EPJ Web of Conferences 97, 00020 (2015)
DOI: 10.1051/epjconf/20159700020

Lambda-Lambda Correlation in Relativistic Heavy Ion Collisions

Kenji Morita[1,2,a], Takenori Furumoto[3], and Akira Ohnishi[1,b]

[1] *Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*
[2] *Institute of Theoretical Physics, University of Wroclaw, PL-50204 Wrocław, Poland*
[3] *National Institute of Technology, Ichinoseki College, Ichinoseki, Iwate 021-8511, Japan*

Abstract. We investigate $\Lambda-\Lambda$ correlation function in relativistic heavy ion collisions to extract their interaction. Using an expanding source model for heavy ion collisions, we disentangle effects of collective expansion and those of the interaction at low relative momenta. Then, we discuss the influence of the feed-down correction for Σ^0 decay and indicate the possible existence of a residual correlation at high relative momenta. Consequently, the present STAR data suggest a weakly attractive interaction for the $\Lambda\Lambda$ pairs which is represented by the scattering length, $1/a_0 < -0.8$ fm^{-1}.

1 Introduction

The $\Lambda-\Lambda$ interaction plays an important role in various aspects of modern nuclear physics. For instance, it is expected that hyperons can emerge at already at moderate baryon densities within neutron star cores [1]. One needs more information on the interaction among hyperons to understand whether this picture is consistent with recently observed massive neutron stars [2] or not. Also in hadron physics, the existence of the H particle Jaffe pointed out in 1977 [3] depends on whether $\Lambda\Lambda$ can be deeply bound or not.

While an observation of a double hypernucleus $^{6}_{\Lambda\Lambda}$He and its weak decay (Nagara event)[4] ruled out the possibility of the deeply bound state, the bond energy of $\Lambda\Lambda$, $B_{\Lambda\Lambda}$, extracted from the $^{6}_{\Lambda\Lambda}$He indicates a weakly attractive $\Lambda\Lambda$ interaction, characterized by the scattering length and the effective range in the $^1S_0$ channel as $(a_0, r_{\rm eff}) = (-0.77\text{fm}, 6.59\text{fm})$ [5] or $(a_0, r_{\rm eff}) = (-0.575\text{fm}, 6.45\text{fm})$ [6] as extracted from structure calculations. Nevertheless, a single number $B_{\Lambda\Lambda}$ is not sufficient to determine the interaction characterized by the scattering length and the effective range. Recently, an alternative possibility has been explored in relativistic heavy ion collisions at Relativistic Heavy Ion Collider (RHIC). The STAR collaboration measured $\Lambda-\Lambda$ momentum correlation $C(Q)$ in Au+Au collisions at $\sqrt{s_{NN}} = 200$GeV, as a function of relative momentum $Q = |p_1 - p_2|$ [7]. In high energy heavy ion collisions, the momentum intensity correlation functions have been measured for various particles to access the space-time extent of the hot matter via Hanbury-Brown Twiss (HBT) effect. It has been pointed out that one may also extract information on interaction between particles when the source size and the effective range of the interaction are comparable [8–11].

[a] e-mail: kmorita@yukawa.kyoto-u.ac.jp
[b] e-mail: ohnishi@yukawa.kyoto-u.ac.jp

The goal of this work is to determine $\Lambda\Lambda$ interaction from the measured data at RHIC. For this purpose, we need to understand several effects on the $\Lambda\Lambda$ correlation function other than interaction. In this proceeding, we highlight results from our recent work [12].

2 $\Lambda\Lambda$ correlation from an expanding source

We compute the $\Lambda\Lambda$ correlation function $C(\mathbf{Q},\mathbf{K})$ as a function of relative momentum $\mathbf{Q} = \mathbf{p_1} - \mathbf{p_2}$ and average one $\mathbf{K} = (\mathbf{k_1} + \mathbf{k_2})/2$ using a formula [13]:

$$C(\mathbf{Q},\mathbf{K}) = \frac{\int d^4x_1 d^4x_2 S(x_1,\mathbf{K}) S(x_2,\mathbf{K}) |\Psi_{12}(\mathbf{Q}, \mathbf{x_1} - \mathbf{x_2} - (t_2 - t_1)\mathbf{K}/m)|^2}{\int d^4x_1 d^4x_2 S(x_1,\mathbf{k_1}) S(x_2,\mathbf{k_2})}. \tag{1}$$

