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Spectroscopic properties of the B meson

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Abstract. Investigation of the $B(b\bar{q}); q = u, d$ meson properties is carried out using variational method within phenomenological quark antiquark potential(coulomb plus power) model using the Gaussian wave function. $O(1/m)$ correction to the potential energy term and relativistic corrections to the kinetic energy term of the hamiltonian are incorporated. Spin-orbit, spin-spin and tensor interactions are employed to obtain the mass spectra. Various other properties such as the decay constants, e1 and m1 transitions are also obtained

1 Introduction

Potential models based on the asymptotic freedom and the confinement features of QCD have been quite successful in the study of mesons composed of quarks which are heavy. The $B(b\bar{q})$; $q = u, d$, consists of a heavy and a light quark[1]. Therefore the validity of a potential model approach for such a system should be judged by its predictions. The B meson has been extensively studied in various theoretical schemes including potential models[2–7]. In general there is a fair agreement between the experimentally measured ground state masses and the various models. However for the $L = 1$ 1^3P_0 state one finds significant differences.

In this paper we employ a conventional potential model scheme that incorporates kinematic relativistic corrections to the kinetic energy term as well as $O(1/m)$ corrections to the potential energy term[8]. The paper is organized as following. In section 2 theoretical formulation as well as results for the mass spectra are outlined. Subsequently in section 3 decay constants and in section 4 electromagnetic transition widths are obtained. Finally the conclusion is made in section 5.

2 Theoretical formulation

For the study of the B meson we consider the relativistic Hamiltonian in which motion of the quarks inside the meson is relativistic[9–11]

$$H = \sqrt{\mathbf{p}^2 + m_b^2} + \sqrt{\mathbf{p}^2 + m_{\bar{q}}^2} + V(\mathbf{r}) \tag{1}$$

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where $\mathbf{p}$ is the relative momentum of the quark-antiquark and m_b is the b quark mass and $m_{\bar{q}}$ is the $q = u, d$ quark mass. The Hamiltonian in Eq(1) represents the energy of the meson in the meson rest frame. We expand the kinetic energy(K.E.) part of the Hamiltonian, as

$$K.E. = \frac{\mathbf{p}^2}{2}\left(\frac{1}{m_b} + \frac{1}{m_{\bar{q}}}\right) - \frac{\mathbf{p}^4}{8}\left(\frac{1}{m_b^3} + \frac{1}{m_{\bar{q}}^3}\right) + O(p^6) \tag{2}$$

and $V(\mathbf{r})$ is the quark-antiquark potential[12],

$$V(r) = V^{(0)}(r) + \left(\frac{1}{m_b} + \frac{1}{m_{\bar{q}}}\right) V^{(1)}(r) + O\left(\frac{1}{m^2}\right); \tag{3}$$

where[13–15],

$$V^{(0)}(r) = -\frac{\alpha_c}{r} + Ar + V_0 \tag{4}$$

A is the potential parameter and V_0 is a constant. $\alpha_c = (4/3)\alpha_S\left(M^2\right)$, $\alpha_S\left(M^2\right)$ is the strong running coupling constant. The non-perturbative form of $V^{(1)}(r)$ is not yet known, but leading order perturbation theory yields

$$V^{(1)}(r) = -C_F C_A \alpha_s^2/4r^2; \tag{5}$$

where $C_F = 4/3$ and $C_A = 3$ are the Casimir charges of the fundamental and adjoint representation, respectively[12]. The value of the QCD coupling constant $\alpha_s(M^2)$ is determined through the simplest model with freezing[16, 17], namely

$$\alpha_s(M^2) = \frac{4\pi}{\left(11 - \frac{2}{3}n_f\right)\ln\frac{M^2+M_B^2}{\Lambda^2}} \tag{6}$$

where $M = 2m_Q m_{\bar{q}}/\left(m_Q + m_{\bar{q}}\right)$, $M_B = 0.95$ GeV[16, 17], and $\Lambda = 0.413$ GeV[18].

We have used the gaussian wave function in the present study. The gaussian wave function in position space has the form

$$R_{nl}(\mu, r) = \mu^{3/2}\left(\frac{2(n-1)!}{\Gamma(n+l+1/2)}\right)^{1/2}(\mu r)^l e^{-\mu^2 r^2/2} L_{n-1}^{l+1/2}(\mu^2 r^2) \tag{7}$$

and in momentum space has the form

$$R_{nl}(\mu, p) = \frac{(-1)^n}{\mu^{3/2}}\left(\frac{2(n-1)!}{\Gamma(n+l+1/2)}\right)^{1/2}\left(\frac{p}{\mu}\right)^l \times e^{-p^2/2\mu^2} L_{n-1}^{l+1/2}\left(\frac{p^2}{\mu^2}\right) \tag{8}$$

Here, μ is the variational parameter and L is Laguerre polynomial.

