

# Spectra and decay rates of $b\bar{b}$ meson using Gaussian wave function

Ajay Kumar Rai<sup>1,a</sup>, Nayneshkumar Devlani<sup>2,b</sup>, and Virendrasinh H. Kher<sup>2,c</sup>

<sup>1</sup>Department of Applied Physics, S.V. National Institute of Technology, Surat 395007 Gujarat, INDIA

<sup>2</sup>Applied Physics Department, Polytechnic, The M.S. University of Baroda, Vadodara 390002, Gujarat, INDIA

**Abstract.** Using the Gaussian wave function mass spectra and decay rates of  $b\bar{b}$  meson are investigated in the framework of phenomenological quark anti-quark potential (coulomb plus power) model consisting of relativistic corrections to the kinetic energy term. The spin-spin, spin-orbit and tensor interactions are employed to obtain the pseudoscalar and vector meson masses. The decay constants ( $f_{P/V}$ ) are computed using the wave function at the origin. The di-gamma and di-leptonic decays of the  $b\bar{b}$  meson are investigated using Van-Royen Weisskopf formula as well as in the NRQCD formalism.

## 1 Introduction

Now a days, there have been renewed interest in the spectroscopy of the light and heavy flavoured hadrons due to number of experimental facilities CLEO, DELPHI, Belle, BaBar, LHCb etc., which have been continuously providing more accurate and new information about the mesons, baryons and exotic states from light flavour to heavy flavour sector [1–4]. There are many theoretical attempts to understand these states using potential models [5–9]. The success of theoretical model predictions with experiments can provide important information about the quark-antiquark interactions. Such information is of great interest, as it is not possible to obtain the  $Q\bar{Q}$  potential starting from the basic principle of the quantum chromodynamics (QCD) at the hadronic scale [5–23].

The new role of the heavy flavour studies as the testing ground for the non-perturbative aspects of QCD, demands extension of earlier phenomenological potential model studies on quarkonium masses to their predictions of decay widths with the non-perturbative approaches like NRQCD [24].

We present details of the semi-relativistic treatment of the heavy quarks along with the computed results in section-2. The decay constants  $f_{P,V}$  of  $b\bar{b}$  meson incorporating QCD correction is presented in section-3. In section-4 we present the details of the computations of the di-gamma decays of pseudoscalar states and the di-leptonic decay widths of the vector states of the  $b\bar{b}$  quarkonia in the frame work of the NRQCD formalism as well as in the conventional Van-Royen Weisskopf formula. Finally we draw our conclusion in section-5.

---

<sup>a</sup>e-mail: raiajayk@gmail.com

<sup>b</sup>e-mail: nayneshdev@gmail.com

<sup>c</sup>e-mail: vkhher@yahoo.co.in

## 2 Theoretical formulation

For the study of the  $b\bar{b}$  meson we consider the relativistic Hamiltonian in which motion of the quarks inside the meson is relativistic[25–28]

$$H = \sqrt{\mathbf{p}^2 + m_b^2} + \sqrt{\mathbf{p}^2 + m_{\bar{b}}^2} + V(\mathbf{r}) \quad (1)$$

where  $\mathbf{p}$  is the relative momentum of the quark-antiquark and  $m_b$  is the  $b$  quark mass. The Hamiltonian in Eq(1) represents the energy of the meson in the meson rest frame. We expand the kinetic energy(K.E.) part of the Hamiltonian up to  $O(p^4)$  and  $V(\mathbf{r})$  is the quark-antiquark potential[15, 16, 18],

$$V(r) = -\frac{\alpha_c}{r} + Ar^\nu + V_0 \quad (2)$$

$A$  is the potential parameter,  $\nu$  is a general power index, and  $V_0$  is a constant.  $\alpha_c = (4/3)\alpha_s(M^2)$ ,  $\alpha_s(M^2)$  is the strong running coupling constant. The value of the QCD coupling constant  $\alpha_s(M^2)$  is determined through the simplest model with freezing[10, 11], namely

$$\alpha_s(M^2) = \frac{4\pi}{\left(11 - \frac{2}{3}n_f\right) \ln \frac{M^2 + M_B^2}{\Lambda^2}} \quad (3)$$

where  $M = 2m_b m_{\bar{b}} / (m_b + m_{\bar{b}})$ ,  $M_B = 0.95$  GeV[10, 11], and  $\Lambda = 0.413$  GeV[13]. We have used the gaussian wave function in the present study. The gaussian wave function in position space has the form

