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Measuring $a_\mu^{\rm HLO}$ with space-like data

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Abstract. After reviewing the traditional way of computing the leading order hadronic correction to the muon (g-2) through a dispersive approach via time-like data, I will present a novel approach, based on the measurement of the effective electromagnetic coupling in the space-like region extracted from Bhabha scattering data. We argue that this new method may become feasible at flavor factories, resulting in an alternative determination potentially competitive with the accuracy of the present results obtained with the dispersive approach via time-like data

1 α_{em} running and the Vacuum Polarization

Precision physics requires appropriate inclusion of higher order effects and the knowledge of very precise parameters of the electroweak Standard Model SM. One of the basic input parameters is the fine structure constant which depends logarithmically on the energy scale [1]. At zero momentum transfer, the QED coupling constant $\alpha(0)$ is very accurately known from the measurement of the anomalous magnetic moment of the electron and from solid-state physics measurements:

$$\alpha^{-1}(0) = 137.03599976(50). \quad (1)$$

Vacuum polarization effects lead to a partial screening of the charge in the low energy limit (Thomson limit) while at higher energies the strength of the electromagnetic interaction grows. This is due to virtual lepton and quark loop corrections to the photon propagator.
This effect can also be understood as an increasing penetration of the polarized cloud of virtual particles which screen the bare electric charge of a particle.
Thus, at large distances, we observe a reduced bare charge due to this effect of screening. As we probe closer we penetrate into the cloud of virtual particles, decreasing the screening effect and observing more of the bare charge and thus a strengthening of the coupling The charge has to be replaced by a *running charge*:

$$e^2 \to e^2(q^2) = \frac{e^2 Z}{1+\Pi_\gamma'(q^2)}\,.$$

The wave function renormalization factor Z is fixed by the condition that for $q^2 \to 0$ one obtains the classical charge (charge renormalization in the Thomson limit) [2]. Thus the renormalized charge is:

$$e^2 \to e^2(q^2) = \frac{e^2}{1+(\Pi_\gamma'(q^2)-\Pi_\gamma'(0))} \quad (2)$$

where, in the perturbation theory, the lowest order diagram which contributes to $\Pi_\gamma'(q^2)$ is:

which describes virtual creation and reabsorption of fermion pairs:

$$\gamma^* \to e^+e^-, \mu^+\mu^-, \tau^+\tau^-, u\bar{u}, d\bar{d}, ... \to \gamma^*$$

in leading order. In terms of the fine structure constant $\alpha = e^2/4\pi$:

$$\alpha(q^2) = \frac{\alpha}{(1-\Delta\alpha)}; \Delta\alpha = -\Re(\Pi_\gamma'(q^2) - \Pi_\gamma'(0)). \quad (3)$$

The shift $\Delta\alpha$ is large due to the large change in scale going from zero momentum to the Z-mass scale $\mu = M_Z$ and due to the many species of fermions contributing. Zero momentum more precisely means the light fermion mass thresholds [1]. The various contributions to the shift in the fine structure constant come from the leptons (lep=e,μ and τ) the 5 light quarks (u,b,s,c and the corresponding hadrons =had) and from the top quark:

$$\Delta\alpha = \Delta\alpha_{lep} + \Delta^{(5)}\alpha_{had} + \Delta\alpha_{top} + ...$$

While the leptonic contributions are calculable with very high precision in QED by the perturbation theory, the hadronic ones are more problematic as they involve the quark masses and hadronic physics at low momentum scales.
Because of the importance of the running coupling to physics, it is crucial to observe experimentally. Indeed

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this was performed in past, where TOPAZ [3] experiment probed the running in the time-like region (positive Q^2) while VENUS [4], L3 [5] and OPAL [6] measured the running in space-like regions. Given the limited energy region covered in these measurements, there is a big opportunity to improve them at future high-energy e^+e^- collider(s) [7, 8].

2 The Muon g-2

The muon anomaly $a_\mu = (g-2)/2$ is a low-energy observable, which can be both measured and computed to high precision [9, 10]. Therefore it provides an important test of the Standard Model (SM) and it is a sensitive search for new physics [11]. Since the first precision measurement of a_μ from the E821 experiment at BNL in 2001 [12], there has been a discrepancy between its experimental value and the SM prediction. The significance of this discrepancy has been slowly growing due to reductions in the theory uncertainty. Figure 1 (taken from [13]) shows a recent comparison of the SM predictions of different groups and the BNL measurement for a_μ. The a_μ determinations of the different groups are in very good agreement and show a consistent $\approx 3\,\sigma$ discrepancy [13–15], despite many recent iterations in the SM calculation. It should be noted that with the final E821 measurement and advances in the theoretical SM calculation that both the theory and experiment uncertainties have been reduced by more than a factor two in the last ten years [16].

