

# Numerical simulation of ZrO<sub>2</sub>(Y<sub>2</sub>O<sub>3</sub>) ceramic plate penetration by cylindrical plunger

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**Abstract.** In this paper dynamic fracture process due to high-speed impact of steel plunger into ceramic sample is simulated. The developed numerical model is based on finite element method and a concept of incubation time criterion, which is proven to be applicable in order to predict brittle fracture under high-rate deformation. Simulations were performed for ZrO<sub>2</sub>(Y<sub>2</sub>O<sub>3</sub>) ceramic plates. To characterize fracture process quantitatively fracture surface area parameter is introduced and controlled. This parameter gives area of new surface created during dynamic fracture of a sample and is essentially connected to energetic peculiarities of fracture process. Multiple simulations with various parameters made it possible to explore dependencies of fracture area on plunger velocity and material properties. Energy required to create unit of fracture area at fracture initiation (dynamic analogue of Griffith's surface energy) was evaluated and was found to be an order of magnitude higher as comparing to its static value.

## 1. Introduction

Dynamic fracture properties of ceramic materials are primarily studied due to applications where ceramic/multi-layered ceramics or ceramic composites serve as a protection against mechanical and thermal impacts on an object [1,2] however theoretical problems primarily include understanding the mechanisms driving static and dynamic fracture of this special class of materials (extremely brittle, very high strength and low toughness [3]).

In this paper an approach that was previously successfully applied to simulate dynamic fracture in other classes of quasi-brittle materials [4,5] was applied to study penetration of ceramic plate by a cylindrical plunger. The focus is primarily on energetic peculiarities of the process, including investigations of histories of fracture surface evolution in fractured ceramic material, calculations of specific fracture energy at dynamic rupture initiation and studies of correlation between new fracture surface within ceramic material created as a result of a plunger impact and initial plunger energy (velocity) as well as the properties of the ceramic material.

This study is deliberately restricted to the simplest possible geometry and contact problem formulation. Thermal effects are not considered in this work. In the considered case, an elastic cylindrical plunger with properties typical for steel normally hits a circular homogeneous ceramic plate, behaving as linear elastic material up to the moment of fracture initiation.

## 2. Problem formulation

Both the plunger and the target are supposed to be linear elastic with stress-strain state given by the Lamé equations and the Hook's law:

$$\rho \frac{\partial^2 U_i}{\partial t^2} = (\lambda + \mu) \nabla_i (\nabla \cdot \bar{U}) + \mu \Delta U_i \quad (1)$$

$$\sigma_{i,j} = \delta_{i,j} \lambda \nabla \cdot \bar{U} + \mu \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right). \quad (2)$$

The ceramic plate is supposed to be fixed on its outer radius. The plunger initial velocity  $V$  is in the direction normal to the plate surface. Due to the symmetry of the problem, a 2D problem with axial symmetry is solved.

Material properties for the target are typical for ZrO<sub>2</sub>(Y<sub>2</sub>O<sub>3</sub>) ceramics [6,7]. The plunger is supposed to have properties of steel. Tables 1 and 2 give the material properties utilized for the simulation.

## 3. Fracture criterion

Nowadays it is known and generally recognized that classical fracture criteria (critical stress criterion, critical stress intensity factor criterion, etc.) are inapplicable in order to predict fracture caused by dynamic high-rate loads. Incubation time fracture criterion [8] can be utilized for correct and robust prediction of critical conditions leading to fracture of material loaded by impact loads. In [9,10] this approach was successfully used to simulate dynamic fracture in other classes of quasi-brittle materials. It is supposed that a similar approach can be used to predict fracture initiation, evolution and arrest in ceramic materials [11] for the studied class of problems. The criterion for

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**Table 1.** Target material properties.