There are two ingredients affecting the behavior of the correlation function in Eq. (1). Information on the $\Lambda\Lambda$ interaction is encoded in the relative wave function Ψ_{12}. In the second argument of Eq. (1), $-(t_2 - t_1)\mathbf{K}/m$ with m being the mass of Λ, accounts for possible differences in the emission time of the two Λ particles. Considering only the s-wave interaction, one can write down the relative spatial wave function of the spin-singlet state, which is symmetric with respect to the exchange of the particles (thus totally antisymmetric), as

$$\Psi_s = \sqrt{2}[\cos(\mathbf{Q}\cdot\mathbf{r}/2) + \chi_Q(r) - j_0(Qr/2)] \tag{2}$$

where χ_Q is the s-wave solution of the Schrödinger equation with a given $\Lambda\Lambda$ interaction potential $V(r)$ and the spherical Bessel function $j_0(Qr/2)$ stands for the free s-wave function with $Q = |\mathbf{Q}|$. The spin-averaged total wave function is then given by:

$$\begin{aligned} |\Psi_{12}|^2 &= \frac{1}{4}|\Psi_s|^2 + \frac{3}{4}|\Psi_t|^2 \\ &= 1 - \frac{1}{2}\cos(\mathbf{Q}\cdot\mathbf{r}) + [\chi_Q(r) - j_0(Qr/2)]\cos(\mathbf{Q}\cdot\mathbf{r}/2) + \frac{1}{2}[\chi_Q(r) - j_0(Qr/2)]^2 \end{aligned} \tag{3}$$

For the free wave function, only the first and second terms remain to give $C(\mathbf{Q} = 0) = 1/2$. The correlation below unity reflects the effective repulsion due to Pauli principle for fermions. One notes that $C(\mathbf{Q} = 0) = 2$ for pions, according to the Bose statistics. Thus, one expects that repulsive and attractive interaction give $C(\mathbf{Q} \simeq 0) < 1/2$ and $C(\mathbf{Q} \simeq 0) > 1/2$, respectively. The behavior of the correlation function in the presence of interaction can be systematically studied by employing a simple source function $S(x,\mathbf{K})$ in Eq. (1), which is the other ingredient representing the emission probability of Λ with momentum $\mathbf{K}$ from a space-time point x.

We examined various interaction potentials used in structure calculations of hypernuclei. See Ref. [12] for a tabulated list. Figure 1 displays some of the potentials in the left panel and the corresponding relative spatial wave functions weighted by the source function in the right panel. One sees the difference among the potentials is reflected onto wave function particularly for distances close to the source radius. The difference in the wave function in $r \ll R$ is suppressed by the weight factor. As we shall show below, the correlation function provides a constraint on scattering parameters rather than the detailed form of the potential. The scattering length a_0 and the effective range r_0 are related to the scattering phase shift δ through the shape-independent form:

$$k\cot\delta = -\frac{1}{a_0} + \frac{1}{2}r_{\text{eff}}k^2 + O(k^4). \tag{4}$$

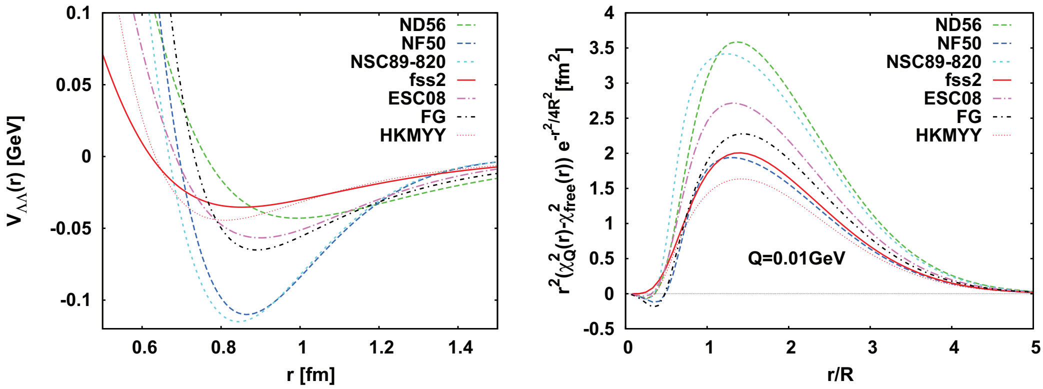

Figure 1. Left: $\Lambda\Lambda$ interaction potentials. Right: difference of relative wave functions from the free one weighted with the static source function at $Q = 0.01$GeV.