$$R_{nl}(\mu, r) = \mu^{3/2} \left( \frac{2(n-1)!}{\Gamma(n+l+1/2)} \right)^{1/2} (\mu r)^l e^{-\mu^2 r^2/2} L_{n-1}^{l+1/2}(\mu^2 r^2); \quad (4)$$

here,  $\mu$  is the variational parameter and  $L$  is Laguerre polynomial. For the present study, we employ the Ritz variational scheme. We obtain the expectation values of the Hamiltonian as

$$H\psi = E\psi \quad (5)$$

For a chosen value of  $\nu$  between 0.5 to 2.0 the variational parameter,  $\mu$  is determined for each state using the Virial theorem[29]. As the interaction potential assumed here does not contain the spin dependent part, Eq(5) gives the spin averaged masses of the system. The calculated spin averaged mass of the ground state is matched with the experimental spin-averaged mass using the equation

$$M_{SA} = M_P + \frac{3}{4}(M_V - M_P) \quad (6)$$

where  $M_V$  and  $M_P$  are the vector and pseudoscalar meson ground state masses taken from ref [1]. This fixes the parameter  $V_0$ . Using this value of  $V_0$  we calculate  $S$ ,  $P$ , and  $D$  wave spin-averaged masses of  $b\bar{b}$  mesons which are listed in Tables 1 and 2. For the comparison for the  $nJ$  state, we compute the spin-averaged or the center of weight mass from the respective theoretical values as [18]

$$M_{CW,n} = \frac{\sum_J 2(2J+1)M_{nJ}}{\sum_J 2(2J+1)} \quad (7)$$

where,  $M_{CW,n}$  denotes the spin-averaged mass of the  $n$  state and  $M_{nJ}$  represents the mass of the meson in the  $nJ$  state.

**Table 1.** S-wave spin averaged masses of  $b\bar{b}$  meson.

$nL$	$\nu$	Gaussian			Expt.[1] (GeV)	Others (GeV)
		$\mu$ (GeV)	$ R(0) $ $GeV^{3/2}$	$E(\mu)$ (GeV)		
1S	0.5	1.019	1.544	9.453	9.453	9.443[14]
	1.0	1.160	1.876	9.453		9.445[12]
	1.5	1.261	2.128	9.453		9.442[30]
	2.0	1.339	2.328	9.453		
2S	0.5	0.548	0.498	9.792		10.015[14]
	1.0	0.771	0.831	10.021		10.015[12]
	1.5	0.965	1.162	10.250		9.996[30]
	2.0	1.131	1.476	10.463		
3S	0.5	0.429	0.308	9.958		10.348[14]
	1.0	0.670	0.602	10.403		10.348[12]
	1.5	0.893	0.926	10.896		10.329[30]
	2.0	1.094	1.256	11.387		
4S	0.5	0.371	0.229	10.077		10.583[14]
	1.0	0.618	0.494	10.723		
	1.5	0.857	0.806	11.487		
	2.0	1.080	1.140	12.286		
5S	0.5	0.335	0.186	10.172		10.864[14]
	1.0	0.585	0.429	11.006		
	1.5	0.834	0.730	12.046		
	2.0	1.073	1.064	13.175		

The value of the radial wave function  $R(0)$  for  $0^{++}$  and  $1^{--}$  states would be different due to their spin dependent hyperfine interaction. The spin hyperfine interaction of the heavy-heavy flavored mesons is small and this can cause a small shift in the value of the wave function at the origin[18, 31]. The parameters used to calculate the low lying masses of the  $b\bar{b}$  meson are  $\alpha_S = 0.33$ ,  $A = 0.24 GeV^{-1}$ ,  $m_{\bar{b}} = 4.88 GeV$  and the value of the constant  $V_0 = -0.363, -0.370, -0.374$  and  $-0.377 GeV$  for  $\nu = 0.5, 1.0, 1.5$  and  $2.0$  respectively. The spin averaged masses for S, P and D states are tabulated in Tables 1 and 2. It can be observed that the spin-averaged masses obtained are in good agreement with experimental and other theoretical predictions at  $\nu = 1$ , but some excited states are over estimated by 100 MeV.

