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Estimation of the restitution coefficient by strain measurement

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Abstract. In order to clarify the collision phenomenon of the traffic machine in motion, such as automobile and aircraft, it is necessary to examine not only the deformation and the behavior of composed structural member but also the dynamic characteristic of friction coefficient and restitution coefficient in the collided contact surface. In this study, the restitution coefficient was estimated from the strain waveform measured by collision test using the Air Gun. The velocity and deformation dependence of the restitution coefficient was examined by carrying out the collision test at the different speed. As a result, in the metal test piece, restitution coefficient decreased by the plastic deformation with the increasing of the collision speed, and the velocity dependence was confirmed.

1. Introduction

It is known that the restitution coefficient between two objects differs by the shape, dimension and material distribution (density distribution, each coefficient distribution of elasticity, plasticity and viscosity, Poisson ratio distribution, etc.) of each object and also the deformation characteristic of contact point [1–3]. However, role and application of the restitution coefficient as basic data which macroscopically and approximately estimates collision behavior between two objects are wide, and it is broadly used in various fields. As rebound behavior evaluation of two objects in the elastic region, the basic research on the estimation of the rebounding characteristics by the numerical solution which used restitution coefficient, mass and moment of inertia of each objects as a parameter is reported [4]. And, in order to generally understand the collision collapse behavior of the whole structure, the method which calculates the restitution coefficient as near zero value is also used by temporarily assuming composing material and member the rigid-plasticity [1,5]. The experimental technique, which evaluates the restitution coefficient in the collision including plastic collapse phenomenon like automobile collision, has not yet been established.

The restitution coefficient is derived from momentum conservation law of the collision object, and it is defined by the speed of the object before and after the collision [6]. Generally, in the case of simple shape object, the restitution coefficient is often calculated from falling height and rebounding height of the object to the other stationary object by the drop type. However, accuracy and reproducibility for this method are low. On the other hand, it is reported that an accurate restitution can get from the approach and restitution impulse by measuring the contact force at the collision contact surface of two objects [7], but it has not measured and evaluated by this technique yet. It is necessary to establish the technique which estimates experimentally the restitution coefficient including plastic collapse phenomenon, such as automobile collision.

In this study, the aim is to establish a technique that measure and estimate standardly the restitution coefficient in the collision which include plastic collapse phenomenon. As a first step, an impulse in the two object collision was estimated from the strain waveform which generated in the stationary object before the collision, and the validity of evaluating method of the restitution coefficient was examined. The velocity and deformation dependence of the restitution coefficient was evaluated from the approach and restitution impulse in the collision test at the different speed by using the Air Gun.

2. Restitution coefficient and impulse

The velocity change of two objects (mass m1 and m2) in one dimensional collision is considered, as shown in Fig. 1. The velocity of two objects before and after the collision are v_1, v_2 and v_1', v_2', respectively. The restitution coefficient e is defined by conservation of momentum as following [6].

$$e = \frac{v_2' - v_1'}{v_1 - v_2}. \tag{1}$$

The whole momentum is conserved before and after the collision because the objects does not forced except for the collision force (in case of Fig. 1). Therefore, the momentum is given by following equation, where v_c is the velocity when two objects becomes the same velocity.

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' = (m_1 + m_2)\, v_c. \tag{2}$$

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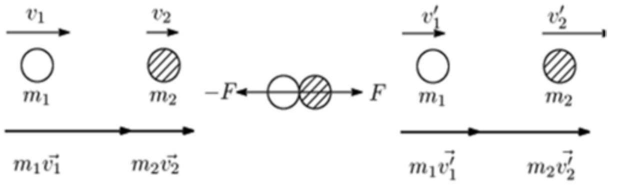

Figure 1. The collision of two objects.