In this definition, the interaction is attractive for $a_0 < 0$ and a bound state exists for $a_0 > 0$. Applying a spherically symmetric static source function, one obtains the constraints: -1.2 fm $< a_0 < -0.8$ fm and 3.2 fm $< r_{\rm eff} < 6.5$ fm from a χ^2 fit to the STAR data [12]. While this result is consistent with the findings from the Nagara event, one needs to investigate the influences from source properties in heavy ion collisions. In particular, the hot matter created in high energy heavy ion collisions exhibits rapid hydrodynamic expansion. At the RHIC energy, the matter consists of deconfined quarks and gluons in the early stage of the collision process. Hydrodynamics indicates the matter undergoes transition into hadronic matter around 5 fm/c after the impact. We assume that Λ particles are produced at the hadronization process. Since single particle levels of Λ including those of deep s-states are clearly observed [14], one expects the interaction of Λ with pionic environment is weak, in contrast to the π and proton cases. Thus we assume that effects of interaction with surrounding particles on the correlation function can be neglected. One needs to invoke dynamical simulations to check the validity of the assumption. Nevertheless, one needs to consider an appropriate source function which takes into account the collective expansion and the relevant geometry. We employ a cylindrically symmetric source model with a longitudinal boost invariant and transverse Hubble-type expansions and is used for studying $\pi\pi$ HBT correlation in [15]. Then the source function $S(x, \mathbf{K})$ reads:

$$S(x,\mathbf{K}) = \frac{m_T \cosh(y - Y_L)}{(2\pi)^3\sqrt{2\pi(\Delta\tau)^2}} n_f(u\cdot k, T) \exp\left[-\frac{(\tau-\tau_0)^2}{2(\Delta\tau)^2} - \frac{x^2+y^2}{2R^2}\right]. \tag{5}$$

with $y = \ln\sqrt{(E_k + p_z)/(E_k - p_z)}$ and $m_T = \sqrt{p_t^2 + m^2}$ being rapidity and transverse momentum of Λ particles, respectively. The expansion velocity u^μ is parameterized in terms of longitudinal rapidity Y_L and transverse one Y_T as $u^\mu = (\cosh Y_T \cosh Y_L, \sinh Y_T \cos\phi, \sinh Y_T \sin\phi, \cosh Y_T \sinh Y_L)$. The boost invariant expansion is then realized by $Y_L = \eta = \ln\sqrt{(t+z)/(t-z)}$ [16]. The transverse flow is given by $Y_T = \eta_f r/R$ where η_f is a strength parameter. Considering the emission of Λ at the hadronization, we fix the freeze-out temperature $T = 160$MeV, freeze-out proper time $\tau_0 = 5$ fm and emission duration $\Delta\tau = 2$ fm. Then through the invariant single particle distribution

$$E\frac{dN}{d^3\mathbf{k}} = \int d^4x S(x,\mathbf{K}), \tag{6}$$

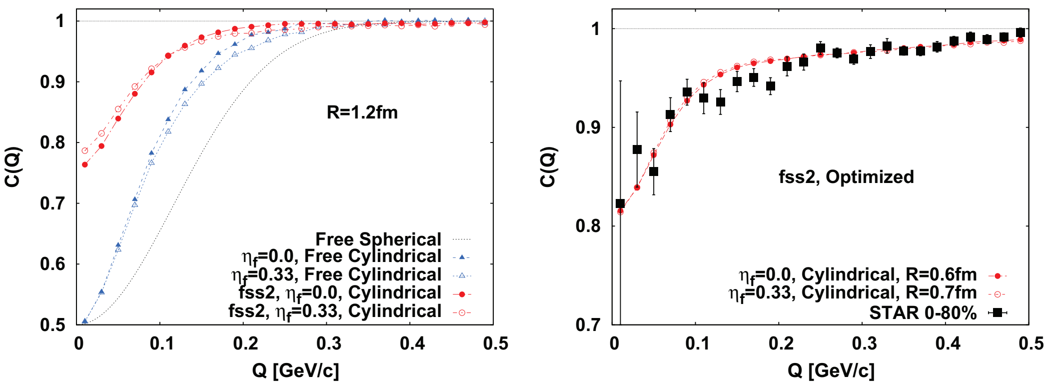

Figure 2. $\Lambda\Lambda$ correlation function for $R = 1.2$ fm. Left: Comparison of the free cases and interacting cases with and without flow. Right: R is optimized for the STAR data.

we can determine $\eta_f = 0.33$ by fitting to p_t spectrum [17].