## 2.1 Excited states

In the case of quarkonia the quark-antiquark bound states are represented by  $n^{2S+1}L_J$ , identified with the  $J^{PC}$  values, with  $\vec{J} = \vec{L} + \vec{S}$ ,  $\vec{S} = \vec{S}_Q + \vec{S}_{\bar{Q}}$ , parity  $P = (-1)^{L+1}$  and the charge conjugation  $C = (-1)^{L+S}$  with  $(n, L)$  being the radial quantum numbers[32]. For computing the mass differences between different degenerate  $Q\bar{Q}$  meson states, the spin dependent part of the potential employed is given by

$$V_{SD}(r) = V_{SS}(r) \left[ S(S+1) - \frac{3}{2} \right] + V_{LS}(r) (\vec{L} \cdot \vec{S}) + V_T(r) \left[ S(S+1) - \frac{3(\vec{S} \cdot \vec{r})(\vec{S} \cdot \vec{r})}{r^2} \right]; \quad (8)$$

**Table 2.** P and D-wave Spin averaged masses of  $b\bar{b}$  meson.

$nL$	$\nu$	Gaussian		Expt.[1] (GeV)	Others (GeV)
		$\mu$ (GeV)	$E(\mu)$ (GeV)		
1P	0.5	0.617	9.757	9.899	9.900[14]
	1.0	0.836	9.899		9.901[12]
	1.5	0.907	10.673		9.873[30]
	2.0	1.167	11.035		
2P	0.5	0.453	9.936	10.260	10.260[14]
	1.0	0.695	10.300		10.261[12]
	1.5	0.865	11.268		10.231[30]
	2.0	1.078	11.920		
3P	0.5	0.384	10.059		10.544[14]
	1.0	0.632	10.627		
	1.5	0.840	11.831		
1D	0.5	0.500	9.895		10.163[14]
	1.0	0.742	10.168		10.158[12]
	1.5	0.849	11.591		10.127[30]
	2.0	1.071	12.396		
2D	0.5	0.407	10.029		
	1.0	0.656	10.512		
	1.5	NS			
	2.0	NS			

where the spin-orbit, tensor and spin-spin interactions can be written in terms of the vector and scalar parts of the  $V(r)$  as

$$V_{LS}(r) = \frac{1}{2m_Q m_{\bar{Q}}} \left( 3 \frac{dV_V}{dr} - \frac{dV_S}{dr} \right), \quad (9)$$

$$V_T(r) = \frac{1}{6m_Q m_{\bar{Q}}} \left( 3 \frac{d^2 V_V}{dr^2} - \frac{1}{r} \frac{dV_V}{dr} \right), \quad (10)$$

$$V_{SS}(r) = \frac{1}{3m_Q m_{\bar{Q}}} \nabla^2 V_V = \frac{16\pi\alpha_S}{9m_Q m_{\bar{Q}}} \delta^3(\vec{r}), \quad (11)$$

where

$$V_V = -\frac{4\alpha_S}{3r} \quad \text{and} \quad V_S = Ar^{\nu}. \quad (12)$$

The spin dependent part Eq(8) is added to the Hamiltonian Eq(5) to calculate the excited state masses. Masses are tabulated in Table-3.

### 3 Decay constants

The decay constants of mesons are important parameters in the study of leptonic or non-leptonic weak decay processes. In the non-relativistic limit, we compute the decay constants using the Van-Royen-Weisskopf formula[33],

$$f_{P/V}^2 = \frac{12 |\psi_{P/V}(0)|^2}{M_{P/V}} \bar{C}^2(\alpha_S); \quad (13)$$

