Detector-Response Correction of Two-Dimensional $\gamma$-Ray Spectra from Neutron Capture

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Abstract. The neutron-capture reaction produces a large variety of $\gamma$-ray cascades with different $\gamma$-ray multiplicities. A measured spectral distribution of these cascades for each $\gamma$-ray multiplicity is of importance to applications and studies of $\gamma$-ray statistical properties. The DANCE array, a 4π ball of 160 BaF$_2$ detectors, is an ideal tool for measurement of neutron-capture $\gamma$-rays. The high granularity of DANCE enables measurements of high-multiplicity $\gamma$-ray cascades. The measured two-dimensional spectra ($\gamma$-ray energy, $\gamma$-ray multiplicity) have to be corrected for the DANCE detector response in order to compare them with predictions of the statistical model or use them in applications. The detector-response correction problem becomes more difficult for a 4π detection system than for a single detector. A trial and error approach and an iterative decomposition of $\gamma$-ray multiplets, have been successfully applied to the detector-response correction. Applications of the decomposition methods are discussed for two-dimensional $\gamma$-ray spectra measured at DANCE from $\gamma$-ray sources and from the $^{10}$B($n$, $\gamma$) and $^{113}$Cd($n$, $\gamma$) reactions.

1 Introduction

The ($n$, $\gamma$) resonances de-excite via $\gamma$-ray cascades with various multiplicities. The $\gamma$-ray spectra for each $\gamma$-ray multiplicity ($M_\gamma$) have different intensity distributions ($I_\gamma$) characteristic for the product nucleus from the reaction and the ($n$, $\gamma$) resonance. All these $\gamma$-ray spectra can be represented by a single two-dimensional (2D) spectrum $I_\gamma(M_\gamma$, $E_\gamma$), which includes the $\gamma$-ray intensities emitted from a given resonance of the ($n$, $\gamma$) reaction. The 2D spectra are important for determining the $\gamma$-ray strength functions and for Monte Carlo transport codes simulating the neutron-capture reaction.

The Detector for Advanced Neutron Capture Experiments (DANCE) is an excellent tool to measure the spectra from the ($n$, $\gamma$) resonances because of its high granularity, high efficiency and near 4π coverage. DANCE is installed at the moderated spallation neutron target of the Los Alamos Neutron Science Center. DANCE is a 4π calorimeter measuring the total $\gamma$-ray energy released in one $\gamma$-ray cascade, which is equal to the $Q$ value of the ($n$, $\gamma$) reaction. A $Q$-value cut of the total measured energy helps with the background reduction by removing $\gamma$-rays from other ($n$, $\gamma$) reactions as the capture of neutron by the barium isotopes of the BaF$_2$ crystals, for example. The fast timing of the BaF$_2$ crystals helps with the background reduction too, by setting a narrow coincidence window of 6-10 ns eliminating $\gamma$-rays from the ambient and beam-induced backgrounds. DANCE is located at 20.25 m from the water moderator allowing measurement of the incident-neutron energy using the time-of-flight technique and thus selecting the $\gamma$-rays de-exciting an ($n$, $\gamma$) resonance of interest.

The measured spectra at DANCE are smeared by the poor energy resolution of the BaF$_2$ detectors and the intensity distribution is distorted by the detector efficiency. The spectra have to be corrected for the detector response in order to obtain the $\gamma$-ray intensities emitted by the target. We consider two approaches (i) the forward approach, known as a “trial and error” method, and (ii) the backward approach based on a direct de-convolution. We will present applications of these approaches to DANCE spectra in the following sections.

2 Trial and error approach

The forward approach is based on suggesting a distribution of the emitted $\gamma$-rays from the target, folding them with the detector response of DANCE, and comparing this predicted spectrum with the measured one. If they differ then the suggested $\gamma$-ray distribution is modified and the spectra are compared again. The iterations repeat until the predicted spectrum represents closely the measured one. $\chi^2$ can be used as a metric for the comparison.

The $\gamma$-ray distribution from the capture of thermal neutrons by $^{113}$Cd is of a great importance to applications and detector development. We carried out a measurement at DANCE with a small-mass cadmium target highly enriched to $^{113}$Cd collecting the $\gamma$-rays from the strongest resonance of the $^{113}$Cd($n$, $\gamma$)$^{114}$Cd reaction at $E_{\gamma}$ = 0.178 eV. The $\gamma$-ray cascades in $^{114}$Cd were simulated with the code DICEBOX [1] using the results for the photon strength...
function (PSF) from our study of the neutron-capture γ-rays from $^{112}$Cd and $^{114}$Cd [2]. A DICEBOX realization represents a level scheme of the nucleus and assignment of all transition widths from a given state to all levels below it. A second nuclear realization is created by changing the seed of the random generator, giving a new set of levels and transitions widths, but using the same input average values of the PSF and the level density. When creating many nuclear realizations, there is a chance for one of them to have a widths distribution very close to the one from a given $(n, \gamma)$ resonance and to provide a γ-ray spectrum similar to the measured one.