Figure 2-left illustrates the effects of the collective expansion on $C(Q)$ and its interplay with the interaction. The dotted line shows the correlation function from the spherically symmetric Gaussian source with $R = 1.2$ fm. Filled triangles denotes the results from the source function (5) with the same transverse size R, but without transverse flow (i.e, $\eta_f = 0$) and interaction. One notes that the width of $C(Q)$ is reduced, because the source has a large longitudinal extent due to the boost-invariant expansion along the collision axis. If the transverse flow is turned on, the width is decreased as seen in the open triangles, but the effect is not significant. See Refs. [15, 18] for detailed discussion on the effects of collective expansion on the pion HBT correlation. Closed and open circles plot $C(Q)$ with fss2 interaction, which exhibits weak attraction characterized by $a_0 = -0.81$ fm and $r_{\rm eff} = 3.99$ fm. One sees an enhancement of the correlation as expected for an attractive interaction, but the enhancement is found to be stronger in the presence of transverse flow, because of the effectively smaller source size caused by the transverse expansion. To extract the interaction from the measured $\Lambda\Lambda$ correlation function data by the STAR collaboration, we perform a χ^2 analysis with R being a free parameter. In Fig. 2-right, the $C(Q)$ obtained using the fss2 interaction and corresponding to the optimized value of R are shown. One finds that both $C(Q)$, with and without transverse flow, do not differ after optimization. The minimization of χ^2 leads to slightly smaller size R in the presence of the transverse flow. Thus, we conclude that the effect of the transverse flow is absorbed into the size parameter through the fitting procedure.

Figure 3 summarizes the results from the source model (5). Although the optimized size parameter is decreased and the value of $\chi^2/N_{\rm dof}$ changes in each of potentials, we find that the favored potentials are the same as those in the static source model. Even in the presence of collective flow, the low Q behavior of $C(Q)$ fairly reflects differences among the interactions. The stronger the attraction becomes, the closer to unity $C(Q)$ is. This shows feasibility of $\Lambda\Lambda$ correlation in heavy ion collisions to discriminate the nature of the interaction.

3 Feed-down correction

In the previous section, we have assumed that Λ particles are directly produced from the medium modeled by the source function (5). In reality, however, some of Λ come from decay of heavier particles.

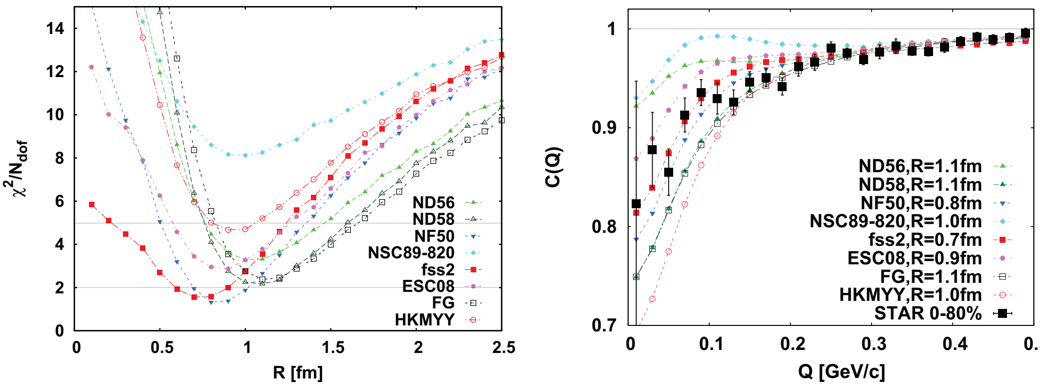

Figure 3. $\chi^2/N_{\rm dof}$ as functions of R (left) and resultant $\Lambda\Lambda$ correlation function (right) for the optimized source sizes, for various potentials.