**Table 3.** Masses of the  $b\bar{b}$  mesons(in GeV).

States	Our	Expt.[1]	Ref.[5]	Ref.[7]	Ref.[14]	Ref.[12]	Ref.[30]
$1^1S_0$	9.421	9.391	9.399	9.409	9.398	9.400	9.377
$1^3S_1$	9.464	9.460	9.461	9.458	9.460	9.460	9.464
$1^3P_0$	9.879	9.859	9.872	9.858	9.859	9.863	9.834
$1^3P_1$	9.892	9.893	9.893	9.906	9.892	9.892	9.864
$1^1P_1$	9.899	9.898	9.898	9.911	9.900	9.901	9.873
$1^3P_2$	9.908	9.912	9.906	9.924	9.912	9.913	9.886
$2^1S_0$	10.014		9.969	9.996	9.990	9.993	9.963
$2^3S_1$	10.023	10.023	10.006	10.012	10.023	10.023	10.007
$1^3D_1$	10.165		10.149	10.149	10.154	10.153	10.120
$1^3D_2$	10.168	10.164	10.153	10.154	10.161	10.158	10.126
$1^1D_2$	10.168		10.154	10.154	10.163	10.158	10.127
$1^3D_3$	10.171		10.156	10.156	10.166	10.162	10.130
$2^3P_0$	10.282	10.232	10.238	10.209	10.233	10.234	10.199
$2^3P_1$	10.294	10.255	10.257	10.250	10.255	10.255	10.224
$2^1P_1$	10.300	10.259	10.261	10.254	10.260	10.261	10.231
$2^3P_2$	10.307	10.269	10.268	10.265	10.268	10.268	10.242
$3^1S_0$	10.400		10.328	10.334	10.329	10.328	10.298
$3^3S_1$	10.404	10.355	10.351	10.345	10.355	10.355	10.339
$2^3D_1$	10.508		10.447	10.433	10.435		
$2^3D_2$	10.511		-	10.439	10.443		
$2^1D_2$	10.512		-	10.439	10.445		
$2^3D_3$	10.514		-	10.441	10.449		
$3^3P_0$	10.613		10.527	10.491	10.521		
$3^3P_1$	10.622		10.544	10.527	10.541		
$3^1P_1$	10.627		10.548	10.531	10.544		
$3^3P_2$	10.633		10.555	10.541	10.550		
$4^1S_0$	10.720		-	10.612	10.573		
$4^3S_1$	10.723	10.579	-	10.623	10.586		
$5^1S_0$	11.005		-	10.865	10.851		
$5^3S_1$	11.007	10.876	-	10.870	10.869		

where  $\bar{C}(\alpha_S)$  is the QCD correction factor given by[34]

$$\bar{C}^2(\alpha_S) = 1 - \frac{2\alpha_S}{\pi} \quad (14)$$

The computed  $f_P$  and  $f_V$  for  $b\bar{b}$  meson using equation (13) are tabulated in Table 4. The value in parenthesis is the decay constant with QCD correction. The Eq. (13) also gives the inequality[29]

$$\sqrt{m_v}f_v \geq \sqrt{m_p}f_p \quad (15)$$

Our results are in accordance with Eq. (15).

**Table 4.** Decay constants of the  $b\bar{b}$  meson(in GeV).

State	$f_P$		State	$f_V$	
	Present work	[35]		Present Work	[36]
1S	0.595(0.471)	0.711	1S	0.597(0.472)	$0.708 \pm 0.008$
2S	0.256(0.203)		2S	0.257(0.203)	
3S	0.182(0.144)		3S	0.182(0.144)	
4S	0.147(0.116)		4S	0.147(0.117)	
5S	0.126(0.100)		5S	0.126(0.100)	
6S	0.112(0.088)		6S	0.112(0.088)	

#### 4 Di-gamma and di-leptonic decay rates

The di-gamma decay of  $^1S_0$  state and the di-leptonic decay of  $^3S_1$  state using the conventional Van-Royen Weisskopf formula[33] is

$$\Gamma_0 = \frac{12\alpha^2 e_Q^4}{M_P^2} R_P^2(0), \quad (16)$$

$$\Gamma_{VW} = \frac{4\alpha^2 e_Q^4}{M_V^2} R_V^2(0) \quad (17)$$

Omitting  $\mathcal{O}(v^4\Gamma)$  the NRQCD formula for the decays can be written as[24]

$$\Gamma(^1S_0 \rightarrow \gamma\gamma) = \frac{2\text{Im}f_{\gamma\gamma}(^1S_0)}{\pi m_Q^2} |R_{1S_0}|^2 - \frac{N_c \text{Im}g_{\gamma\gamma}(^1S_0)}{\pi m_Q^4} \text{Re}(\bar{R}_s^* \nabla^2 \bar{R}_s) \quad (18)$$

$$\Gamma(^3S_0 \rightarrow e^+e^-) = \frac{N_c \text{Im}f_{ee}(^3S_1)}{\pi m_Q^2} |R_{3S_1}|^2 - \frac{N_c \text{Im}g_{ee}(^3S_1)}{\pi m_Q^4} \text{Re}(\bar{R}_s^* \nabla^2 \bar{R}_s) \quad (19)$$