Material property	Value
Elastic module $E$ , GPa	200
Poisson ratio	0.25
Ultimate stress $\sigma_c$ , MPa	750
Fracture toughness $K_{Ic}$ , $\text{MPa}\sqrt{m}$	13.3
Density $\rho$ , $\text{kg}/\text{m}^3$	6000

**Table 2.** Plunger material properties.

Material property	Value
Elastic module $E$ , GPa	200
Poisson ratio	0.25
Density $\rho$ , $\text{kg}/\text{m}^3$	7860

fracture at a point  $x$ , at time  $t$ , reads as:

$$\frac{1}{\tau} \int_{t^*-\tau}^{t^*} \frac{1}{d} \int_{x^*-d}^{x^*} \sigma(x, t) dx dt \geq \sigma_c, \quad (3)$$

where  $\tau$  is the microstructural time of a fracture process (or fracture incubation time) – a parameter characterizing the response of the material to applied dynamical loads (i.e.  $\tau$  is constant for a given material and does not depend on problem geometry, the way a load is applied, the shape of a load pulse or its amplitude).  $d$  is the characteristic size of a fracture process zone and is constant for the given material and chosen scale.  $\sigma(x, t)$  is stress at a point, changing with time, and  $\sigma_c$  is its critical value (ultimate stress or critical tensile stress found in quasi-static conditions).

Assuming:

$$d = \frac{2}{\pi} \frac{K_{Ic}^2}{\sigma_c^2}, \quad (4)$$

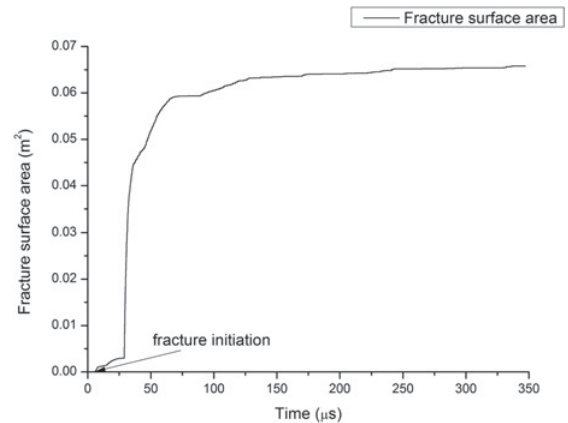
where  $K_{Ic}$  is the critical stress intensity factor for mode I loading (mode I fracture toughness), measured in quasi-static experimental conditions.

## 4. Simulation technique

Fracture criterion (3) was integrated into finite element computational scheme. By the choice of the element size to be equal to  $d$  – characteristic size of the fracture process zone, the linear increment of a single fracture event will also be equal to  $d$ , as implied by the incubation time approach [12].

Once fracture condition is executed somewhere in the plate, rupture should take place and this is simulated by splitting the nodes in the point where the fracture criterion was executed. Regular mesh with rectangular elements allows for fracture in two perpendicular directions. Each element of the target mesh has its own set of nodes – neighbouring elements do not have common nodes. However, nodes coincident in location have their degrees of freedom (DOF's) coupled, which is giving the same numerical formulation as a common node. Once the fracture condition is executed in one of the perpendicular directions, the restriction on DOF's in the same direction is removed – a new surface is created.

The solution output provides a possibility to monitor the change of fracture surface area after every solution

**Figure 1.** Fracture surface area as a function of time.

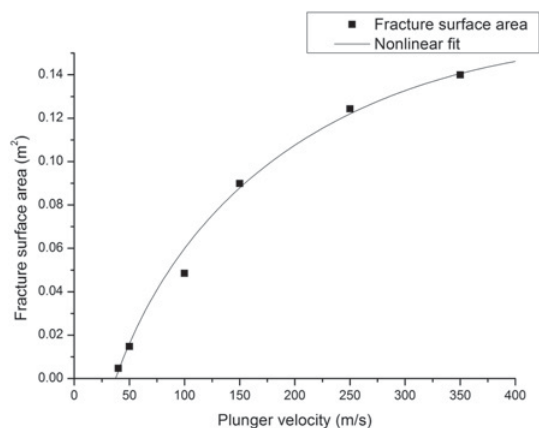
step. After the solution completion time dependence of the created fracture area can be provided for analysis. Since the problem is solved in two-dimensional formulation, the fracture surface is calculated considering axial symmetry of the problem. In first case the total length of microcrack faces is calculated – once elements are separated the length of their newly appeared free edges is added to the total fracture length. On another hand, if the axial symmetry is considered one should multiply the length of microcrack faces by, where is the distance from the microcrack to the symmetry axis and thus, the fracture surface area created in the bulk of the plate is estimated. Both approaches do not describe real three-dimensional fracture pattern, but the second one seems to be more realistic. Typical fracture surface area – time dependence is shown in Fig. 1.

It should be noticed here, that parts of the ceramic target separated in course of the fracture process do not interact with each other, which is, of course, a significant simplification of the model.