Since this situation also applies to pion production in heavy ion collisions, effects of resonances have been studied in $\pi\pi$ correlations [19]. Since the source function for daughter particles are extended by the decay length, short-lived decay parents affect the shape of the correlation function at low Q. Long-lived ones effectively reduce the intercept parameter λ which is defined by $C(\mathbf{Q} = 0)$ because too long relative distance of pairs causes a strong correlation at too small Q values to be resolved. While the long-lived parents can be neglected when one focuses on the source size, one needs to consider the effects in order to study the interaction. As discussed above, the nature of $\Lambda\Lambda$ interaction is encoded in $C(Q \simeq 0)$, thus the reduction of λ due to long-lived resonances may affect the interpretation of the data.

For the total Λ yield $N_{\rm tot}$ and the yields stemming from long-lived parents N^p, the reduced intercept λ is given by [20]:

$$\lambda = \left(1 - \frac{N^p}{N_{\rm tot}}\right)^2 . \tag{7}$$

The dominant contribution to the Λ yield comes from $\Sigma(1385)$, Σ^0 and Ξ as well as primordial Λ. Since the decay width of the $\Sigma(1385)$ resonance is 36-40 MeV, it can be treated as short-lived resonances contributing to $N_{\rm tot}$ only. We find that the Ω contribution is negligible, thus we consider Σ^0 and Ξ as the main contribution when evaluating N^p.

Σ^0 contributes to Λ samples through $\Sigma^0 \to \Lambda\gamma$, which is not easy to reconstruct. We make use of an experimental results of Σ^0 yields, $N_{\Sigma^0}/N_\Lambda = 0.278$ in p+Be collisions at $p_{\rm lab} = 28.5$GeV [21]. In spite of intrinsic differences of p+A collisions from A+A, we note that this ratio is consistent with thermal model calculations. Ξ yields have been found to be 15% of Λ in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV [17]. According to their decay length, we assume that part of Ξ decay contribution is excluded by the Λ sample selection (DCT < 0.4cm) in the STAR measurement [7]. Thus, taking into account only the Σ^0 contribution to N^p, we find $\lambda = (0.67)^2$. If we also include Ξ, $\lambda = (0.572)^2$. Since the selection might not reject all the Ξ decay contribution to Λ, these numbers can be regarded as possible upper and lower limits. In the following, we present results only for $\lambda = (0.67)^2$, but we also confirm that the conclusion does not change if one adopts $\lambda = (0.572)^2$. Then, the $\Lambda\Lambda$ correlation function can be corrected for the feed-down contribution as:

$$C_{\rm corr}(Q) = 1 + \lambda(C_{\rm bare}(Q) - 1), \tag{8}$$

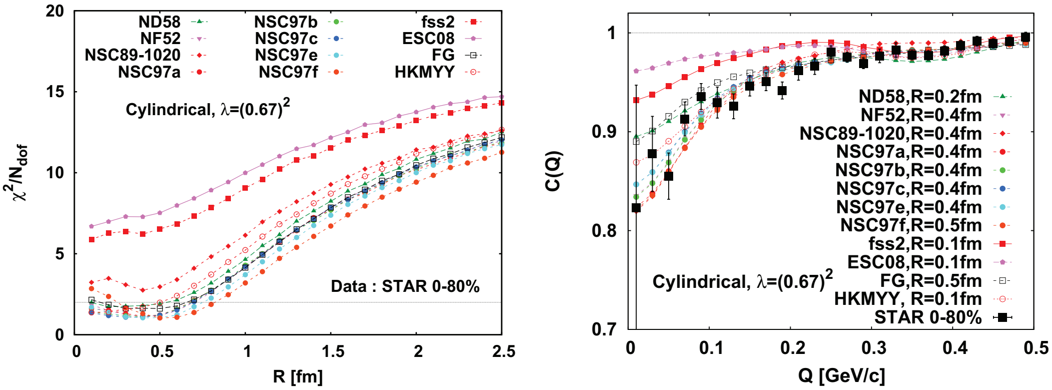

Figure 4. Same as Fig. 3, but with feed-down correction for Σ^0 decay (8).

where $C_{\rm bare}$ denotes the bare correlation function calculated from Eq. (1) with the wave function (3) and the source function (5).