The impulse during the collision is calculated from Eq. (2).

$$P_1 = m_1(v_1 - v_c) = m_2(v_c - v_2)$$
$$P_2 = m_1(v_c - v_1') = m_2(v_2' - v_c) \tag{3}$$

where P_1 and P_2 are the impulse when the two objects are approaching and restituting, respectively. The restitution coefficient is shown in the ratio of the restitution impulse to the approach impulse from Eq. (3), as following equation.

$$\frac{P_2}{P_1} = \frac{v_2' - v_1'}{v_1 - v_2} = e. \tag{4}$$

The force F generated in the object is given by following relation.

$$F = \sigma A = \varepsilon E A \tag{5}$$

where A, E, σ and ε are the cross-sectional area, longitudinal modulus of elasticity, stress and strain of the object, respectively. An impulse P is defined in the product of the force and the action time of that force, and it given by following equation from Eq. (5).

$$P = \int F dt = \int \varepsilon E A dt. \tag{6}$$

The approach and restitution period from the contact to the separation of the objects in the collision, and the strain generated in the object during that period are considered from the strain waveform, as shown in Fig. 2. When the object is collided by other object (t = 0), the strain wave generates in object. The strain takes maximum value at time $t = t_1$ and the objects approach extremely. The restitution of objects starts at $t = t_2$ after the duration of the maximum strain. The contact of objects finishes at $t = t_3$ (objects are separated and strain takes zero). The approach impulse P_1 (period from 0 to t_1) and restitution impulse P_2 (period from t_2 to t_3) are calculated by following equation from Eq. (6).

$$P_1 = \int_0^{t_1} \varepsilon_1(t) E A dt = EA \int_0^{t_1} \varepsilon_1(t) dt$$
$$P_2 = \int_{t_2}^{t_3} \varepsilon_2(t) E A dt = EA \int_{t_2}^{t_3} \varepsilon_2(t) dt. \tag{7}$$

Consequently, the restitution coefficient e is estimated by following equation from Eqs. (4) and (7).

$$e = \frac{P_2}{P_1} = \frac{EA \int_{t_2}^{t_3} \varepsilon_2(t) dt}{EA \int_0^{t_1} \varepsilon_1(t) dt} = \frac{\int_{t_2}^{t_3} \varepsilon_2(t) dt}{\int_0^{t_1} \varepsilon_1(t) dt}. \tag{8}$$

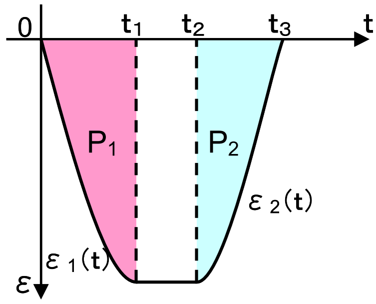

Figure 2. Strain waveform and impulse.

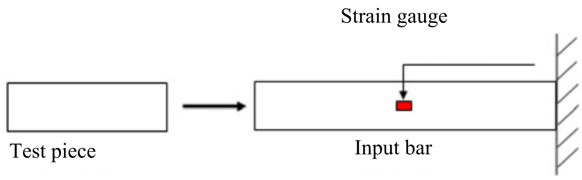

Figure 3. Schematic of collision test.

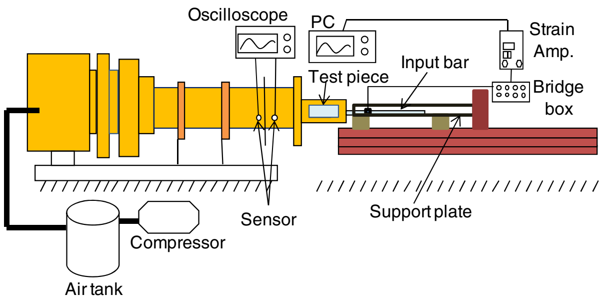

Figure 4. Schematic of the experimental equipments.

By measuring the strain wave generated by collision, the evaluation of the restitution coefficient is possible from the strain change and time during the approach and restitution period.

3. Collision test

3.1. Test piece

The restitution coefficient of test piece is estimated by measuring the deformation of elastic body from one dimensional collision test (wave propagation), as shown in Fig. 3. The test piece made of aluminum alloy (A5052) is carried out the collision test. The dimensions of test piece are 29.5 mm diameter and 100 mm length. The collision side of test piece is machined in R160 arc shape. Stainless steel (SUS304) of 20 mm diameter and 500 mm length is used for the input bar.

3.2. Test procedure

In the collision test used Air Gun (Fig. 4), the test piece which is accelerated by compression air is collided with the input bar. The strain gauge was bonded to the input bar (100 mm away from collision side). The stress wave which propagated in the input bar was measured by the oscilloscope in of 100 MHz sampling [8].

