EPJ Web of Conferences 95, 05005 (2015)
DOI: 10.1051/epjconf/20159505005

Study of various spectroscopic properties of the D_s meson

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Abstract. Spectroscopic parameters of the $D_s(c\bar{s})$ meson are obtained using phenomenological quark antiquark potential(coulomb plus power) model consisting of $O(1/m)$ correction to the potential. Within Variational scheme Gaussian wave function is employed with a hamiltonian incorporating kinematic relativistic corrections to obtain various properties such as the mass spectra, decay constants, electromagnetic transitions. The results are compared with various experimental measurement as well as other theoretical predictions.

1 Introduction

The D_s meson is a light-heavy quark structure composed of the charm and the strange quark[1]. The ground state masses as well as the 1P state masses of the D_s meson have been measured quite accurately[1]. Mass spectrum as well as many other spectroscopic properties of the D_s meson have been extensively studied in various theoretical schemes[2]. There exists mutual variation in the masses of 1P states predicted by these models as well as with the experimental measurements. Recently some of the higher excited states such as the $D_{s1}(2710)$, $D_{sJ}(2860)$ and $D_{sJ}(2040)$ have been experimentally measured. Being a light-heavy quark meson system the D_s meson requires a relativistic treatment. In this paper we employ a potential model scheme that incorporates kinematic relativistic corrections to the kinetic energy term as well as $O(1/m)$ corrections to the potential energy term of the hamiltonian. This allows for an opportunity to test the applicability and validity of such a model to a light-heavy quark bound system.

The paper is organized as followingly. Section 2 provides the theoretical background and the calculation of the mass spectrum of the D_s meson. In section 3 we obtain the decay constants. In section 4 electromagnetic transition rates are estimated while in section 5 we conclude the present work.

2 Theoretical formulation

For the study of the D_s meson we consider the relativistic Hamiltonian in which motion of the quarks inside the meson is relativistic[3–5]

$$H = \sqrt{\mathbf{p}^2 + m_c^2} + \sqrt{\mathbf{p}^2 + m_{\bar{s}}^2} + V(\mathbf{r}) \tag{1}$$

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where $\mathbf{p}$ is the relative momentum of the quark-antiquark and m_c is the c quark mass and $m_{\bar{s}}$ is the s quark mass. The Hamiltonian in Eq(1) represents the energy of the meson in the meson rest frame. We expand the kinetic energy(K.E.) part of the Hamiltonian, as

$$K.E. = \frac{\mathbf{p}^2}{2}\left(\frac{1}{m_c} + \frac{1}{m_{\bar{s}}}\right) - \frac{\mathbf{p}^4}{8}\left(\frac{1}{m_c^3} + \frac{1}{m_{\bar{s}}^3}\right) + O(p^6) \tag{2}$$

and $V(\mathbf{r})$ is the quark-antiquark potential[6, 7],

$$V(r) = V^{(0)}(r) + \left(\frac{1}{m_c} + \frac{1}{m_{\bar{s}}}\right)V^{(1)}(r) + O\left(\frac{1}{m^2}\right); \tag{3}$$

where[8–10],

$$V^{(0)}(r) = -\frac{\alpha_c}{r} + Ar + V_0 \tag{4}$$

A is the potential parameter and V_0 is a constant. $\alpha_c = (4/3)\alpha_S\left(M^2\right)$, $\alpha_S\left(M^2\right)$ is the strong running coupling constant. The non-perturbative form of $V^{(1)}(r)$ is not yet known, but leading order perturbation theory yields

$$V^{(1)}(r) = -C_F C_A \alpha_s^2/4r^2; \tag{5}$$

where $C_F = 4/3$ and $C_A = 3$ are the Casimir charges of the fundamental and adjoint representation, respectively[6]. The value of the QCD coupling constant $\alpha_s(M^2)$ is determined through the simplest model with freezing[11, 12], namely

$$\alpha_s(M^2) = \frac{4\pi}{\left(11 - \frac{2}{3}n_f\right)\ln\frac{M^2+M_B^2}{\Lambda^2}} \tag{6}$$

where $M = 2m_c m_{\bar{s}}/\left(m_c + m_{\bar{s}}\right)$, $M_B = 0.95$ GeV[11, 12], and $\Lambda = 0.413$ GeV[13].

