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Compiling results from the count of oscimodes, analytical for light flavored mesons and from Fortran programs for like (anti-)baryons

Material shown in part at the Poster Session

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Abstract. The construction of oscillatory modes of $q\ \overline{q}^{\ \prime}$ and $q\ ,\ q^{\ \prime}\ ,\ q^{\ \prime\prime}$ with $q\ ,\ q^{\ \prime}\ ,\ q^{\ \prime\prime}$ = u, d, s – modes in mesons and baryons is presented as an abbreviated outline .

1 Introduction

This outline presents results from the count of osillatory modes of light flavored u, d , s quarks inside hadrons, based on material prepared in evaluating analytical formulae and Fortran programs dedicated to this goal. The first such: t-66.f , ref. 1f = [1f-2014] , is presented in detail in a special section called 'Description of Fortran programs' at the end ofl this work. The traceback of derivations , which led to the elaboration of the Fortran file t-66, in ref. 1f = [1f-2014] , ascending in time , goes through refs. 3 = [2-2013] (56 slide-pages) and 6 = [3-2013] (150 slide-pages) .
The development of oscillatory modes of valence quarks and antiquarks in mesons was subject of lectures/exercises by one of us (P.M.) in 1978 . Derivations are given in condensed form in ref. 4 = [4-1980].
This set up the problem of extension to (three) valence quarks in baryons (and antiquarks in antibaryons), which took a while, to be answered in ref. 4 = [4-1980], where the discussion of baryons was restricted to only two light flavors : u, d . It was noted in Table 2 , p. 267, op.cit. ref. 4 , that starting with the main oscillatory quantum number N = 2 there were (nonstrange) baryon states missing in the PDG tables of 1980 .
In order to bring up to date the phenomenological situation and the comparison with the meson and baryon states for 3 light flavors of valence quarks with candidate resonances listed by the PDG ref. 5 = [5-2012] an extension to a notefile, shown in part, but only for baryons and antibaryons at the occasion of two seminars in 2013 : 24. April at the Los Alamos National Laboratory and 1. - 8. May at Caltech was made in ref. 6 = [6-2013] .
A usefool tool in assessing the $N = 1, 3$ u, d, s baryons is the PDG review, ref. 7 = [7-2012].
The use of a broken extended symmetry combining

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$$SU2\left(\sum_q \vec{S}_q\right)\times SO3\left(\sum_q \vec{L}_q\right) \rightarrow SU6$$

was vigorously pursued up tp the end of the 60-ies, whence the three valence quarks wave function is conceived in their c.m. frame. Other orbital 3-quark states were also considered, as e.g. the $p_z \rightarrow \infty$ states, where a boosted $SU6_w$ symmetry is introduced instead of $SU6$. In this respect the main quantum number N was not related to genuine oscillator degrees of freedom but instead inferred from recurrences along Regge trajectories from the relation

$$\alpha' M^2(J) = J + J_0 \text{ with } \alpha' \Delta M^2 = \Delta J = \Delta N \tag{1}$$

In eq. 1 $\vec{J} = \vec{L} + \vec{S}$ denotes total angular momentum.
The literature quoted in ref. 4 = [4-1980] is very limited, to which we add here papers by R. Dashen and M. Gell-Mann, ref. 8 = [8-1965] , by G. Zweig , ref. 9 = [9-1968] and J. Schwinger , ref. 9 = [10-1964] , illustrating the search for local field variables underlying the strong interactions at that time.
The counting of oscillatory modes of light flavored u, d, s quarks is a new investigation, which began in 2013, with a report to the ICNFP2013 conference in ref. 1 = [1-2013] .
On 12. July 2014 a new *extension* of the t-66.f associated programs was established, compiled and executed for the first time at CERN. This brought about a change in this report, removing sections 3 and 4 to the sidelines, while the actual outline contains exclusively the new results obtained with the extended set of programs : t-66.f , t-osci.f , t-osciPM.f .