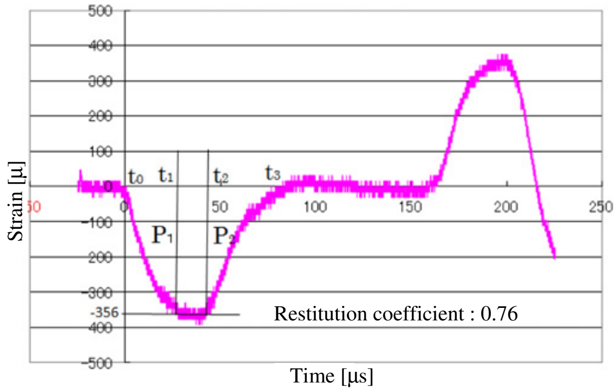

Figure 5. Example of strain waveform. (collision velocity 4.5 m/s).

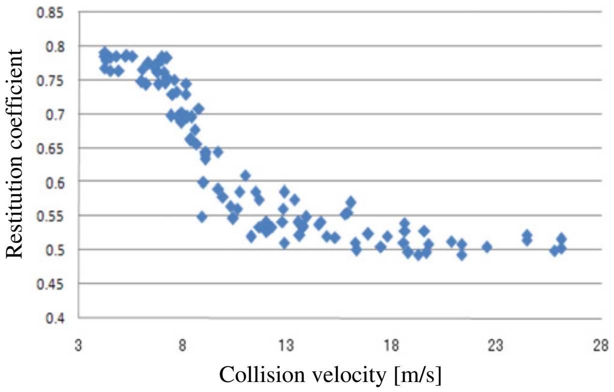

Figure 6. Dependence of restitution coefficient on collision velocity (Aluminum alloy).

The approach impulse P_1 and restitution impulse P_2 were estimated from the before and after maximum strain value in the measured strain waveform, and the restitution coefficient was obtained from Eq. (8). The collision test was carried out in 4–26 m/s velocity range, and the effectiveness of this technique which estimated the restitution coefficient by the strain measurement was examined.

4. Results and discussions

An example of the strain waveform measured at the collision test is shown in Fig. 5. The moving average processing was conducted to the measurement waveform, since the error by some noise and digitizations is contained in the signal. Maximum strain value, the closest approach time t_1, the restitution start time t_2 and the contact finish time t_3 was estimated from measured waveform, and then the restitution coefficient was calculated from the approach impulse P_1 and restitution impulse P_2.

The restitution coefficient of aluminum alloy for stainless steel at each collision velocity is shown in Fig. 6. The value of the restitution coefficient calculated by strain measurement in the elastic region (low collision velocity) was equivalent to the value shown in literature [9].

The restitution coefficient rapidly decreases after approximately 7 m/s collision velocities, though it takes constant value until that velocity. R160 arc shape which was the impact side of test piece was partially deformed plastically at the collision velocity 7 m/s to 10 m/s. The plastic deformation increased with the increase in collision velocity in this velocity range. It is considered that the kinetic energy of test piece was spent in the energy of plastic deformation and the restitution coefficient decreased with that plastic deformation. And also, in the collision over 10 m/s velocity, the whole arc part of the test piece perfectly deforms plastically. Further deformation is accumulated over the rod cross section and more large collision force is required for the plastic deformation. Therefore, the drop in the restitution coefficient seems to be slowly.

Measured strain also considered the plastic deformation, and it was confirmed that this technique was effective for the estimation of restitution coefficient containing the plastic collapse phenomenon.

5. Conclusions

In this study, the restitution coefficient was estimated from strain and impulse in the collision object measured by collision test at the different velocity, and the velocity and deformation dependence of the restitution coefficient were investigated. Consequently, in metal test piece, the restitution coefficient decreased by the plastic deformation with the increase in collision velocity, and the velocity dependence was confirmed. It was validated that the estimation of restitution coefficient by this technique was effective for the measurement in both of the elastic and plastic region. As the future work, the effect of shape and material of test piece on restitution coefficient and accuracy improvement of this technique will be attempted by the collision test at the wide velocity range.

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