We simulated the γ-ray cascades in $^{114}$Cd with DICEBOX using 10000 nuclear realizations. The γ-ray cascades from each realization were used as input to GEANT4 simulating predicted 2D spectra. A comparison of the measured spectrum with the predicted one providing the smallest $\chi^2$ from all 10000 realizations is given in Fig. 1. The γ-ray transition intensities as a function of the excitation energy and the γ-ray energy, extracted from the nuclear realization describing best the measured spectrum, are shown in Fig. 2. This matrix can be directly implemented in a random generator to generate γ-ray cascades in $^{114}$Cd that reproduce the energy and multiplicity distributions correctly. This work is described in more detail in Ref. [3].

Figure 1. Measured γ-ray spectra (in black) with DANCE from the 0.178 eV resonance of the $^{113}$Cd$(n, \gamma)$ reaction for γ-ray multiplicities $M_\gamma=2$ to 7. Predicted spectra corresponding to a DICEBOX realization which represent best the cascade transitions from the 0.178 eV resonance are shown in red. The figure is taken from Ref. [3].

Figure 2. Gamma-ray transition intensities, simulated with DICEBOX, as a function of the excitation ($E_x$) and γ-ray ($E_\gamma$) energies representing the cascades transitions in $^{114}$Cd de-exciting the resonance at 0.178 eV in the reaction $^{113}$Cd$(n, \gamma)$. The binning of $E_x$ and $E_\gamma$ is 100 keV/channel. The color scale represents the transition intensity for $10^7$ neutron captures. The figure is taken from Ref. [3].
3 Direct decomposition

The measured spectrum $y=I_\gamma(M_\gamma, E_\gamma)$ is a product of the detector-response matrix $R$ and the $\gamma$-ray spectrum $x$ emitted from the target: $y=R \cdot x$. The “backward” method of decomposition solves the inverse equation and finds a solution for $x$. An advantage of using the backward method is that it does not require nuclear-physics input as level density and PSF, which in general are not known with a good accuracy.

We simplify the problem by transforming the measured spectrum and representing it as a sequence of one dimensional spectra for each multiplicity. Then the response matrix will be a two-dimensional matrix. We build the response matrix by simulating spectra with GEANT4 corresponding to emitted from the target single-transition $\gamma$-rays, two-step $\gamma$-ray cascades, three-step $\gamma$-ray cascades, etc. Each of these combinations of emitted $\gamma$-rays is identified by a unique number (ID). Each ID defines a cascade of given $\gamma$-ray transitions. The response matrix is a “stack” of such simulated spectra, each row corresponding to a given ID of emitted $\gamma$-rays. The solution of the inverse problem will be the percentage contribution of each ID to the total $\gamma$-ray distribution emitted from the target. Indeed, the solution provides more than a detector-response corrected spectrum, it gives the intensity of different $\gamma$-ray cascades and thus a correlation between $E_\gamma$ and $M_\gamma$. The simulated spectra are transformed too, analogously to the measured spectrum, and smeared to account for the energy resolution of the BaF$_2$ detectors. An illustrative example of the structures of the measured spectrum and the response matrix is given in Fig. 3. We use the following conditions to reduce the dimension of the response matrix: (i) the sum of all energies of the $\gamma$-rays in a cascade has to be equal to the $Q$ value of the $(n, \gamma)$ reaction (ii) the $\gamma$-ray energies have to be ordered $E_{\gamma 1} < E_{\gamma 2} < ... < E_{\gamma n}$ to avoid IDs consisting the same $\gamma$-rays (iii) the $\gamma$-ray energies are discretized using a bin size $\Delta E$. 

Figure 3. Transformed measured spectrum (a) from the $^{113}$Cd$(n, \gamma)$ reaction (cf. Fig. 1). The spectra for each $\gamma$-ray multiplicity are denoted with red arrows. A response matrix (b) for the $^{113}$Cd$(n, \gamma)$ measurement built from simulated spectra with GEANT4 for various combinations of $\gamma$-rays labeled with ID numbers. The multiplicity structure of these spectra is the same as the one of the measured spectrum (a).