The short distance coefficients  $f$ 's and  $g$ 's computed in the order of  $\alpha^2$  as[24]

$$\text{Im}f_{\gamma\gamma}(^1S_0) = \pi Q^4 \alpha^2 \quad (20)$$

$$\text{Im}g_{\gamma\gamma}(^1S_0) = -\frac{4\pi Q^4}{3} \alpha^2 \quad (21)$$

$$\text{Im}f_{ee}(^3S_1) = \frac{\pi Q^2 \alpha^2}{3} \quad (22)$$

$$\text{Im}g_{ee}(^3S_1) = -\frac{4\pi Q^2 \alpha^2}{9} \quad (23)$$

where color factor  $C_F = (N_c^2 - 1) / (2N_c) = 4/3$ . Details of NRQCD is nicely given in ref[24].

**Table 5.**  $0^{-+} \rightarrow \gamma\gamma$  and  $1^{--} \rightarrow l^+l^-$  decay rates(in keV).

Models	$0^{-+} \rightarrow \gamma\gamma$			$1^{--} \rightarrow l^+l^-$		
	$\Gamma_0$	$\Gamma_{\text{NRQCD}}$	$\Gamma_{\text{Others}}$	$\Gamma_0$	$\Gamma_{\text{NRQCD}}$	$\Gamma_{\text{Expt.}}$
BT[37]	0.569	0.278	0.364[24]	1.717	0.833	$1.340 \pm 0.018[1]$
PL[38]	0.408	0.195	0.497[39]	1.216	0.588	
Log[40]	0.437	0.211		1.303	0.633	
Cornell[41]	1.258	0.597		3.719	1.801	
Present	0.311	0.179		0.933	0.946	

## 5 Conclusion

In present work,  $b\bar{b}$  meson is studied in the general frame work of potential model. The mass spectra, decay constants and decay rates are calculated using the Gaussian wave function. The potential model parameters and the masses of the  $b\bar{b}$  meson are employed to study their decay properties in the frame work of NRQCD formalism as well as using the conventional Van-Royen-Weisskopf formula.

The mass spectra obtained in the present scheme is reasonably close to experimental values as well other theoretical results. The some excited states like 3S, 4S and 5S are little overestimated. The mass predicted for  $\eta_b(2S)$  is very close to the recent measurements of ref[4]. The decay constants of the pseudoscalar ( $f_P$ ) and the vector meson ( $f_V$ ) are computed with and without QCD corrections, is found to be 100 MeV off with the recent predictions [31, 35, 36, 42]. The departure from the predicted  $f_P$  and  $f_V$  need more refined mechanism related to their wave functions incorporating the confinement and hyperfine splitting. Using the predicted masses and radial wave functions at the origin, the di-gamma and di-leptonic, decay rates of bottomonium are studied and tabulated in Table-(5). The di-gamma decay rate calculated using Van-Rayan Weisskopf formula close to the other predicted values, but NRQCD results are well off from others. In the case of the di-leptonic decay both the results are underestimated. For the di-gamma prediction results calculated with potentials [37, 38, 40] are in good agreement with other theoretical predictions. For the di-leptonic decay rates, only Log potential is giving good result [40]. Finally, predicted results suggested that it required relativistic corrections to the potential, which will improve the present results.

## Acknowledgements

A. K. Rai acknowledge the financial support extended by D.S.T., India under SERB fast track scheme SR/FTP/PS-152/2012 and also thankful to Sardar Vallabhbai National Institute of Technology (Institute Research Grant (Dean (R&C)/1488/2013-14)) for providing financial assistance.

## References

- [1] J. Beringer, et. al., (Particle Data Group) Phys. Rev. **D86**, 010001 (2012)
- [2] K.A. Olive, et. al., (Particle Data Group) Chinese Phys. **C38**, 1 (2014)
- [3] I. Adachi et al. (Belle Collaboration), Phys. Rev. Lett. **108**, 032001 (2012)
- [4] S. Sandilya et al. (Belle Collaboration), Phys. Rev. Lett. **111**, 112001 (2013)
- [5] W.W. Repko, M.D. Santia, S.F. Radford, Nucl.Phys. **A924**, 65 (2014)
- [6] S.F. Radford, W.W. Repko, M.J. Saelim, Phys. Rev. D **80**, 034012 (2009)
- [7] T. Wei-Zhao, C. Lu, Y. You-Chang, C. Hong, Chin.Phys. **C37**, 083101 (2013), 1308.0960