2 The main quantities to count for lightflavored valence – $q\,\overline{q}\,'$ mesons and $q\,,\,q'\,,\,q''$ baryons – $q\,,\,q'\,,\,q'' =$ u, d, s

We report the derivations leading to the asymptotic form for the eigenvalues of the masssquare operator in the meson c.m. frame from refs. 4 = [4-1980] and 12 = [12-1978] extending eq. 1 below

$$\begin{aligned} &\alpha' M^2(J) = J + J_0 \text{ with } \alpha' \Delta M^2 = \Delta J = \Delta N \\ &\left(\alpha'\right)^{-1} = 1.06 \pm 5\% \text{ GeV}^2 \\ &\alpha' \; : \text{ slope of Regge trajectories excepting the Pomeron} \end{aligned} \tag{2}$$

In order to keep the systematics of barycentric coordinates as defined for general $N_c \equiv \mathbb{N}$, also appropriate for baryons discussed in ref. 4 = [4-1980] , we set for the kinematically simpler $q\,\overline{q}\,'$ mesons

$$\begin{aligned} &\vec{z} = \vec{z}_1 = \tfrac{1}{\sqrt{2}}\left(\vec{x}_1 - \vec{x}_2\right) \textbf{ and} \\ &\vec{y} = \vec{x}_1 - \vec{x}_2 = \sqrt{2}\,\vec{z} \end{aligned} \tag{3}$$

Next we consider the Lagrangean

$$\begin{aligned} &\mathcal{L}_{q\overline{q}\,'} = -\left[\begin{array}{c} m_1(\vec{y}; M_1)\sqrt{1 - \vec{v}_1^{\,2}} \\ + m_2(\vec{y}; M_2)\sqrt{1 - \vec{v}_2^{\,2}} \end{array}\right] \\ &M_{1,2} \; : \text{ masses of } q_1 \, , \, \overline{q}_2 \text{ respectively} \\ &\vec{v}_{1,2} = (d/dt)\,\vec{x}_{1,2} \; ; \text{ in the meson c.m. system} \\ &t \; : \text{ overall synchronized time in the c.m. system ,} \\ &\qquad \text{using units such that } c = 1 \end{aligned} \tag{4}$$

In the next step we choose the chiral limit as a valid approximation at large distances , to be specified subsequently.

Then eq. 4 simplifies through the relations

$$\begin{array}{l} m_1 \left(\vec{y} ;\, M_1 \rightarrow 0 \right) \rightarrow m_1 \left(\vec{y} \right) \\ m_2 \left(\vec{y} ;\, M_2 \rightarrow 0 \right) \rightarrow m_2 \left(\vec{y} \right) \\ m_1 \left(\vec{y} \right) = m_2 \left(\vec{y} \right) = m \left(\vec{y} \right) \\ \vec{v}_1 = - \vec{v}_2 = \vec{v} \end{array} \tag{5}$$

and $\mathcal{L}_{q\overline{q}\,'}$ in eq. 4 becomes

$$\begin{array}{l} \mathcal{L}_{q\overline{q}\,'} = - \overline{m} \left(\vec{y} \right) \sqrt{1 - \vec{v}^{\,2}} \\ \left(\overline{m} = m_1 + m_2 = 2\, m \right) \left(\vec{y} \right) \\ \vec{v} = \frac{1}{2} \dot{\vec{y}} \;;\; \dot{} = d \,/\, dt \end{array} \tag{6}$$

We expand the derivations following ref. 4 = [4-1980] .
The canonical momentum relative to $\frac{1}{2} \vec{y}$ becomes

$$\begin{array}{l} \vec{p} = \vec{p}_1 - \vec{p}_2 = 2\, \vec{p}_{c.m.} = \left(\mathcal{L}_{q\overline{q}\,'} \right)_{,\, \vec{v}} = \overline{m} \dfrac{\vec{v}}{\sqrt{1 - \vec{v}^{\,2}}} \\ \mathcal{H}_{(2)} = \vec{v} \left(\mathcal{L}_{q\overline{q}\,'} \right)_{,\, \vec{v}} - \mathcal{L}_{q\overline{q}\,'} = \dfrac{\overline{m}}{\sqrt{1 - \vec{v}^{\,2}}} \\ \vec{p}^{\,2} = \overline{m}^{\,2} \dfrac{v^2}{1 - v^2} = \mathcal{H}^{\,2}_{(2)} - \overline{m}^{\,2} \;;\; v^2 = \vec{v}^{\,2} \longrightarrow \\ \vec{p}^{\,2} + \overline{m}^{\,2} = \mathcal{H}^{\,2}_{(2)} \end{array} \tag{7}$$