## 5. Results and discussion

Utilized fracture criterion (3) requires a material strength parameter to be evaluated in an independent test (ex. [13]). Such an experimental study was not yet performed for ceramic materials and the incubation time is unknown for the materials used in the simulation. This opens a possibility to investigate the influence of the incubation time on behaviour of the fractured media. Reasonable range for possible incubation time variation was chosen keeping in mind values typical for other brittle materials [10]. Initial velocity of the plunger was another parameter that was alternated in course of simulations. As a result, dependencies of fracture process on the incubation time value and the plunger initial velocity were received.

The main parameter characterizing fracture in this study is the induced damage – the new surface created in a result of the interaction between the ceramic sample and the plunger. Induced damage was calculated according to the procedure described in the previous section. Figure 2 gives the received final damage as the plunger velocity  $V$  is alternated. Residual velocity of the plunger was also registered as another parameter of the process providing opportunity to find threshold initial velocity



**Figure 2.** Fracture surface area as a function of plunger initial velocity.

corresponding to the situation when the plunger is completely stopped by the target.

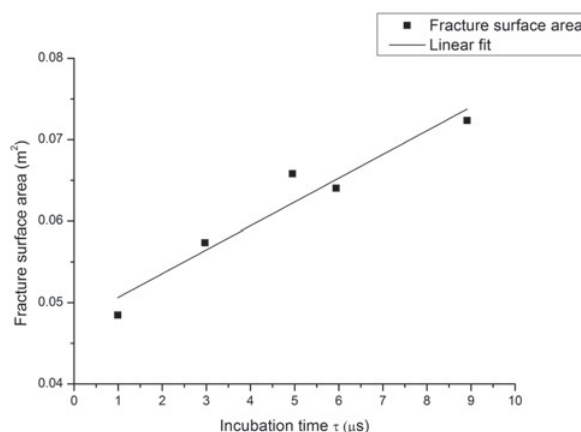
Having the initial velocity of the plunger, its kinetic energy can be easily calculated as  $E = mV^2/2$ , where  $m$  is the plunger mass. Kinetic energy of the plunger after the interaction with the target can be calculated analogously using its residual velocity. Difference between these energies is transferred to the ceramic plate and can be used for fracture. Knowledge of this energy and the corresponding area of fracture created in the result of the interaction provide a possibility to calculate a value of specific dynamic fracture surface energy  $\gamma_d$  analogous to specific fracture surface energy introduced by Griffith [14].

This energy can be calculated as:

$$\gamma_d = \left. \frac{dE}{dS} \right|_{s=0}, \quad (5)$$

where  $E$  stands for energy transferred to the ceramic plate and  $S$  is the area of fracture surface created in a result of interaction. Calculation of  $\gamma_d$  using (6) gives  $\gamma_d = 1343 \frac{J}{m^2}$  for the studied case. This value is considerably higher (approximately an order of magnitude) than fracture surface energy evaluated in quasi-static loading conditions [15]. The similar behaviour of dynamic analogue of Griffith's surface energy was previously observed for PMMA in [16].

Variation of the incubation time used in fracture criterion (3) in fact, means the variation of the material as the incubation time is a material property responsible for material reaction on dynamic loading. Figure 3 gives the final fracture surface area  $S$  as a function of incubation time  $\tau$ . Presented data was obtained for the same projectile velocity – 100 m/s. Greater incubation times lead to higher values of final fracture surface area. This is explained by the fact that for lower values of incubation time, fracture condition is executed earlier, resulting in fracture concentrated in the areas close to the contact between the ceramic plate and the plunger while for bigger values of the incubation time fracture is spread further from the contact area.



**Figure 3.** Final fracture surface area versus incubation time dependence.

## 6. Conclusions

The presented investigation is the first attempt to simulate penetration of ceramics utilizing incubation time fracture criterion controlling fracture of ceramics. Relatively simple model geometry and constitutive laws gave a possibility to concentrate the study on the most important fracture process characteristics such as dependency of final fracture surface area on plunger velocity and calculation of dynamic analogue of Griffith's specific fracture energy. The use of incubation time fracture criterion provided an opportunity to examine fracture process parameters with consideration to microstructural processes preceding macroscopic fracture of the target. Moreover, the dependence of fracture surface area (and hence the amount of damage induced) on incubation time was studied.

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