Like the effective fine-structure constant at the scale M_Z, the SM determination of the anomalous magnetic moment of the muon a_μ is presently limited by the evaluation of the hadronic vacuum polarisation effects, which cannot be computed perturbatively at low energies. However, using analyticity and unitarity, it was shown long ago that this term can be computed from hadronic e^+e^- annihilation data via the dispersive integral [17]:

$$a_\mu^{\rm HLO} = \left(\frac{1}{4\pi^3}\right)\int_{m_\pi^2}^{\infty} ds\, K(s)\sigma^0(s) = \left(\frac{\alpha^2}{3\pi^2}\right)\int_{m_\pi^2}^{\infty} ds\, K(s)R(s)/s\,. \quad (4)$$

The kernel function $K(s)$ decreases monotonically with increasing s. This integral is similar to the one entering the evaluation of the hadronic contribution $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ [1]. Here, however, the weight function in the integrand gives a stronger weight to low-energy data.

Two recent compilations of e^+e^- data give [18, 19]:

$$a_\mu^{\rm had;LO} = (6\,923 \pm 42)\times 10^{-11}\,, \quad (5)$$

$$a_\mu^{\rm had;LO} = (6\,949 \pm 43)\times 10^{-11}\,, \quad (6)$$

respectively.

Important earlier global analyses include those of Hagiwara et al. [20], Davier, et al., [21], Jegerlehner and Nyffler [14].

Therefore the leading-order hadronic vacuum polarization contribution, $a_\mu^{\rm HLO}$ is known with a fractional accuracy of 0.7%, i.e. to about 0.4 ppm. The O(α^3) hadronic light-by-light contribution, $a_\mu^{\rm HLbL}$, is the second dominant error in the theoretical evaluation. It cannot at present be determined from data, and relies on using specific models. Although its value is almost two orders of magnitude smaller than $a_\mu^{\rm HLO}$, it is much worse known (with a fractional error of the order of 30%) and therefore it still give a significant contribution to $\delta a_\mu^{\rm TH}$ (between 2.5 and 4 $\times 10^{-10}$).

From the experimental side, the error achieved by the BNL E821 experiment is $\delta a_\mu^{\rm EXP} = 6.3\times 10^{-10}$ (0.54 ppm) [22]. This impressive result is still limited by the statistical errors, and a new experiment, E989 [23], to measure the muon anomaly to a precision of 1.6×10^{-10} (0.14 ppm) is under construction at Fermilab. If the central value remains unchanged, then the statistical significance of the discrepancy with respect to the SM prediction would then be over 5σ, see Ref. [10], and would be larger than this with the expected improvements in the theoretical calculation.

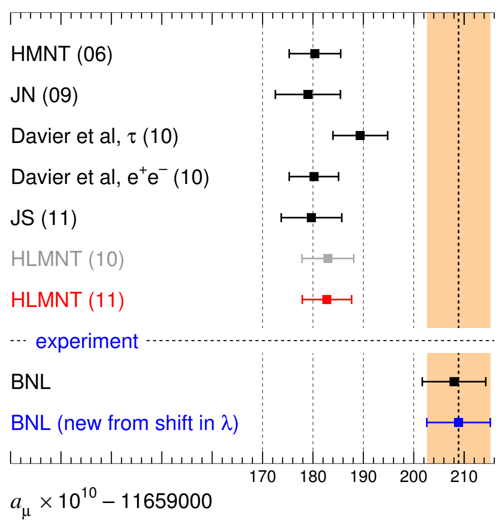

Figure 1. Standard model predictions of a_μ by several groups compared to the measurement from BNL (taken from [13]).