It follows from the relations in eq. 7 that $\mathcal{H}_{(2)}$ is a constant of motion in both classical and quantum mechanical interpretations of the two body $q\,\overline{q}'$ system considered . The Euler-Lagrange equations become

$$\begin{array}{rcl} \dot{\vec{p}} & = & \mathcal{H}_{(2)} \ddot{\vec{v}} = \mathcal{H}_{(2)} \frac{1}{2} \ddot{\vec{y}} = \left(\mathcal{L}_{q\overline{q}\,'} \right)_{,\, \frac{1}{2} \vec{y}} \\ & = & - \sqrt{1 - \vec{v}^{\,2}}\; 2\, grad_{\,\vec{y}}\, \overline{m} = - \mathcal{H}^{-1}_{(2)}\, \overline{m}\, grad_{\,\vec{y}}\, \overline{m}^{\,2} \\ \longrightarrow \quad \mathcal{H}_{(2)} \frac{1}{2} \ddot{\vec{y}} & = & - \left(\mathcal{H}_{(2)} \right)^{-1} 2\, \overline{m}\, grad_{\,\vec{y}}\, \overline{m} \\ & = & - \left(\mathcal{H}_{(2)} \right)^{-1} grad_{\,\vec{y}}\, \overline{m}^{\,2} \end{array} \tag{8}$$

We can see how the mass-square dynamical variables arise in the classical interpretation of the equations of motion introducing the notation

$$\mathcal{M}^{\,2}_{meson} = \mathcal{H}^{\,2}_{(2)} \tag{9}$$

Then eq. 8 takes the form

$$\mathcal{M}^{\,2}_{meson}\, \ddot{\vec{y}} = - 2\, grad_{\,\vec{y}}\, \overline{m}^{\,2} \tag{10}$$

The oscillatory modes inherit the central scale of QCD for light (u, d , s-) flavors of quark as implied by the trace anomaly – ref. 13 = [13-1975] , in the adopted chiral limit through the parameter Λ , not to be confused with Λ_{QCD}, valid in the perturbatively accessible region, in the Ansatz for the mass function for large values of $|\vec{y}|$, of dimension mass-square

$$\overline{m}^{2}(\vec{y}) \sim \underset{|\vec{y}|\to\infty}{\longrightarrow} \sim \tfrac{1}{4}\Lambda^{2}|\vec{y}|^{2}\left[1 + O\left(\frac{M_{q}}{\Lambda|\vec{y}|}\right)\right] \tag{11}$$

In eq. 11 M_q denote the physical quark-mass parameters Inserting the asymptotic large $|\vec{y}|$ part of the mass-square function $\overline{m}^{2}(\vec{y})$ into eq. 10 we obtain

$$\begin{aligned} \mathcal{M}^{2}_{meson}\ddot{\vec{y}} &= - \sim \Lambda^{2}\vec{y} \leftrightarrow \omega_{cl} = \frac{\Lambda}{\mathcal{H}_{(2)}} \\ \omega^{2}_{cl} &= \frac{\Lambda^{2}}{\mathcal{M}^{2}} \end{aligned} \tag{12}$$

We here turn to the quantum mechanical description following from the Lagrangean given in eq. 6

$$\widehat{\vec{p}} = \tfrac{1}{i}\nabla_{\frac{1}{2}\vec{y}} = \left(\mathcal{L}_{q\overline{q}'}\right)_{,\vec{v}} = \left(\overline{m}\frac{\vec{v}}{\sqrt{1-v^{2}}}\right)_{ordered} \tag{13}$$

In eq. 13 the suffix *ordered* shall indicate that the operators $\overline{m}$ and $\frac{\vec{v}}{\sqrt{1-v^{2}}}$, written as product, do not commute, which necessitates the ordering guaranteeing a self-adjoint operator beeing represented by the product .

2-1 Counting oscillatory modes of valence quarks and antiquarks $q,\overline{q}^{\,\prime}$; $q,q^{\prime}$ = u, d, s in mesons

We first determine the number density at given main quantum number N , which amounts to calculate the power of the set of occupation numbers n_1, n_2, n_3 of the associated 3 oscillators .
p is given by the number of partitions

$$p(N) = \{n_1, n_2, n_3 |\ n_1 + n_2 + n_3 = N\ ;\ n_{1,2,3} = 0, 1, 2\cdots N\} \tag{14}$$

and multiplied with the multiplicity of $SU6(spin \times N_{fl}) = 36$.
The power of the set $p(N)$ is readily written as a sum over n_3

$$\begin{aligned} p(N) &= \textstyle\sum_{n_3=0}^{N} p(n_1, n_2; \nu) = \sum_{\nu=0}^{N} p(n_1, n_2; \nu) \\ \nu &= N - n_3 = 0, 1, \cdots, N\ ;\ n_{1,2} = 0.1, 2\cdots N \\ p(n_1, n_2; \nu) &= \{n_1, n_2 |\ n_1 + n_2 = \nu\} = \nu + 1 \end{aligned} \tag{15}$$

