrix elements, namely, in the shell-model [1, 2], and the quasi-particle random-phase approximation (QRPA) [3] and in the microscopic interacting boson approximation (IBM) [4]. However, there still remain large ambiguities in estimating those nuclear matrix elements in various methods [1–4]. In this paper, we carry out shell-model calculations and pair-truncated shell-model calculations to check the model dependence on nuclear matrix elements in comparison with other models. In this work, we calculate the nuclear matrix elements of the $0\nu\beta\beta$ decay for the transition from $^{82}$Se to $^{82}$Kr in two different formulations.

2 Shell-model calculations for Nuclear structure

Systematic studies were carried out for even-even and odd-mass nuclei in the mass $A \sim 80$ region in terms of the

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Comparison between the experimental energy spectra (expt.) and those of the shell-model (SM) for $^{82}$Se and $^{82}$Kr.}
\end{figure}

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$S$-subspace only with $S$ pairs, the $SD$-subspace without $G$ pairs and the $SDG$-subspace including $G$ pairs. In the following calculations of nuclear matrix elements we perform PTSM calculations in $S$, $SD$- and $SDG$-subspaces using the same Hamiltonian employed in the full shell model.

### 3 Theoretical framework and results for neutrinoless double beta decay

The $0\nu\beta\beta$ decay half-life is given by

$$T_{1/2}^{0\nu} = \frac{G_{0\nu}}{\langle m_e \rangle^2} \left( \frac{m_\nu}{m_e} \right)^2,$$  

where $G_{0\nu}$ is a phase-space factor, $\langle m_\nu \rangle$ is the effective mass of the electron neutrino, $m_e$ is the electron mass, and $M^{(0\nu)}$ is the nuclear matrix element between wave functions of two particular nuclei. Here we focus our attention on the nuclear matrix element:

$$M^{(0\nu)}_{1s} = \langle 82^{\text{Se}} | \hat{V}_{1s2s}^{(G)} | 82^{\text{Kr}} \rangle.$$  

Three types of matrix elements play a particularly important role, Fermi ($F$), Gamow-Teller ($GT$), and tensor ($T$) matrix elements. The $F$, $GT$, and $T$ transition operators can be expressed using the neutrino potential $V^{(G)}_{1s2s}$ by [4]. Other details are given in Ref. [4].

Table 1. Neutrinoless matrix elements in units of fm$^{-1}$ for the SM, the PTSM, the IBM [4] and the QRPA [3] in Tomoda’s formulation for mass $A = 82$. For the PTSM calculations, the results within the $SDG$ subspace ($SDG$) and those within the $SD$ subspace ($SD$) and those within the $S$ subspace ($S$) are presented.

<table>
<thead>
<tr>
<th>Model</th>
<th>$M^{(0\nu)}_F$</th>
<th>$M^{(0\nu)}_{GT}$</th>
<th>$M^{(0\nu)}_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SM</td>
<td>$-0.076$</td>
<td>$+0.115$</td>
<td>$+0.163$</td>
</tr>
<tr>
<td>PTSM ($SDG$)</td>
<td>$-0.118$</td>
<td>$+0.128$</td>
<td>$+0.203$</td>
</tr>
<tr>
<td>PTSM ($SD$)</td>
<td>$-0.126$</td>
<td>$+0.105$</td>
<td>$+0.185$</td>
</tr>
<tr>
<td>PTSM ($S$)</td>
<td>$-0.192$</td>
<td>$+0.231$</td>
<td>$+0.353$</td>
</tr>
<tr>
<td>IBM</td>
<td>$-0.211$</td>
<td>$+0.346$</td>
<td>$+0.481$</td>
</tr>
<tr>
<td>QRPA</td>
<td>$-0.131$</td>
<td>$+0.293$</td>
<td>$+0.377$</td>
</tr>
</tbody>
</table>

In Šimkovic’s formulation [6], the nuclear matrix elements are given as

$$M^{(0\nu)} = -\left( \frac{g_\nu}{g_A} \right)^2 M^{(0\nu)}_F + M^{(0\nu)}_{GT},$$  

where all the details are given in Ref. [6]. The results are shown in Table 2 in comparison with those of the IBM [4]. The obtained results are similar to those in Tomoda’s formulation. In general contributions from Tensor type are not large, but its importance is enhanced in the shell model since other type ($F$ and $GT$) contributions become relatively small in comparison with the IBM results.

Table 2. Neutrinoless matrix elements in dimensionless units for the SM, the PTSM and the IBM [4] in Šimkovic’s formulation for mass $A = 82$. For the PTSM calculations, the results within the $SDG$ subspace ($SDG$) and those within the $SD$ subspace ($SD$) and those within the $S$ subspace ($S$) are presented.

<table>
<thead>
<tr>
<th>Model</th>
<th>$M^{(0\nu)}_F$</th>
<th>$M^{(0\nu)}_{GT}$</th>
<th>$M^{(0\nu)}_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SM</td>
<td>$-0.800$</td>
<td>$+1.164$</td>
<td>$-0.384$</td>
</tr>
<tr>
<td>PTSM ($SDG$)</td>
<td>$-1.230$</td>
<td>$+1.316$</td>
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<tr>
<td>PTSM ($SD$)</td>
<td>$-1.308$</td>
<td>$+1.105$</td>
<td>$-0.667$</td>
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<tr>
<td>PTSM ($S$)</td>
<td>$-1.984$</td>
<td>$+2.366$</td>
<td>$-0.809$</td>
</tr>
<tr>
<td>IBM</td>
<td>$-2.197$</td>
<td>$+3.260$</td>
<td>$-0.254$</td>
</tr>
</tbody>
</table>

### 4 Summary

In the present study, we evaluate the nuclear matrix elements of neutrinoless double beta decay in terms of the SM and the PTSM. Compared to other models, such as, the QRPA and the IBM, our model predicts smaller transition matrix elements, indicating the strong sensitivity on nuclear matrix elements calculated by using different wave functions of various nuclear models. Compared to the IBM results, our shell model calculations predict roughly ten times longer half lives for the neutrinoless double beta decay from $^{82}\text{Se}$ to $^{82}\text{Kr}$.

### References