Figure 4. A comparison of the measured spectra with an $^{88}$Y source for multiplicities from 1 to 5 with the solution of the decomposition problem folded with the simulated DANCE detector response. The convoluted spectra were scaled to the measured ones by a single normalization factor. The decomposition provided two solutions of $\gamma$-ray pairs: 23% intensity of the pair $E_{\gamma 1}=0.8$ MeV and $E_{\gamma 2}=1.8$ MeV and 77% intensity of the pair $E_{\gamma 1}=0.9$ MeV and $E_{\gamma 2}=1.8$ MeV.
The decomposition method of Gold [4] is positive defined providing always positive solutions. It was successfully used for detector-response correction of γ-ray spectra measured with GAMMASPHERE [5]. In our application to the DANCE spectra we used a similar method [6] that including a regularization parameter to speed up the iteration procedure.

We tested the decomposition procedure with $^{60}$Co and $^{88}$Y source measurements, providing two γ-rays in a cascade, and a measurement with a $^{22}$Na source, which can mimic a three-step cascade (1275 keV + 2x511 keV). Note, the coincidence window at DANCE was set to 10 ns collecting events only from the decay of the para positronium. The decomposition found solutions with only two γ-rays in coincidence for the $^{60}$Co and $^{88}$Y measurements, but due to the relatively large binning, we obtained two similar solutions instead of only one. The result of the decomposition is 29% intensity of the pair $E_\gamma^1$=1.1 MeV and $E_\gamma^2$=1.3 MeV and 71% intensity of the pair $E_\gamma^1$=1.2 MeV and $E_\gamma^2$=1.3 MeV for the $^{60}$Co source and 23% intensity of the pair $E_\gamma^1$=0.8 MeV and $E_\gamma^2$=1.8 MeV and 77% intensity of the pair $E_\gamma^1$=0.9 MeV and $E_\gamma^2$=1.8 MeV for the $^{88}$Y source. An example of the measured spectra from the $^{88}$Y source compared with the obtained solution folded with the DANCE detector response is given in Fig. 4. It demonstrates that the decomposition procedure is reliable, finding solutions that reproduce well both the shape of the spectra and the absolute intensity, which varies by a few orders of magnitude with increasing $M_f$.

We extended the test of the backward method with an in-beam measurement of the neutron-capture γ-rays from the $^{10}$B($n,\gamma$) reaction. The de-excitations of $^{11}$B, known from the literature, provide a single γ-ray transition, two- and three-step cascades. The $^{10}$B($n,\gamma$) cascades are summarized in Table 1. The solution of the decomposition is compared with the intensity of the known cascades in $^{11}$B in Fig. 5. Overall, the backward method can obtain the correct intensity distribution. The high intensity in the first bin of the spectrum is a result of the 150-keV threshold of the measured spectra and the 250-keV bin size. The upgrade of the DANCE data acquisition [7] will help to omit this problem by performing more accurate measurement of the low-energy γ-rays.

### Table 1. Gamma-ray cascades from the $^{10}$B($n,\gamma$) reaction, known from the literature, and their intensities.

<table>
<thead>
<tr>
<th>Intensity</th>
<th>$E_\gamma$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.7%</td>
<td>11.447</td>
</tr>
<tr>
<td>55.3%</td>
<td>4.444+7.007</td>
</tr>
<tr>
<td>17.9%</td>
<td>4.711+6.740</td>
</tr>
<tr>
<td>13.4%</td>
<td>2.533+8.917</td>
</tr>
<tr>
<td>7.7%</td>
<td>2.297+4.444+4.711</td>
</tr>
<tr>
<td>1.0%</td>
<td>2.533+4.444+4.475</td>
</tr>
</tbody>
</table>

The $J^\pi = 1^+$ resonance at 0.178 eV of the $^{113}$Cd($n,\gamma$)$^{114}$Cd reaction de-excites via many different cascades with an average multiplicity of 4 [2]. To reduce the size of the response matrix, we considered γ-ray cascades with multiplicity up to 4 and bin width $\Delta E = 0.5$ MeV. This particular response matrix is shown in Fig. 3 (b). The intensity spectrum result of the decomposition is shown in Fig. 5. It is compared with the intensity of the cascades with multiplicity $M_f \leq 4$ obtained from the forward method showing that the two independent methods provide very close results.

### 4 Summary

We applied the forward and backward methods to neutron-capture γ-ray spectra measured at DANCE. Each of the approaches has disadvantages: the forward method relies on accurate knowledge of the nuclear level density and PSF, while the backward method may not provide a unique solution. In a systematic study, we demonstrated that the direct decomposition method gives reliable results very close to the intensity distributions known from the literature. We will extend the work on the direct decomposition method, in future, by reducing the binning size of the response matrix and parallelizing the computation process.

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References