Thus $p(N)$ defined in eq. 14 becomes

$$\begin{aligned} p(N) &= \textstyle\sum_{\nu=0}^{N}(\nu + 1) = \sum_{\nu_+=1}^{N+1}\nu_+ \\ &= \tfrac{1}{2}(N+1)(N+2) \\ \nu_+ &= \nu + 1 \end{aligned} \tag{16}$$

and $z(N) = 36\,p(N)$ is

$$\begin{aligned} z(N) &= \partial\varrho / \partial\left(\alpha' M^{2}\right) = 18(N+1)(N+2) \\ \alpha' M^{2} &= \alpha'\Delta M^{2} + N_0 = N + N_0 \end{aligned} \tag{17}$$

We give here the derivation yielding the sum of squares of integers from the difference of sums of cubes

$$S_2(N^*) = \sum_{\nu=0}^{N^*} \nu^2 \ , \ S_3(N^*) = \sum_{\nu=0}^{N^*} \nu^3 \tag{18}$$

which makes use of the recursive relation

$$\begin{aligned} & S_3(N^* + 1) - S_3(N^*) = \\ & = \sum_{\nu=0}^{N^*} \left[(\nu + 1)^3 - \nu^3 \right] \\ & = (N^* + 1)^3 \\ & = 2 S_2(N^*) + \tfrac{3}{2} N^*(N^* + 1) + N^* + 1 \end{aligned} \tag{19}$$

Then it follows

$$S_2(N^*) = \tfrac{1}{6} N^*(N^* + 1)(2N^* + 1) \tag{20}$$

We display the function $S_2(N^*)$, $N^* = 1$ to 5 in table 1 below

N^*	$S_2(N^*)$
1	1
2	5
3	14
4	30
5	55

(21)

Table 1

The sum of the squares $S_2(N^*)$ is in the present context an *auxiliary* function necessary for the calculation of the total number of meson states with $N \le N^*$.
We thus find for the number of u,d,s meson states with $N \le N^*$

$$\begin{aligned} Z(N^*) & = \\ & = 18 \left(S_2(N^* + 1) + \tfrac{1}{2} (N^* + 1)(N^* + 2) \right) \\ & = 18 \left(\begin{array}{l} \frac{1}{6} (N^* + 1)(N^* + 2)(2N^* + 3) \\ + \frac{1}{2} (N^* + 1)(N^* + 2) \end{array} \right) \\ & = 18 \left((N^* + 1)(N^* + 2) \left(\tfrac{1}{3} (N^* + 1) \right) \right) \\ & = 6 (N^* + 1)(N^* + 2)(N^* + 3) \end{aligned} \tag{22}$$

The main results of this subsection are contained in eqs. 17 for the number density as a function of the main oscillator quantum number N: $z(N) = \partial \varrho / \partial \left(\alpha' M^2 \right)$ and 22 for the total number of oscillatory modes below and including the limiting main quantum number N^* : $Z(N^*)$, both equations recapitulated for u, d, s mesons $q \overline{q}'$ below

$$\begin{aligned} & z(N) = \partial \varrho / \partial \left(\alpha' M^2 \right) = 18 (N + 1)(N + 2) \\ & \alpha' M^2 = \alpha' \Delta M^2 + N_0 = N + N_0 \end{aligned} \tag{23}$$

$$Z(N^*) = 6 (N^* + 1)(N^* + 2)(N^* + 3) \tag{24}$$

2-2 Oscillatory modes of valence quarks and antiquarks $q\,,\overline{q}\,'\,;\; q, q\,'$ = u. d, s in mesons (continued)

We recall here the relations in eqs. 5 and 6 , repeated below

$$\begin{aligned} & m_1\,(\,\vec{y}\,;\; M_1 \,\to\, 0\,) \,\to\, m_1\,(\,\vec{y}\,) \\ & m_2\,(\,\vec{y}\,;\; M_2 \,\to\, 0\,) \,\to\, m_2\,(\,\vec{y}\,) \\ & m_1\,(\,\vec{y}\,) \,=\, m_2\,(\,\vec{y}\,) \,=\, m\,(\,\vec{y}\,) \\ & \vec{v}_1 \,=\, -\,\vec{v}_2 \,=\, \vec{v} \end{aligned} \tag{25}$$

$\mathcal{L}_{q\overline{q}\,'}$ in eq. 6 becomes

$$\begin{aligned} & \mathcal{L}_{q\overline{q}\,'} \,=\, -\,\overline{m}\,(\,\vec{y}\,)\;\sqrt{1 \,-\, \vec{v}^{\,2}} \\ & (\,\overline{m} \,=\, m_1 \,+\, m_2 \,=\, 2\,m\,)\,(\,\vec{y}\,) \\ & \vec{v} \,=\, \tfrac{1}{2}\,\dot{\vec{y}}\;\;;\;\; \dot{} \,=\, d\,/\,dt \end{aligned} \tag{26}$$