- [8] A.K. Rai, J. Pandya, P. Vinodkumar, Indian J.Phys. **A80**, 387 (2006)
- [9] A.K. Rai, J. Pandya, P. Vinodkumar, Nucl.Phys. **A782**, 406 (2007)
- [10] A.M. Badalian, A.I. Veselov, B.L.G. Bakker, Phys. Rev. D **70**, 016007 (2004)
- [11] Y.A. Simonov, Physics of Atomic Nuclei **58**, 107 (1995)
- [12] D. Ebert, R. Faustov, V. Galkin, Phys. Rev. **D67**, 014027 (2003)
- [13] D. Ebert, R.N. Faustov, V.O. Galkin, Phys. Rev. D **79**, 114029 (2009)
- [14] D. Ebert, R. Faustov, V. Galkin, Eur. Phys. J. **C71**, 1825 (2011)
- [15] A.K. Rai, R.H. Parmar, P.C. Vinodkumar, J. Phys. G: Nucl. Part. Phys. **28**, 2275 (2002)
- [16] A.K. Rai, J.N. Pandya, P.C. Vinodkumar, J. Phys. G: Nucl. Part. Phys. **31**, 1453 (2005)
- [17] A.K. Rai, P. Vinodkumar, Pramana **66**, 953 (2006)
- [18] A.K. Rai, B. Patel, P.C. Vinodkumar, Phys. Rev. C **78**, 055202 (2008)
- [19] A.K. Rai, J. Pandya, P. Vinodkumar, Eur.Phys.J. **A38**, 77 (2008)
- [20] A.K. Rai, P. Vinodkumar, AIP Conf.Proc. **1257**, 316 (2010)
- [21] A.K. Rai, AIP Conf.Proc. **1343**, 415 (2011)
- [22] A.K. Rai, J.Phys.Conf.Ser. **374**, 012017 (2012)
- [23] A.K. Rai, N. Devlani, PoS **Hadron2013**, 045 (2013)
- [24] G.T. Bodwin, E. Braaten, G.P. Lepage, Phys. Rev. D **51**, 1125 (1995)
- [25] N. Devlani, A.K. Rai, Phys. Rev. D **84**, 074030 (2011)
- [26] N. Devlani, A.K. Rai, Eur. Phys. J. **A48**, 104 (2012)
- [27] N. Devlani, A.K. Rai, Int. J. Theor. Phys. **52**, 2196 (2013)
- [28] N. Devlani, V. Kher, A.K. Rai, Eur. Phys. J. **A50**, 154 (2014)
- [29] D.S. Hwang, G.H. Kim, Phys. Rev. D **55**, 6944 (1997)
- [30] E.J. Eichten, C. Quigg, Phys. Rev. D **49**, 5845 (1994)
- [31] B. Patel, P.C. Vinodkumar, J. Phys. G. **36**, 035003 (2009)
- [32] S. Gershtein, V. Kiselev, A. Likhoded, A. Tkabladze, Phys. Usp. **38**, 1 (1995)
- [33] R. Van Royen, V. Weisskopf, Nuovo Cim. **A50**, 617 (1967)
- [34] E. Braaten, S. Fleming, Phys. Rev. D **52**, 181 (1995)
- [35] J. Pandya, P. Vinodkumar, Pramana **57**, 821 (2001)
- [36] C.W. Hwang, Z.T. Wei, J. Phys. G **G34**, 687 (2007), hep-ph/0609036
- [37] W. Buchmüller, S.H.H. Tye, Phys. Rev. D **24**, 132 (1981)
- [38] A. Martin, Phys.Lett. **B93**, 338 (1980)
- [39] M. Shah, A. Parmar, P.C. Vinodkumar, Phys. Rev. D **86**, 034015 (2012)
- [40] C. Quigg, J.L. Rosner, Phys.Lett. **B71**, 153 (1977)
- [41] E. Eichten, K. Gottfried, T. Kinoshita, K.D. Lane, T.M. Yan, Phys. Rev. D **17**, 3090 (1978)
- [42] A. Parmar, B. Patel, P. Vinodkumar, Nucl. Phys. **A848**, 299 (2010)