Figure 4 shows $\chi^2/N_{\rm dof}$ and $C(Q)$ with the optimized value of R, similar to Fig. 3. One sees that good fits can be achieved only for unphysically small R. The reason is clearly seen if one looks into $C(Q)$ at high Q. Since the data has a long tail, smaller size is favored to fit the tail. On the other hand, such smaller size enhances the effect of attractive interaction. As feed-down correction introduces a larger intercept, $C(Q=0) = 0.776$ for $\lambda = (0.67)^2$, strong attraction leads to overshooting the data at low Q. Consequently, the data can be fitted only with the small size and very weak interaction potential which was excluded in the analyses without feed-down correction. There seems to exist an additional source of the correlation which produces the long tail at high Q.

Here we try to subtract this residual correlation in the data by employing an additional Gaussian term

$$C_{\rm res}(Q) = a_{\rm res} e^{-r_{\rm res}^2 Q^2}, \tag{9}$$

such that the total correlation function is given by $C(Q) = C_{\rm corr}(Q) + C_{\rm res}(Q)$. This term was also introduced in the analysis by the STAR collaboration and found to improve the quality of the fit [7], but the two quantities $(a_{\rm res}, r_{\rm res})$ were treated as additional fitting parameters. Because the origin of this correlation is not known, we first investigate systematics of the fit against the variation of these parameters.

We evaluate the minimum of $\chi^2/N_{\rm dof}$ against the parameters $(a_{\rm res}, r_{\rm res})$ for a fixed R and repeated it for a variation of R between 0 and 2.5 fm. Figure 5-left displays the values of $\chi^2/N_{\rm dof}$. For shown potentials, one sees $\chi^2/N_{\rm dof} \simeq 1$ for $R > 0.5$fm. Although one may think this insensitivity could indicate no information on the interaction due to the connection between the size and the attraction discussed above, one can see the influence of the interaction on the behavior of $a_{\rm res}$ and $r_{\rm res}$. For small R, the bare correlation has a long tail owing to collective expansion and low Q behavior is strongly affected by the interaction. As one sees in the middle and right panel of Fig. 5, interaction dependence of the two residual correlation parameters can be understood as a consequence of compensation by the residual correlation to fit the data. On the other hand, at large R, these two parameters converge to common values. In this case, low Q behavior is dominated by the interaction and the flow effect, and the residual correlation (9) is responsible for the long tail at high Q. Since the source size cannot be too small according to consistency with HBT data of other particles, this results may indicate an

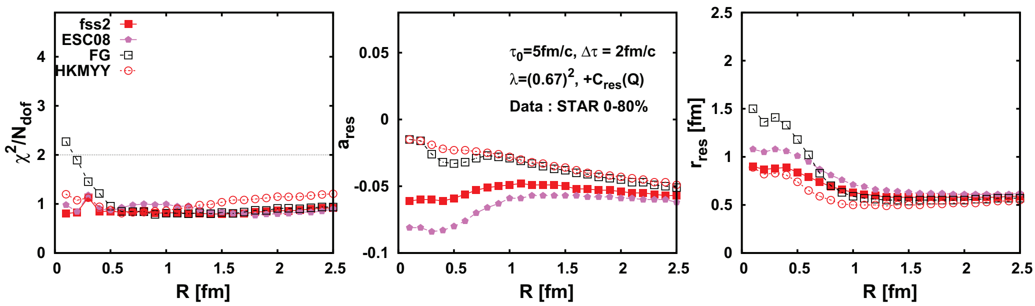

Figure 5. Left: Minima of $\chi^2/N_{\rm dof}$ against $(a_{\rm res}, r_{\rm res})$ for each R and selected $\Lambda\Lambda$ potentials. Center and right: $a_{\rm res}$ and $r_{\rm res}$ at the minimum of $\chi^2/N_{\rm dof}$, respectively.

existence of the residual correlation which cannot be explained by conventional effects which are examined in this work.

Repeating the same analysis with other potentials, we find that all the potential favored in Sec. 2 is still favored after taking into account the residual correlation as well as the feed-down correction. The reduction of $C(Q = 0)$ due to the feed-down correction, however, makes the correlation less sensitive to the interaction. In particular, we cannot find any constraints on the effective range $r_{\rm eff}$ but the scattering length as $1/a_0 < -0.8$ fm^{-1}.