3 Recent results and expected improvement on the hadronic contribution

In contrast to the QED and Electroweak contributions to a_μ, which can be calculated using perturbation theory, and therefore are well under control, the hadronic contributions (LO VP and HLbL) cannot be computed reliably using perturbative QCD. The hadronic contribution $a_\mu^{\rm HLO}$ can can be computed from hadronic e^+e^- annihilation data via a dispersion relation, and therefore its uncertainty strongly depends on the accuracy of the experimental data. For the Hadronic Light-by-Light contribution $a_\mu^{\rm HLbL}$ there is no direct connection with data and therefore only model-dependent estimates exist. As the hadronic sector dominates the uncertainty on the theoretical prediction $a_\mu^{\rm TH}$, it has been the subject of considerable recent activity from both experimental and theoretical groups, with the following outcomes:

- A precise determination of the hadronic cross sections at the e^+e^- colliders (VEPP-2M, DAΦNE, BEPC, PEP-II and KEKB) has allowed a determination of $a_\mu^{\rm HLO}$ with a fractional accuracy below 1%. These efforts have led to the development of dedicated high precision theoretical tools such as the addition of Radiative Corrections (RC) and the non-perturbative hadronic contribution to the running of α (i.e. the vacuum polarisation, VP) into the Monte Carlo (MC) programs used for the analysis of the experimental data [24];
- The use of '*Initial State Radiation*' (ISR) data [25–27] which has opened a new way to precisely obtain the electron-positron annihilation cross sections into hadrons at particle factories operating at fixed beam-energies [28, 29].
- A dedicated effort on the evaluation of the Hadronic Light-by-Light contribution (see for example [30]), where two different groups [14, 31] have obtained consistent values (with slightly different errors), and therefore strengthened the confidence in the reliability of these estimates.
- Impressive progress on the lattice, where an accuracy of $\sim 4\%$ has been reached on the four-flavour calculation of $a_\mu^{\rm HLO}$ [32];
- A better agreement between the e^+e^-– and the τ– based evaluation of $a_\mu^{\rm HLO}$, due to improved isospin corrections [15]. These two sets of data are now broadly in agreement (with τ data moving towards e^+e^-– data) after including vector meson and $\rho-\gamma$ mixing [33, 34].

Further improvements are expected on the calculations of the hadronic contribution to a_μ on the timescale of the new g-2 experiments at Fermilab and J-PARC and this will be augmented, on the experimental side, by more data from current and future e^+e^- colliders. From the theoretical side, the lattice calculation has already reached a mature stage and has real prospects to match the experimental precision. From both activities a further reduction of the error on $a_\mu^{\rm HLO}$ can be expected and thus progress on $a_\mu^{\rm HLbL}$ will be required. Although for the HLbL contribution there isn't a direct connection with data, $\gamma-\gamma$ measurements performed at e^+e^- colliders will help constrain the on-shell form factors [35] and lattice calculations will help better determine the off shell contributions.

4 Measuring $a_\mu^{\rm HLO}$ with space-like data

The leading-order hadronic contribution to the muon g-2 is given by the well-known formula [9, 17, 36]

$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi^2}\int_0^\infty \frac{ds}{s}\, K(s)\, {\rm Im}\Pi_{\rm had}(s+i\epsilon), \qquad (7)$$

where $\Pi_{\rm had}(s)$ is the hadronic part of the photon vacuum polarization, $\epsilon > 0$,

$$K(s) = \int_0^1 dx\, \frac{x^2(1-x)}{x^2+(1-x)(s/m_\mu^2)} \qquad (8)$$

is a positive kernel function, and m_μ is the muon mass.

Equation (7) can be rewritten in the form [2, 37]

$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\rm had}[t(x)]\,. \qquad (9)$$

where

$$t(x) = \frac{x^2m^2}{x-1} < 0 \qquad (10)$$

is a space-like squared four-momentum.

Equation (9), involving the hadronic contribution to the running of the effective fine-structure constant at space-like momenta, can be computed by measurements of the effective electromagnetic coupling in the space-like region.