Next we recall eq. 13 repeated below

$$\widehat{\vec{p}} \,=\, \tfrac{1}{i}\,\nabla_{\frac{1}{2}\vec{y}} \,=\, \left(\,\mathcal{L}_{q\overline{q}\,'}\,\right)_{,\,\vec{v}} \,=\, \left(\,\overline{m}\;\frac{\vec{v}}{\sqrt{1\,-\,v^{2}}}\,\right)_{ordered} \tag{27}$$

Eq. 7 adapted to the quantum mechanical logic becomes

$$\begin{aligned} & \widehat{\vec{p}} = \widehat{\vec{p}}_1 - \widehat{\vec{p}}_2 = 2\widehat{\vec{p}}_{c.m.} = \left(\mathcal{L}_{q\overline{q}\,'}\right)_{,\,\vec{v}} = \overline{m}\,\frac{\vec{v}}{\sqrt{1-\vec{v}^{\,2}}}\Bigg|_{ordered} \\ & \widehat{\mathcal{H}}_{(2)} \,=\, \vec{v}\,\left(\,\mathcal{L}_{q\overline{q}\,'}\,\right)_{,\,\vec{v}} \,-\, \mathcal{L}_{q\overline{q}\,'} \,=\, \frac{\overline{m}}{\sqrt{1\,-\,\vec{v}^{\,2}}}\Bigg|_{ordered} \end{aligned} \tag{28}$$

$$\begin{aligned} & \widehat{\vec{p}}^{\;2} = \overline{m}^{\,2}\;\frac{v^2}{1-v^2}\Bigg|_{ordered} = \widehat{\mathcal{H}}^2_{(2)} - \overline{m}^{\,2}\;;\; v^2 = \vec{v}^{\,2} \;\longrightarrow \\ & \widehat{\vec{p}}^{\;2} \,+\, \overline{m}^{\,2} \,=\, \widehat{\mathcal{H}}^{\,2}_{(2)} \,=\, \widehat{\mathcal{M}}^{\,2}\;\;;\;\; \widehat{\vec{p}}^{\;2} \,=\, -\,\Delta_{\frac{1}{2}\vec{y}} \,=\, -\,4\,\Delta_{\vec{y}} \\ & \overline{m}^{\,2}\,(\,\vec{y}\,) \,\sim\, \underset{|\vec{y}|\to\infty}{\longrightarrow} \,\sim\, \tfrac{1}{4}\,\Lambda^{2}\,|\,\vec{y}\,|^{\,2}\left[\,1 \,+\, O\left(\frac{M_q}{\Lambda\,|\,\vec{y}\,|}\right)\right] \end{aligned} \tag{29}$$

Eq. 29 shows the main result of this section , in particular the second order wave equation (on the second line)

$$\widehat{\mathcal{M}}^{\,2} \,=\, \left[\,-\,4\,\Delta_{\vec{y}} \,+\, \tfrac{1}{4}\,\Lambda^{2}\,|\,\vec{y}\,|^{\,2}\,\right] \tag{30}$$

If we determine it from the positive parity Λ trajectory from the present PDG tables [5-2012]

$$\begin{array}{rcccc} \Lambda\,,\,J^{P} & : & \frac{1}{2}^{\,+} & \frac{5}{2}^{\,+} & \frac{9}{2}^{\,+} \\ M_j & : & 1.115683 & 1.820 & 2.350 \\ M_j^{\,2} & : & 1.2447485 & 3.3124 & 5.5225 \\ \frac{1}{2}\,\Delta\,M^{2} & : & & 1.034 & 1.105 \end{array} \tag{31}$$

and average the two half mass square difference entries in the last line of eq. 31 with weights two to one we obtain

$$1 / \alpha' = \frac{1}{3} \left(M_2^2 - M_1^2 \right) + \frac{1}{6} \left(M_3^2 - M_2^2 \right) \sim 1.06 \text{ GeV}^2 \tag{32}$$

We remark that in ref. 5 = [5-2012] $\Lambda \frac{9}{2}^+$ has only three stars , and furthermore the trajectory contains only three entries , whereas one of us (P.M.) thinks to remember that it contained four sometimes back [1]. $\Lambda \frac{13}{2}^+$ would extrapolate to 2.755 GeV using eq. 31 .
In order to exhibit the oscillator variables we substitute rescaled coordinates and derivatives relative to the spatial variable $\vec{y}$

$$\vec{y} = \lambda^{-1} \vec{\zeta} \;,\; \nabla_{\vec{y}} = \lambda \nabla_{\vec{\zeta}} \tag{33}$$