We have used the gaussian wave function in the present study. The gaussian wave function in position space has the form

$$R_{nl}(\mu, r) = \mu^{3/2}\left(\frac{2\,(n-1)!}{\Gamma\,(n+l+1/2)}\right)^{1/2}(\mu r)^l\, e^{-\mu^2 r^2/2} L_{n-1}^{l+1/2}(\mu^2 r^2) \tag{7}$$

and in momentum space has the form

$$R_{nl}(\mu, p) = \frac{(-1)^n}{\mu^{3/2}}\left(\frac{2\,(n-1)!}{\Gamma\,(n+l+1/2)}\right)^{1/2}\left(\frac{p}{\mu}\right)^l e^{-p^2/2\mu^2} L_{n-1}^{l+1/2}\left(\frac{p^2}{\mu^2}\right) \tag{8}$$

Here, μ is the variational parameter and L is Laguerre polynomial.

For the present study, we employ the Ritz variational scheme. We obtain the expectation values of the Hamiltonian as

$$H\psi = E\psi \tag{9}$$

The variational parameter, μ is determined for each state using the Virial theorem[14]. Gaussian wavefunction in position space has been employed to obtain the expectation value of the potential energy part in the Virial theorem while momentum space wave function has been used to obtain the kinetic energy part.

Table 1. Spin averaged masses of D_s meson.

State	μ(GeV)	$\|R(0)\|$GeV$^{3/2}$	M_{SA}(GeV)					
			Present work	Expt.[1]	[16]	[17]	[18]	[2]
1S	0.451	0.454	2.076	2.076	2.076	2.082	2.074	2.075
2S	0.322	0.224	2.712		2.779	2.700	2.706	2.720
3S	0.268	0.152	3.279		3.323	3.165	3.076	3.236
4S	0.237	0.117	3.806				3.356	3.665
5S	0.216	0.096	4.311					
6S	0.201	0.082	4.802					
1P	0.351		2.533	2.514	2.568	2.531	2.538	2.537
2P	0.283		3.096		3.142	3.008	2.954	3.119
1D	0.313		2.865		2.917	2.873	2.850	2.950
2D	0.264		3.395		3.288	3.161		3.436

As the interaction potential assumed here does not contain the spin dependent part, Eq(9) gives the spin averaged masses of the system. The calculated spin averaged mass os the ground state is matched with the experimental spin-averaged mass using the equation [10]

$$M_{SA} = M_P + \frac{3}{4}(M_V - M_P) \tag{10}$$

;where M_V and M_P are the vector and pseudoscalar meson ground state masses taken from ref [1]. This fixes the parameter V_0. Using this value of V_0 we calculate S, P, and D wave spin-averaged masses of D_s mesons which are listed in Table 1. For the comparison for the nJ state, we compute the spin-averaged or the center of weight mass from the respective theoretical values as [10]

$$M_{CW,n} = \frac{\Sigma_J 2(2J+1)M_{nJ}}{\Sigma_J 2(2J+1)} \tag{11}$$

where, $M_{CW,n}$ denotes the spin-averaged mass of the n state and M_{nJ} represents the mass of the meson in the nJ state.

The value of the radial wave function $R(0)$ for 0^{-+} and 1^{--} states would be different due to their spin dependent hyperfine interaction. The spin hyperfine interaction of the heavy-light flavored mesons is small and this can cause a small shift in the value of the wave function at the origin[10, 15].