5 $\Delta\alpha_{\rm had}(t)$ from Bhabha scattering data

The hadronic contribution to the running of α in the space-like region, $\Delta\alpha_{\rm had}(t)$, can be extracted comparing Bhabha scattering data to Monte Carlo (MC) predictions. The LO Bhabha cross section receives contributions from t- and s-channel photon exchange amplitudes. At NLO in QED, it is customary to distinguish corrections with an additional virtual photon or the emission of a real photon (photonic NLO) from those originated by the insertion of the vacuum polarization corrections into the LO photon propagator (VP). The latter goes formally beyond NLO when the Dyson resummed photon propagator is employed, which simply amounts to rescaling the α coupling in the LO s- and t-diagrams by the factor $1/(1-\Delta\alpha(q^2))$. In MC codes, e.g. in `BabaYaga` [38], VP corrections are also applied to photonic NLO diagrams, in order to account for a large part of the effect due to VP insertions in the NLO contributions. Beyond NLO accuracy, MC generators consistently include also the exponentiation of (leading-log) QED corrections to provide a more realistic simulation of the process and to improve the theoretical accuracy. We refer the reader to Ref. [24] for an overview of the status of the most recent MC generators employed at flavor factories. We stress that, given the inclusive nature of the measurements, any contribution to vacuum polarisation which is not explicitly subtracted by the MC generator will be part of the extracted $\Delta\alpha(q^2)$. This could be the case, for example, of the contribution of hadronic states including photons (which, although of higher order, are conventionally included in $a_\mu^{\rm HLO}$), and that of W bosons or top quark pairs.

Before entering the details of the extraction of $\Delta\alpha_{\rm had}(t)$ from Bhabha scattering data, let us consider a few simple points. In fig. 2 (left) we plot the integrand $(1-x)\Delta\alpha_{\rm had}[t(x)]$ of Eq. (9) using the output of the routine `hadr5n12` [39] (which uses time-like hadroproduction

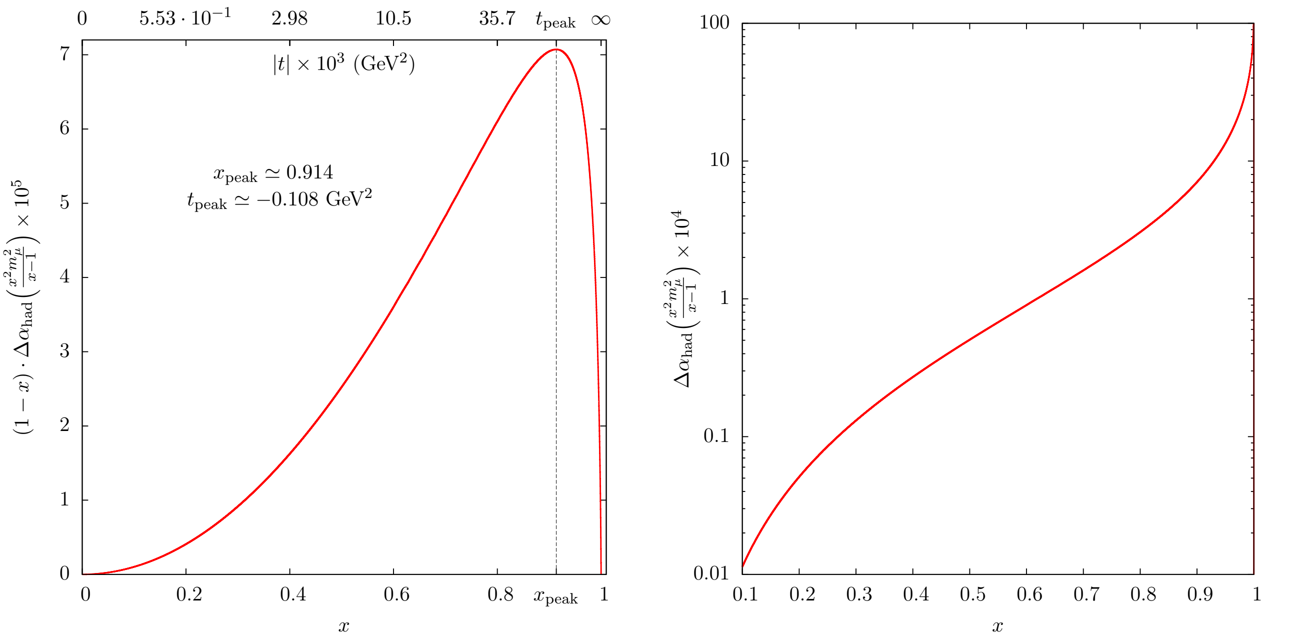

Figure 2. Left: The integrand $(1-x)\Delta\alpha_{had}[t(x)] \times 10^5$ as a function of x and t. Right: $\Delta\alpha_{had}[t(x)] \times 10^4$.