For the present study, we employ the Ritz variational scheme. We obtain the expectation values of the Hamiltonian as

$$H\psi = E\psi. \tag{9}$$

The variational parameter, μ is determined for each state using the Virial theorem[19]. Gaussian wavefunction in position space has been employed to obtain the expectation value of the potential energy part in the Virial theorem while momentum space wave function has been used to obtain the kinetic energy part.

Table 1. Spin averaged masses of B meson.

State	μ(GeV)	$\lvert R(0)\rvert\left(\text{GeV}^{3/2}\right)$	M_{SA}(GeV)					
			Present work	Expt.[1]	[2]	[7]	[4]	[3]
1S	0.494	0.521	5.314	5.314	5.314	5.313	5.318	5.313
2S	0.317	0.218	5.878		5.902	5.842	5.860	5.912
3S	0.260	0.145	6.329		6.385	6.131	6.232	6.340
4S	0.231	0.112	6.731		6.785	6.347		
1P	0.340		5.758		5.745	5.696	5.695	5.717
2P	0.270		6.210		6.249	6.030	6.105	6.184
1D	0.297		6.036		6.106	5.924	5.970	6.007
2D	0.251		6.448		6.540	6.183		6.306

As the interaction potential assumed here does not contain the spin dependent part, Eq(9) gives the spin averaged masses of the system. The calculated spin averaged mass of the ground state is matched with the experimental spin-averaged mass using the equation [15]

$$M_{SA} = M_P + \frac{3}{4}(M_V - M_P) \tag{10}$$

;where M_V and M_P are the vector and pseudoscalar meson ground state masses taken from ref [1]. This fixes the parameter V_0. Using this value of V_0 we calculate S, P, and D wave spin-averaged masses of B mesons which are listed in Table 1. For the comparison for the nJ state, we compute the spin-averaged or the center of weight mass from the respective theoretical values as [15]

$$M_{CW,n} = \frac{\Sigma_J 2(2J+1)M_{nJ}}{\Sigma_J 2(2J+1)} \tag{11}$$

where, $M_{CW,n}$ denotes the spin-averaged mass of the n state and M_{nJ} represents the mass of the meson in the nJ state.

The value of the radial wave function $R(0)$ for 0^{-+} and 1^{--} states would be different due to their spin dependent hyperfine interaction. The spin hyperfine interaction of the heavy-light flavored mesons is small and this can cause a small shift in the value of the wave function at the origin[15, 20].

The parameters used to calculate the low lying masses of the B meson are $A = 0.1\ GeV^{-1}$, $m_{\bar{q}} = 0.45\ GeV$, $m_b = 4.88\ \ GeV$ and the value of the constant $V_0 = -0.132$ GeV. The spin averaged masses for S, P and D states are tabulated in Table 1. It can be observed that the spin-averaged masses obtained are in good agreement with experimental and other theoretical predictions.

2.1 Excited states

We add separately (in Eq.(9)) the spin-dependent part of the usual one gluon exchange potential (OGEP) between the quark anti quark for computing the hyperfine and spin-orbit shifting of the low-lying S, P and D-states. Thus to take into account the spin dependent and spin-orbit interaction, causing the splitting of the nL levels one introduces additional term in the Hamiltonian[21–23]

$$\begin{aligned} V_{SD}(\mathbf{r}) \quad = \quad & \left(\frac{\mathbf{L}\cdot\mathbf{S_b}}{2m_b^2} + \frac{\mathbf{L}\cdot\mathbf{S_{\bar{c}}}}{2m_{\bar{c}}^2}\right)\left(-\frac{dV^{(0)}(r)}{rdr} + \frac{8}{3}\frac{\alpha_S}{r^3}\right) + \frac{4}{3}\frac{\alpha_S}{m_b m_{\bar{c}}}\frac{\mathbf{L}\cdot\mathbf{S}}{r^3} + \frac{8\alpha_S}{9m_b m_{\bar{c}}}\mathbf{S_b}\cdot\mathbf{S_{\bar{c}}}4\pi\delta(\mathbf{r}) \\ & + \frac{4}{3}\alpha_S\frac{1}{m_b m_{\bar{c}}}\left\{3(\mathbf{S_b}\cdot\mathbf{n})(\mathbf{S_{\bar{c}}}\cdot\mathbf{n}) - (\mathbf{S_b}\cdot\mathbf{S_{\bar{c}}})\right\}\frac{1}{r^3}, \qquad \mathbf{n} = \frac{\mathbf{r}}{r} \end{aligned} \tag{12}$$

where $V^{(0)}(r)$ is the phenomenological potential, the first terms takes into account the relativistic corrections to the potential $V(r)$, the second term accounts spin orbital interaction, third term is usual spin-spin interaction part which is responsible for pseudoscalar and vector meson splitting(Eq. (15) & (16)) and fourth term stands for tensor interaction.