The parameters used to calculate the low lying masses of the D_s meson are $A = 0.135\ GeV^{-1}$, $m_{\bar{s}} = 0.55\ GeV$, $m_c = 1.35\ GeV$ and the value of the constant $V_0 = -0.123$ GeV.The spin averaged masses for S, P and D states are tabulated in Table 1. It can be observed that the spin-averaged masses obtained are in good agreement with experimental and other theoretical predictions.

2.1 Excited states

We add separately (in Eq.(9)) the spin-dependent part of the usual one gluon exchange potential (OGEP) between the quark anti quark for computing the hyperfine and spin-orbit shifting of the low-lying S, P and D-states. Thus to take into account the spin dependent and spin-orbit interaction, causing the splitting of the nL levels one introduces additional term in the Hamiltonian[19–21]

$$\begin{aligned} V_{SD}(\mathbf{r}) \quad = \quad & \left(\frac{\mathbf{L}\cdot\mathbf{S_c}}{2m_c^2} + \frac{\mathbf{L}\cdot\mathbf{S_{\bar{s}}}}{2m_{\bar{s}}^2}\right)\left(-\frac{dV^{(0)}(r)}{rdr} + \frac{8}{3}\frac{\alpha_S}{r^3}\right) + \frac{4}{3}\frac{\alpha_S}{m_c m_{\bar{s}}}\frac{\mathbf{L}\cdot\mathbf{S}}{r^3} + \frac{8\alpha_S}{9m_c m_{\bar{s}}}\mathbf{S_c}\cdot\mathbf{S_{\bar{s}}}4\pi\delta(\mathbf{r}) \\ & + \frac{4}{3}\alpha_S\frac{1}{m_c m_{\bar{s}}}\left\{3(\mathbf{S_c}\cdot\mathbf{n})(\mathbf{S_{\bar{s}}}\cdot\mathbf{n}) - (\mathbf{S_c}\cdot\mathbf{S_{\bar{s}}})\right\}\frac{1}{r^3}, \quad \mathbf{n} = \frac{\mathbf{r}}{r} \end{aligned} \tag{12}$$

where $V^{(0)}(r)$ is the phenomenological potential, the first terms takes into account the relativistic corrections to the potential $V(r)$, the second term accounts spin orbital interaction, third term is usual spin-spin interaction part which is responsible for pseudoscalar and vector meson splitting(Eq. (15) & (16)) and fourth term stands for tensor interaction.

In the case of quark and antiquark of unequal mass charge-conjugation parity is no longer a good quantum number and so the states with $J = L$, are mixtures of spin-triplet $\left|^3L_L\right\rangle$ and spin-singlet $\left|^1L_L\right\rangle$ states: $J = L = 1,\ 2,\ 3, \ldots$

$$|\psi_J\rangle \quad = \quad \left|^1L_L\right\rangle \cos\phi + \left|^3L_L\right\rangle \sin\phi \tag{13}$$

$$\left|\psi'_J\right\rangle \quad = \quad -\left|^1L_L\right\rangle \sin\phi + \left|^3L_L\right\rangle \cos\phi \tag{14}$$

where ϕ is the mixing angle and the primed state has the heavier mass. Such mixing occurs due to the nondiagonal spin-orbit and tensor terms in Eq (9). The masses of the physical states were obtained by diagonalizing the mixing matrix. The calculated values of the mass spectra of D_s meson are listed in Table 2. We are following spectroscopic notation $n^{2S+1}L_J$ in Table 2. Overall the mass spectrum is in satisfactory agreement with others.

Table 2. Masses of the D_s mesons(in GeV).