The differential operator on the right hand side of eq. 30 becomes

$$-4 \Delta_{\vec{y}} + \frac{1}{4} \Lambda^2 | \vec{y} |^2 = -4 \lambda^2 \Delta_{\vec{\zeta}} + \frac{1}{4} \lambda^{-2} \Lambda^2 | \vec{\zeta} |^2 \tag{34}$$

The parameter λ , of dimension mass, shall be chosen such that

$$4 \lambda^2 = \frac{1}{4} \lambda^{-2} \Lambda^2 \rightarrow 2 \lambda = \frac{1}{2} \lambda^{-1} \Lambda \rightarrow 4 \lambda^2 = \Lambda \tag{35}$$

Substituting the last equation in eq. 35 in eq. 34 we obtain

$$-4 \Delta_{\vec{y}} + \frac{1}{4} \Lambda^2 | \vec{y} |^2 = \Lambda \left[- \Delta_{\vec{\zeta}} + | \vec{\zeta} |^2 \right] \tag{36}$$

The deliberations leading eventually to the definition of $\widehat{\mathcal{M}}^2$ given in eq. 30 go back to ref. 14 = [14-2010] in 1970, published in 1971. They take shape as oscillatory modes for light flavored mesons first in 1975 and in published form in ref. 12 = [12-1978] in 1978, oscillatory modes for light flavored mesons and baryons were first presented in ref. 4 = [4-1980] .

2-3 Oscillatory modes of valence quarks and antiquarks $q, \overline{q}' \; ; \; q, q' =$ u. d, s in mesons (proper)

The dimensionless rescaled spatial variables $\vec{\zeta}$ defined through eqs. 33 and 35

$$\vec{\zeta} = \lambda \vec{y} \;;\; \lambda = \frac{1}{2} (\Lambda)^{\frac{1}{2}} \tag{37}$$

and their canonically conjugate momenta

$$\widehat{\vec{p}}_{\vec{\zeta}} = \frac{1}{i} \nabla_{\vec{\zeta}} = \frac{1}{i} \partial_{\vec{\zeta}} \tag{38}$$

in components

$$\zeta^m , \widehat{p}_n = \partial_{\zeta^n} \;;\; [\widehat{p}_n , \zeta^m] = \frac{1}{i} \delta_n{}^m \;; m, n = 1, 2, 3 \tag{39}$$

generate the 3 oscillator absorption and creation operators absorption and creation operators

$$\begin{aligned} a^m &= \frac{1}{\sqrt{2}} (\zeta^m + i \widehat{p}_m) \quad & a^{*m} &= \frac{1}{\sqrt{2}} \left(\zeta^m - \partial_{\zeta^m} \right) \\ &= \frac{1}{\sqrt{2}} \left(\zeta^m + \partial_{\zeta^m} \right) \; , & &= \frac{1}{\sqrt{2}} \left(\zeta^m - \partial_{\zeta^m} \right) \\ m &= 1, 2, 3 \end{aligned} \tag{40}$$

obeying the commutation rules

$$[a^m , a^{*n}] = \delta^{mn} ¶ \;;\; [a^m , a^n] = [a^{*m} , a^{*n}] = 0 \tag{41}$$

[1] **"Tempora mutantur nos et mutamus in illis ."**

The oscillator algebra displayed in eq. 41 is common to any (3) canonical pairs of operators , associated with a threedimensional uncurved space, as is the case here. It shows directly the U3-invariance of the commutation rules .
What is very special is the relation for the dynamical form of the mass square operator $\widehat{\mathcal{M}}^{2}$ as given in eq. 36 , which becomes

$$\begin{aligned} \widehat{\mathcal{M}}^{2} &= \Lambda \left[- \Delta_{\vec{\zeta}} + \vec{\zeta}^{\,2} \right] = 2 \Lambda \left[\widehat{N} + \tfrac{3}{2} \P \right] \\ \widehat{N} &= \sum_{m=1}^{3} \widehat{N}_{m} \ ; \ \widehat{N}_{n} = a^{*n} a^{n} \end{aligned} \tag{42}$$

For an individual n we have

$$\begin{aligned} 2 a^{*n} a^{n} &= \left(\zeta^{n} - \partial_{\zeta^{n}} \right) \left(\zeta^{n} + \partial_{\zeta^{n}} \right) \\ &= - \partial_{\zeta^{n}}^{2} + (\zeta^{n})^{2} - \P \end{aligned} \tag{43}$$

which proves the correctness of the first relation in eq. 42 .