In the case of quark and antiquark of unequal mass charge-conjugation parity is no longer a good quantum number and so the states with $J = L$, are mixtures of spin-triplet $\left|^3L_L\right\rangle$ and spin-singlet $\left|^1L_L\right\rangle$ states: $J = L = 1,\ 2,\ 3, \ldots$

$$|\psi_J\rangle = \left|^1L_L\right\rangle \cos\phi + \left|^3L_L\right\rangle \sin\phi \tag{13}$$

$$\left|\psi'_J\right\rangle = -\left|^1L_L\right\rangle \sin\phi + \left|^3L_L\right\rangle \cos\phi \tag{14}$$

where ϕ is the mixing angle and the primed state has the heavier mass. Such mixing occurs due to the nondiagonal spin-orbit and tensor terms in Eq (9). The masses of the physical states were obtained by diagonalizing the mixing matrix. The calculated values of the mass spectra of B meson are listed in Table 2. We are following spectroscopic notation $n^{2S+1}L_J$ in Table 2. Overall the mass spectrum is in satisfactory agreement with experimental as well as other theoretical predictions.

Table 2. Masses of the B mesons(in GeV).

State	This Work	Expt.[1]	[24]	[7]	[4]	[3]	[6]	[25]
1^1S_0	5.264	5.279	5.280	5.277	5.280	5.279	5.279	5.279
1^3S_1	5.330	5.325	5.326	5.325	5.330	5.324	5.324	5.325
1^3P_0	5.729	$5.732^{\pm0.005}_{\pm0.02}$	5.749	5.678	5.650	5.706	5.689	5.795
$1P'_1$	5.757		5.774	5.699	5.690	5.742	5.744	5.859
$1P_1$	5.755	5.723	5.723	5.686	5.690	5.700	5.731	5.839
1^3P_2	5.766	5.743	5.741	5.704	5.710	5.714	5.759	5.875
2^1S_0	5.870		5.890	5.822	5.830	5.886	5.892	
2^3S_1	5.881		5.906	5.848	5.870	5.920	5.924	
1^3D_1	6.069		6.119	6.005	5.970	6.025		
$1D'_2$	6.077		6.121	5.955	5.980	6.037		
$1D_2$	6.007		6.103	5.920	5.960	5.985		
1^3D_3	6.013		6.091	5.871	5.970	5.993		
2^3P_0	6.195		6.221	6.010	6.060	6.163		
$2P_1$	6.206		6.209	6.022	6.100	6.175		
$2P'_1$	6.214		6.281	6.028	6.100	6.194		
2^3P_2	6.213		6.260	6.040	6.120	6.188		
3^1S_0	6.326		6.379	6.117	6.210	6.320		
3^3S_1	6.331		6.387	6.136	6.240	6.347		
2^3D_1	6.425		6.534	6.248	6.240			
$2D'_2$	6.482		6.554	6.207	6.320			
$2D_2$	6.476		6.528	6.179	6.310			
2^3D_3	6.429		6.542	6.140	6.320			
4^1S_0	6.729		6.781	6.335	6.520			
4^3S_1	6.732		6.786	6.351				

3 Decay constants

The decay constants of mesons are important parameters in the study of leptonic or non-leptonic weak decay processes. In the non-relativistic limit, we compute the decay constants using the Van-Royen-Weisskopf formula[26],

$$f_{P/V}^2 = \frac{12\left|\psi_{P/V}(0)\right|^2}{M_{P/V}}\bar{C}^2(\alpha_S); \tag{15}$$

where $\bar{C}(\alpha_S)$ is the QCD correction factor given by[27]

$$\bar{C}^2(\alpha_S) = 1 - \frac{\alpha_S}{\pi}\left[2 - \frac{m_Q - m_{\bar{q}}}{m_Q + m_{\bar{q}}}\ln\frac{m_Q}{m_{\bar{q}}}\right]. \tag{16}$$

The computed f_P and f_V for B meson using equation (15) are tabulated in Table 3. The Eq. (15) also gives the inequality[19]

$$\sqrt{m_v} f_v \geq \sqrt{m_p} f_p \tag{17}$$

Our results are in accordance with Eq. (17).

Table 3. Decay constants of the B meson(in GeV).