data and perturbative QCD). The range $x \in (0, 1)$ corresponds to $t \in (-\infty, 0)$, with $x = 0$ for $t = 0$. The peak of the integrand occurs at $x_{peak} \simeq 0.914$ where $t_{peak} \simeq -0.108$ GeV^2 and $\Delta\alpha_{had}(t_{peak}) \simeq 7.86 \times 10^{-4}$ (see fig. 2 (right)). Such relatively low t values can be explored at e^+e^- colliders with center-of-mass energy $\sqrt{s}$ around or below 10 GeV (the so called "flavor factories") where

$$t = -\frac{s}{2}(1 - cos\theta)\left(1 - \frac{4m_e^2}{s}\right), \tag{11}$$

θ is the electron scattering angle and m_e is the electron mass. Depending on s and θ, the integrand of Eq. (9) can be measured in the range $x \in [x_{min}, x_{max}]$, as shown in fig. 3 (left). Note that to span low x intervals, larger θ ranges are needed as the collider energy decreases. In this respect, $\sqrt{s} \sim 3$ GeV appears to be very convenient, as an x interval $[0.30, 0.98]$ can be measured varying θ between $\sim 2°$ and $28°$. Furthermore, given the smoothness of the integrand, values outside the measured x interval may be interpolated with some theoretical input. In particular, the region below x_{min} will provide a relatively small contribution to a_μ^{HLO}, while the region above x_{max} may be obtained by extrapolating the curve from x_{max} to $x = 1$, where the integrand is null, or using perturbative QCD.

The analytic dependence of the MC Bhabha predictions on $\alpha(t)$ (and, in turn, on $\Delta\alpha_{had}(t)$) is not trivial, and a numerical procedure has to be devised to extract it from the data.[1] In formulae, we have to find a function $\alpha(t)$ such that

$$\left.\frac{d\sigma}{dt}\right|_{data} = \left.\frac{d\sigma}{dt}\Big(\alpha(t), \alpha(s)\Big)\right|_{MC}, \tag{12}$$

where we explicitly kept apart the dependence on the time-like VP $\alpha(s)$ because we are only interested in $\alpha(t)$. We emphasise that, in our analysis, $\alpha(s)$ is an input parameter. Being the Bhabha cross section in the forward region dominated by the t-channel exchange diagram, we checked that the present $\alpha(s)$ uncertainty induces in this region a relative error on the θ distribution of less than $\sim 10^{-4}$ (which is part of the systematic error).

In order to assess the achievable accuracy on $\Delta\alpha_{had}(t)$ with the proposed method, we remark that the LO contribution to the cross section is quadratic in $\alpha(t)$, thus we have:

$$\frac{1}{2}\frac{\delta\sigma}{\sigma} \simeq \frac{\delta\alpha}{\alpha} \simeq \delta\Delta\alpha_{had} \tag{13}$$

Equation (13) relates the *absolute* error on $\Delta\alpha_{had}$ with the *relative* error on the Bhabha cross section. From the theoretical point of view, the present accuracy of the MC predictions [24] is at the level of about 0.5 per-mil, which implies that the precision that our method can, at best, set on $\Delta\alpha_{had}(t)$ is $\delta\Delta\alpha_{had}(t) \simeq 2 \cdot 10^{-4}$. Any further improvement requires the inclusion of the NNLO QED corrections into the MC codes, which are at present not available (although not out of reach) [24].

From the experimental point of view, we remark that a measurement of a_μ^{HLO} from space-like data competitive with the current time-like evaluations would require an $O(1\%)$ accuracy. Statistical considerations show that a 3% fractional accuracy on the a_μ^{HLO} integral can be obtained by sampling the integrand $(1-x)\Delta\alpha_{had}[t(x)]$ in ~ 10 points around the x peak with a fractional accuracy of 10%. Given the value of $O(10^{-3})$ for $\Delta\alpha_{had}$ at $x = x_{peak}$, this implies that the cross section must be known with relative accuracy of $\sim 2 \times 10^{-4}$. Such a statistical accuracy, although challenging, can be obtained at flavor factories, as shown in fig. 3 (right). With an integrated luminosity

[1] This was not the case for example in [6, 40]: there $\alpha(t)$ was extracted from Bhabha data in the very forward region at LEP, where the t channel diagrams are by far dominant and $\alpha(t)$ factorizes.