Identifying 2Λ with the inverse slope of Regge trajectories other than the Pomeron

Since the relation in eq. 42 is only valid for large eigenvalues of the number operator $\widehat{N}$, with eigenvalues $N = n_{1} + n_{2} + n_{3} = 0, 1, 2, \cdots$, where n_{k} , $k = 1, 2, 3$ denote the eigenvalues of the individual counting operators $\widehat{N}_{k}$, we can reparametrize within the same approximation accuracy eq. 42 in the form

$$\widehat{\mathcal{M}}^{2} = 2 \Lambda \left[\widehat{N} + \widehat{N}_{0} \right] \tag{44}$$

In eq. 44 the operator $\widehat{N}_{0}$ contains all effects from short distances and parametrically depends on quark masses . It does not commute with the asymptotically dominating number operators $\widehat{N}$, $\widehat{N}_{k}$; $k = 1, 2, 3$.
We deal with the 'intercept'-related perturbations through the operator $\widehat{N}_{0}$ as characterized in eq. 44 and the text specifying its details in the sense of perturbations of *large* eigenvalues of $\widehat{N}$, the dominant operator for large eigenvalues N of the mass square operator $\widehat{\mathcal{M}}^{2}$, adopting the ansatz for the eigenvalues of $\widehat{\mathcal{M}}^{2}$

$$\mathcal{M}^{2} \sim 2 \Lambda \left(N + N_{0} \right) \ ; \ N = 0, 1, 2 \cdots \text{ yet } \gg 1 \tag{45}$$

We compare the structure of $\mathcal{M}^{2}$ in eq. 45 with the relation to this quantity along a Regge trajectory taken approximately linear

$$\begin{aligned} &\alpha (\mathcal{M}^{2}) = \alpha^{\prime} \mathcal{M}^{2} + \alpha_{0} \rightarrow \alpha^{\prime} : \quad \text{universal slope of Regge trajectories} \\ &\alpha (\mathcal{M}^{2}) \rightarrow J \rightarrow N \ ; \ \mathcal{M}^{2} = \frac{1}{\alpha^{\prime}} \left(N - \alpha_{0} \right) \end{aligned} \tag{46}$$

Comparing th two expressions for $\mathcal{M}^{2}$ in eqs. 45 and 46 , we obtain the sought identification

$$2 \Lambda = \frac{1}{\alpha^{\prime}} \sim 1.06 \pm 0.05 \text{ GeV}^{2} \tag{47}$$

2-4 First results and illustrations from the count of oscillatory modes of valence quarks and antiquarks $q\,,\overline{q}\,'\;;\; q, q\,' \;=$ u. d, s for mesons and baryons

N^*	Z Baryons	Z Mesons
0	56	36
1	266	144
2	2310	360
3	4090	720

(48)

Table 2

In Table 2 we give a comparison of the number of oscillatory modes up to N^* ; $N^* = 0, 1, 2, 3$ for baryons and mesons . The numbers for baryons are from ref. 1 = [1-2013] .
The oscillator structure for mesons shown in eqs. 40 - 43 , which gives rise to the count of partitions, as charecterized in eqs. 44 - 46, is valid for large eigenvalues of the number operator $\widehat{N}$, with eigenvalues $N = n_1 + n_2 + n_3 = 0, 1, 2, \cdots$, where n_k , $k = 1, 2, 3$ denote the eigenvalues of the individual counting operators $\widehat{N}_k$.
The count of these (3-) partitions is illustrated for the simpler case of (2-) partitions ($N = n_1 + n_2 + n_3 = 0, 1, 2, \cdots$) in Fig. 1 , below . It is independent of the 'intercept'-induced shifts which give rise to the parameters $N_0 \leftrightarrow \alpha_0$ shown in eqs. 45 and 46 . The oscillator structure for mesons shown in eqs. 40 - 43 , which gives rise to the count of partitions, as charecterized in eqs. 44 - 46, is valid for large eigenvalues of the number operator $\widehat{N}$, with eigenvalues $N = n_1 + n_2 + n_3 = 0, 1, 2, \cdots$, where n_k , $k = 1, 2, 3$ denote the eigenvalues of the individual counting operators $\widehat{N}_k$.