State	Present	Expt.[1]	[2]	[18]	[17]	[16]	[24]
1^1S_0	1.962	1.968	1.969	1.975	1.940	1.965	1.969
1^3S_1	2.108	2.112	2.111	2.108	2.130	2.113	2.107
1^3P_0	2.436	2.318	2.509	2.455	2.380	2.487	2.344
$1P'_1$	2.536	2.535	2.574	2.522	2.520	2.605	2.510
$1P_1$	2.518	2.460	2.536	2.502	2.510	2.535	2.488
1^3P_2	2.558	2.573	2.571	2.586	2.580	2.581	2.559
2^1S_0	2.684		2.688	2.659	2.610	2.700	2.640
2^3S_1	2.722	2.710^{+12}_{-7}	2.731	2.722	2.730	2.806	2.714
1^3D_1	2.881		2.913	2.838	2.820	2.900	2.804
$1D'_2$	2.867		2.931	2.845	2.860	2.913	2.849
$1D_2$	2.846		2.961	2.856	2.880	2.953	2.788
1^3D_3	2.851	2.862^{+6}_{-3}	2.971	2.857	2.900	2.925	2.811
2^3P_0	3.033		3.054	2.901	2.900	3.067	2.830
$2P_1$	3.086	3.044^{+30}_{-9}	3.067	2.928	3.000	3.114	2.958
$2P'_1$	3.100		3.154	2.942	3.010	3.165	2.995
2^3P_2	3.112		3.142	2.980	3.060	3.157	3.040
3^1S_0	3.265		3.219	3.044	3.090	3.259	
3^3S_1	3.283		3.242	3.087	3.190	3.345	
2^3D_1	3.411		3.383	3.144	3.250		3.217
$2D'_2$	3.396		3.403	3.172	3.280		3.260
$2D_2$	3.380		3.456	3.167	3.290		3.217
2^3D_3	3.382		3.469	3.157	3.310		3.240
4^1S_0	3.798		3.652	3.331			
4^3S_1	3.809		3.669	3.364			

3 Decay constants

The decay constants of mesons are important parameters in the study of leptonic or non-leptonic weak decay processes. In the non-relativistic limit, we compute the decay constants using the Van-Royen-Weisskopf formula[25],

$$f_{P/V}^{2} = \frac{12\left|\psi_{P/V}(0)\right|^{2}}{M_{P/V}}\bar{C}^{2}(\alpha_S); \tag{15}$$

where $\bar{C}(\alpha_S)$ is the QCD correction factor given by[26]

$$\bar{C}^{2}(\alpha_S) = 1 - \frac{\alpha_S}{\pi}\left[2 - \frac{m_Q - m_{\bar{q}}}{m_Q + m_{\bar{q}}}\ln\frac{m_Q}{m_{\bar{q}}}\right]. \tag{16}$$

The computed f_P and f_V for D_s meson using equation (15) are tabulated in Table 3. The value in parenthesis is the decay constant with QCD correction. The Eq. (15) also gives the inequality[14]

$$\sqrt{m_v} f_v \geq \sqrt{m_p} f_p \tag{17}$$

Our results are in accordance with Eq. (17).

Table 3. Decay constants of the D_s meson(in GeV).

		1S	2S	3S	4S
f_P	This work	0.299 (0.200)	0.131 (0.088)	0.082 (0.055)	0.059 (0.039)
	[27]	0.254 ± 0.006			
	[28]	0.248 ± 0.002			
	[29]	0.235 ± 0.024			
f_V	This work	0.312 (0.209)	0.133 (0.089)	0.082 (0.055)	0.059 (0.039)
	[30]	0.335			
	[31]	$0.326^{+0.021}_{-0.017}$			
	[32]	0.254			
	[33]	0.242			

4 Electromagnetic transition widths

4.1 Electric Dipole Transition

The radiative widths are calculated in the dipole approximation. The E1 matrix elements are determined by using the variational radial wave functions of the initial and the final state and explicitly performing the angular integration given by[34]

$$\Gamma_{fi} = \frac{4\alpha}{9}\left(\frac{e_Q m_{\bar{q}} - e_{\bar{q}} m_Q}{m_{\bar{q}} + m_Q}\right)^{2} k^{3} \left|\langle f|r|i\rangle\right|^{2} \frac{E_f}{M_i} \times \begin{cases} 1 & \text{for } {}^3P_J \rightarrow {}^3S_1 \\ 1 & \text{for } {}^1P_1 \rightarrow {}^1S_0 \\ (2J+1)/3 & \text{for } {}^3S_1 \rightarrow {}^3P_J \\ 3 & \text{for } {}^1S_0 \rightarrow {}^1P_1 \end{cases} \tag{18}$$

Here, α is the fine structure constant, k is the photon energy, $e_{\bar{q}}$ and e_Q are the quark charges in units of the proton charge, E_f is the energy of the final meson state, M_i is the mass of the initial meson state, and $m_{\bar{q}}$ and m_Q are the quark masses employed within the present work.