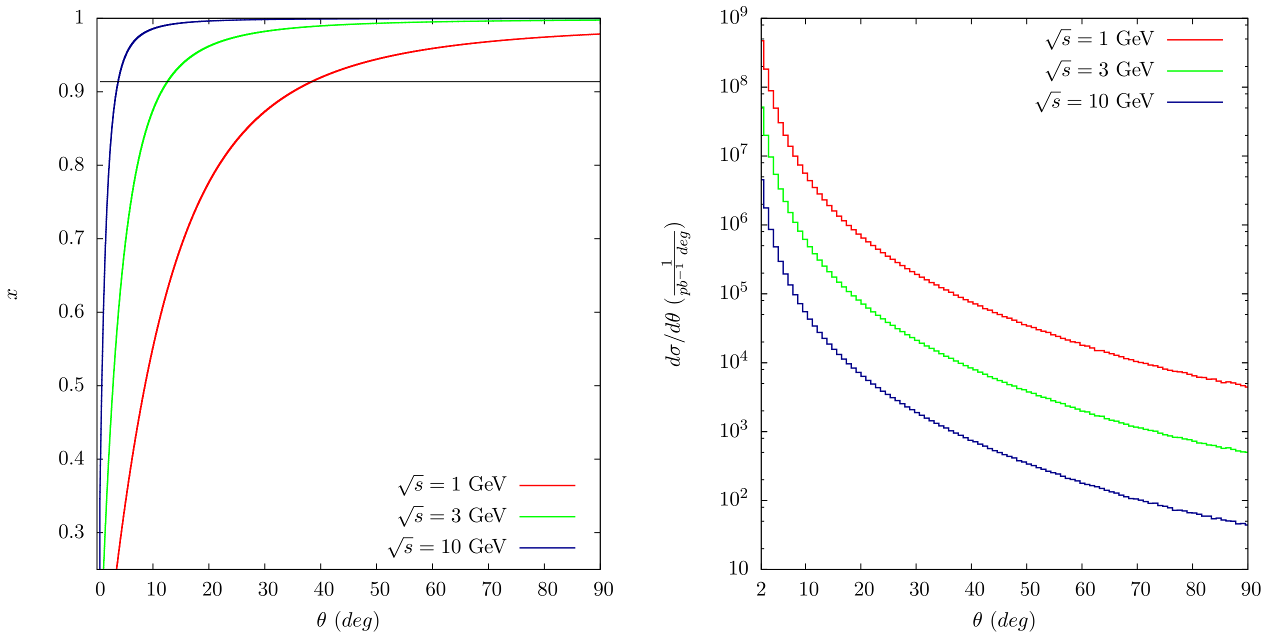

Figure 3. Left: Ranges of x values as a function of the electron scattering angle θ for three different center-of-mass energies. The horizontal line corresponds to $x = x_{\rm peak} \simeq 0.914$. Right: Bhabha differential cross section obtained with `BabaYaga` [38] as a function of θ for the same three values of $\sqrt{s}$ in the angular range $2^\circ < \theta < 90^\circ$.

of $O(1)$, $O(10)$, $O(100)$ fb^{-1} at $\sqrt{s}$ = 1, 3 and 10 GeV, respectively, the angular region of interest can be covered with a 0.01% accuracy per degree. The experimental systematic error must match the same level of accuracy.

A fraction of the experimental systematic error comes from the knowledge of the machine luminosity, which is normalized by calculating a theoretical cross section in principle not depending on $\Delta\alpha_{\rm had}$. We devise two possible options for the normalization process:

1. using the $e^+e^- \to \gamma\gamma$ process, which has no dependence on $\Delta\alpha_{\rm had}$, at least up to NNLO order;

2. using the Bhabha process at $t \sim 10^{-3}$ GeV^2 ($x \sim 0.3$), where the dependence on $\Delta\alpha_{\rm had}$ is of $O(10^{-5})$ and can be safely neglected.

Both processes have advantages and disadvantages; a dedicated study of the optimal choice goes beyond the scope of this paper and will be considered in a future detailed analysis.

6 Conclusions

After reviewing the traditional way to compute the leading hadronic correction to the muon g-2, we present a novel approach based on the measurements of the running of $\alpha(t)$ in the space-like region from Bhabha scattering data. Although challenging, we argued that this alternative determination may become feasible at flavor factories and possibly competitive with the accuracy of the present results obtained with the dispersive approach via time-like data.

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