		1S	2S	3S	4S
f_P	This work	0.219	0.088	0.056	0.042
	[28]	0.191 ± 0.009			
	[29]	0.197 ± 0.009			
	[30]	0.193 ± 0.011			
	[31]	0.198 ± 0.014			
f_V	This work	0.221	0.088	0.056	0.042
	[32]	$0.190^{+0.028}_{-0.027}$			
	[33]	0.190			
	[34]	0.164			
	[35]	0.194 ± 0.008			

4 Electromagnetic transition widths

4.1 Electric Dipole Transition

The radiative widths are calculated in the dipole approximation. The E1 matrix elements are determined by using the variational radial wave functions of the initial and the final state and explicitly performing the angular integration given by[36]

$$\Gamma_{fi} = \frac{4\alpha}{9}\left(\frac{e_Q m_{\bar{q}} - e_{\bar{q}} m_Q}{m_{\bar{q}} + m_Q}\right)^2 k^3 \left|\langle f\,|r|\,i\rangle\right|^2 \frac{E_f}{M_i} \times \begin{cases} 1 & \text{for } {}^3P_J \to {}^3S_1 \\ 1 & \text{for } {}^1P_1 \to {}^1S_0 \\ (2J+1)/3 & \text{for } {}^3S_1 \to {}^3P_J \\ 3 & \text{for } {}^1S_0 \to {}^1P_1 \end{cases} \tag{18}$$

Here, α is the fine structure constant, k is the photon energy, $e_{\bar{q}}$ and e_Q are the quark charges in units of the proton charge, E_f is the energy of the final meson state, M_i is the mass of the initial meson state, and $m_{\bar{q}}$ and m_Q are the quark masses employed within the present work.

The E1 radiative transition widths are listed in tables (4).

4.2 Magnetic Dipole Transitions

The M1 rate for transitions between S-wave levels is given by[37, 38]

$$\Gamma_{M1}(i \to f + \gamma) = \frac{16\alpha}{3}\mu^2 k^3 (2J_f + 1) \left|\langle f|\, j_0(kr/2)\, |i\rangle\right|^2 , \tag{19}$$

where the magnetic dipole moment is

$$\mu = \frac{m_{\bar{q}} e_Q - m_Q e_{\bar{q}}}{4 m_{\bar{q}} m_Q} \tag{20}$$

and k is the photon energy. Rates for the allowed transitions between the spin-triplet and the spin-singlet states are given in Table (4).

Table 4. Electromagnetic transition widths in the B meson.

Transition	k(MeV)	Γ(keV)
$1^3P_2 \to 1^3S_1\gamma$	0.419	79.98
$1P'_1 \to 1^3S_1\gamma$	0.411	14.99
$1P'_1 \to 1^1S_0\gamma$	0.472	91.72
$1P_1 \to 1^1S_0\gamma$	0.470	22.40
$1P_1 \to 1^3S_1\gamma$	0.409	59.60
$1^3P_0 \to 1^3S_1\gamma$	0.385	61.91
$2^3S_1 \to 1^3P_2\gamma$	0.114	4.16
$2^3S_1 \to 1^3P_0\gamma$	0.150	1.89
$2^1S_0 \to 1P'_1\gamma$	0.111	5.51
$2^1S_0 \to 1P_1\gamma$	0.113	1.45
$2^3S_1 \to 1P'_1\gamma$	0.123	0.611
$2^3S_1 \to 1P_1\gamma$	0.125	2.61
$1^3S_1 \to 1^1S_0\gamma$	0.066	0.314
$2^3S_1 \to 2^1S_0\gamma$	0.012	0.002
$3^3S_1 \to 3^1S_0\gamma$	0.005	0.000
$2^3S_1 \to 1^1S_0\gamma$	0.585	41.087
$2^1S_0 \to 1^3S_1\gamma$	0.514	85.120

5 Conclusion

The mass spectra, decay constants as well as electromagentic transition widths have been obtained for the B meson within potential model scheme. It can be observed from table 1 that our predictions for the spin-averaged masses are in good agreement with experimental as well as other theoretical estimates. The complete mass spectrum is in general good agreement with experimental measurements as well as with other theoretical predictions as can be seen from table 2. Our prediction for the mass of the 1^3P_0; 5.729 GeV is fairly close the experimental value $5.732^{\pm 0.005}_{\pm 0.02}$ GeV. From table 3 it is found that our predictions for the vector and pseudoscalar decay constants are in general overestimated when compared with estimates from other theoretical models. Due to scarcity of reliable experimental estimates for E1 and M1 transition rates the validity of estimates obtained in the present work cannot be justified.

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