The E1 radiative transition widths are listed in tables (4).

Table 4. E1 transition widths in the D_s meson.

Transition	k(GeV)	Γ(keV)	[35]	[34]	[36]	[24]
$1^3P_2 \to 1^3S_1\gamma$	0.411	2.74	8.8	44.1	19	
$1P_1' \to 1^3S_1\gamma$	0.392	0.05	4.76	8.90	5.6	
$1P_1' \to 1^1S_0\gamma$	0.509	5.11	3.49	54.5	15	
$1P_1 \to 1^1S_0\gamma$	0.495	0.09	4.9	12.8	6.2	
$1P_1 \to 1^3S_1\gamma$	0.377	2.07	0.13	15.5	5.5	
$1^3P_0 \to 1^3S_1\gamma$	0.306	1.13	1.0	4.92	1.9	
$2^3S_1 \to 1^3P_2\gamma$	0.158	0.14				0.1
$2^3S_1 \to 1^3P_0\gamma$	0.271	0.73				6.9
$2^1S_0 \to 1P_1'\gamma$	0.144	0.006				
$2^1S_0 \to 1P_1\gamma$	0.161	0.44				
$2^3S_1 \to 1P_1'\gamma$	0.180	0.004				
$2^3S_1 \to 1P_1\gamma$	0.196	0.27				

4.2 Magnetic Dipole Transitions

The M1 rate for transitions between S-wave levels is given by[24, 37] is

$$\Gamma_{M1}(i \to f + \gamma) = \frac{16\alpha}{3}\mu^2 k^3 (2J_f + 1) \left|\langle f|\, j_0(kr/2)\, |i\rangle\right|^2 , \tag{19}$$

where the magnetic dipole moment is

$$\mu = \frac{m_{\bar{q}} e_Q - m_Q e_{\bar{q}}}{4 m_{\bar{q}} m_Q} \tag{20}$$

and k is the photon energy. Rates for the allowed transitions between the spin-triplet and the spin-singlet states are given in Table (5).

Table 5. M1 transition widths in the D_s meson.

Transition	k(GeV)	Γ(keV)	[34]	[38]
$1^3S_1 \to 1^1S_0\gamma$	0.141	0.085	1.91	0.2
$2^3S_1 \to 2^1S_0\gamma$	0.038	0.002		
$3^3S_1 \to 3^1S_0\gamma$	0.018	0.000		
$2^3S_1 \to 1^1S_0\gamma$	0.654	1.28		
$2^1S_0 \to 1^3S_1\gamma$	0.514	1.82		

5 Conclusion

In this paper we have obtained the mass spectra and various decay properties of the D_s meson within the framework of potential models with kinematic relativistic correction to the kinetic energy term as well as $O(1/m)$ correction to potential energy term. Looking at table 1 it can be seen that the spin averaged masses for the various s, p and d wave states are in good agreement with the experimental measurements as well as other theoretical estimates. From table 2 it can be observed that the entire mass spectrum is also in fair agreement with various models and experimental results. The pseudoscalar and vector decay constants without QCD correction are in satisfactory agreement with other theoretical predictions as can be seen from table 3. The e1 transition rates listed in table 4 as well as M1 transition rates listed in table 5 obtained with the present framework show significant disagreement with other theoretical estimates. In conclusion we find that the present model is adequately predicts the mass spectrum however the decay properties are in disagreement.

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