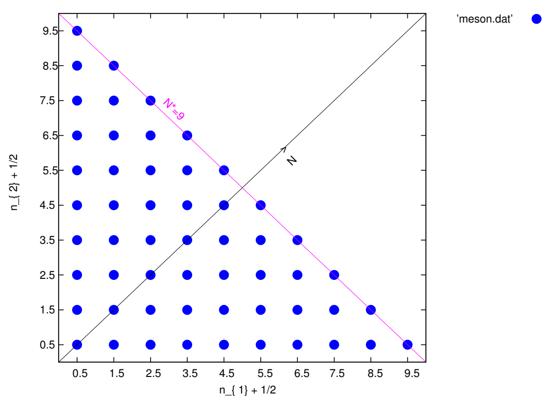

Fig. 1 : Illustration of partitions $N = n_1 + n_2$
simplifying the ones relevant for mesons
$N = n_1 + n_2 + n_3 \longleftrightarrow$

For the so defined (n-) partition count , the unit spatial grid is independent of the operator $\widehat{N}_0$, displayed in eq. 44 . It is reached in momentum space from eq. 47 , which yields the unit in momentum space for each of the 3 components of $\vec{p}$ in the case of mesons.

$$[\,p\,] = \left(\alpha\,'\right)^{-1/2} = (\,1.0296 \pm 2.5\%\,)\ \text{GeV} \tag{49}$$

The comparison of the number density of states per unit mass-square $z\,(\,N\,)$ between baryons and mesons is illustrated in Fig. 2 below

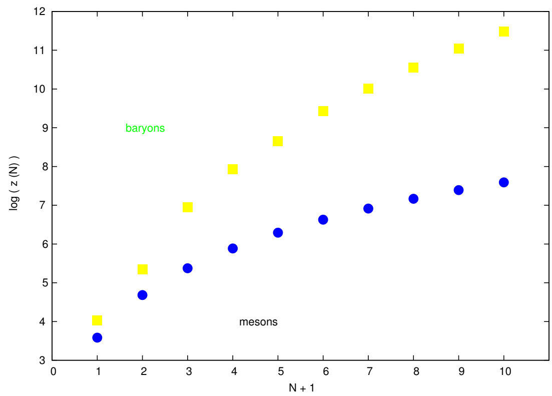

Fig. 2 : Density of states $z\,(\,N\,)$ in logarithmic scale per unit mass-square $\longleftrightarrow$

Fig. 2 serves to show, that the density per unit mass-square count reveals more baryon states than mesonic ones, for all values of N .

Nevertheless in production cross-sections as well as in thermal equilibrium mesons are dominantly produced, for low energies or temperatures .

To this end we calculated the exponentially weighted density per unit mass-square for baryons

$$\partial\,\varrho_{\,baryon}\,/\,\partial\,\left(\,\alpha^{\,\prime}\,M^{\,2}\,\right)\,\times\,\exp\,(\,-\,M\,/\,T\,) \tag{50}$$

for $T\;=\;0.2$ GeV displayed in Fig. 3 below

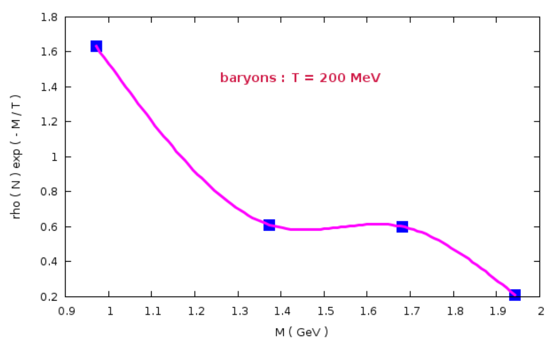

Fig. 3 : Density of states $z\,(\,N\,)$ per unit mass-square for baryons wighted with the exponential factor $exp\,(\,-\,M/T\,)\,;\;T\;:$ Temperature $\longleftrightarrow$

The interpolation between the 4 points displayed in Fig. 3 is done using cubic splines , whereby we have approximated

$$1\,/\,\alpha^{\,\prime}\;\;\sim\;1.0\;\mathrm{GeV}^{\,2} \tag{51}$$

Outlook

We look forward to proceed along the path of counting oscillatory modes of light flavored mesons and baryons , as it is outlined here in its present incomplete stage . The goal is to obtain main consequence for the description of thermal equilibrium as applying to the hadronic phase of QCD . What becomes clear now is that the alleged thermal properties at temperatures not exceeding 200 MeV including the limit of chemical freezout at small or vanishing chemical potentials involves a newly posed assessment of feeding through the decay products of the heavy (heavier) baryons and antibaryons is a serious one affecting mainly pions and kaons. It is conceivable that the difficulties in accounting for the observed abundances at RHIC and with the Alice detector at LHC .

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