4 Concluding remarks

We have studied $\Lambda - \Lambda$ correlation function $C(Q)$ as a function of relative momentum $Q = |p_1 - p_2|$ in relativistic heavy ion collisions by employing various type of interaction potential models and a source function for cylindrically expanding source. Assuming direct production of Λ from the hot source, we obtain a constraint on the interaction by fitting calculations to the data. Comparison of the calculated correlation functions with the STAR data indicates potential feasibility of discriminating the interaction potential even after taking into account expansion of the source in relativistic heavy ion collisions. The sensitivity of the correlation function to the interaction, however, is reduced by feed-down contribution from Σ^0 decay. Moreover, the long tail in the data at high Q seems to indicate existence of additional source of the correlation. By including the additional correlation, we find that the data nevertheless favor weakly attracting $\Lambda\Lambda$ interaction characterized by $1/a_0 < -0.8$fm^{-1}.

Acknowledgements

The authors would like to thank N. Shah and H. Z. Huang for providing them the STAR data. They also would like to thank Y. Akaishi, E. Hiyama, A. Gal, B. Müller, K. Redlich, T. Rijken, S. Shinmura, and Y. Yamamoto for helpful discussions. This work was partially supported by Grants-in-Aid for Scientific Research from JSPS (Nos. (B) 23340067, (B) 24340054, (C) 24540271), by the Grants-in-Aid for Scientific Research on Innovative Areas from MEXT (No. 2404:24105001, 24105008), by the Yukawa International Program for Quark-hadron Sciences, and by the Polish Science Foundation (NCN), under Maestro grant 2013/10/A/ST2/00106.

References

[1] N. K. Glendenning, Astrophys. J. **293**, 470 (1985).

[2] P. Demorest, T. Pennucci, S. Ransom, M. Roberts and J. Hessels, Nature **467**, 1081 (2010).

[3] R. L. Jaffe, Phys. Rev. Lett. **38**, 195 (1977).

[4] H. Takahashi et al., Phys. Rev. Lett. **87**, 212502 (2001).

[5] I. N. Filikhin and A. Gal, Nucl. Phys. **A707**, 491 (2002).

[6] E. Hiyama, M. Kamimura, T. Motoba, T. Yamada and Y. Yamamoto, Phys. Rev. C **66**, 024007 (2002).

[7] L. Adamczyk et al., (STAR Collaboration), Phys. Rev. Lett. **114**, 022301 (2015).

[8] R. Lednicky and V. L. Lyuboshits, Sov. J. Nucl. Phys. **35**, 770 (1982).

[9] W. Bauer, C. K. Gelbke and S. Pratt, Annu. Rev. Nucl. Part. Sci. **42**, 77 (1992).

[10] C. Greiner and B. Müller, Phys. Lett. B **219**, 199 (1989).

[11] A. Ohnishi, Y. Hirata, Y. Nara, S. Shinmura and Y. Akaishi, Nucl. Phys. **A670**, 297 (2000).

[12] K. Morita, T. Furumoto and A. Ohnishi, Phys Rev. C **91**, 024916 (2015).

[13] S. E. Koonin, Phys. Lett. **70B**, 43 (1977); W. G. Gong, W. Bauer, C. K. Gelbke and S. Pratt, Phys. Rev. C **43**, 781 (1991).

[14] O. Hashimoto and H. Tamura, Prog. Part. Nucl. Phys. **57**, 564 (2006).

[15] S. Chapman, P. Scotto and U. Heinz, Heavy Ion Phys. **1**, 1 (1995).

[16] J. D. Bjorken, Phys. Rev. D **27**, 140 (1983).

[17] G. Agakishiev et al., (STAR Collaboration), Phys. Rev. Lett. **108**, 072301 (2012).

[18] N. Makhlin and Yu. M. Sinyukov, Z. Phys. C **39**, 69 (1988); K. Morita, S. Muroya, C. Nonaka and T. Hirano, Phys. Rev. C **66**, 054904 (2002).

[19] U. Wiedemann and U. Heinz, Phy. Rev. C **56**, 3265 (1997).

[20] P. Grassberger, Nucl. Phys. **B120**, 231 (1977); T. Csörgő, B. Lörstad and J. Zimányi, Z. Phys. C **71**, 491 (1996).

[21] M. W. Sullivan et al., Phys. Rev. D **36**, 674 (1987).