EPJ Web of Conferences **99**, 10001 (2015)
DOI: 10.1051/epjconf/20159910001

Shower center of gravity and interaction characteristics

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Abstract. The shower center of gravity is used for studying the interconnection between shower longitudinal profile and hadronic interaction characteristics. The equations for the shower originated by high energy proton in the atmosphere are written and, within certain simplifications, solved for the case of logarithmically decreasing interaction length of hadrons in the air. The obtained expression explicitely splits into center of gravity of the purely electromagnetic cascade at the primary proton energy and modification of that by hadronic cascading and provides transparent view of the way in which hadronic interaction characteristics determine the longitudinal shower development.

1. Introduction

Since long ago, numerous attempts have been undertaken to explicitely connect the air shower longitudinal profile, in particular the shower maximum depth (X_{max}), with hadronic interaction characteristics.

Very approximate approaches were tried, from toy models to extensions of the Heitler model for the electromagnetic shower [1] to the hadronic shower [2–4] and all these have proven to be of not big quantitative help.

A direct way to establish the connection is the use of the cascade theory. A problem is that the shower maximum is an inconvenient quantity for cascade equations. Rather, a convenient quantity is the shower center of gravity (CG).

Although the shift between X_{max} and CG is energy and generator dependent, having been successfully solved equations for CG would allow to look in detail into the dynamics of shower longitudinal development, which governs both X_{max} and CG, and to see explicitely how interaction characteristics enter that dynamics.

2. Expression for the center of gravity

We consider dependence on energy of the shower center of gravity:

$$\overline{X(E)} = \int\limits_0^\infty X\,N(X)\,dX \Big/ \int\limits_0^\infty N(X)\,dX = \int\limits_0^\infty X\,N(X)\,dX \,/\, E, \tag{1}$$

where $N(X)$ is the number of charged particles in a shower.

The proton primary is considered. In simplifying assumptions of i) Feinman scaling, ii) cascading only two types of hadrons, baryons (nucleons) and pions, iii) neglecting pion decay the system of equations for the nominator of (1) $\Phi = \int\limits_0^\infty X\,N(X)\,dX$ looks (`arXiv:1202.4989`):

$$\begin{aligned}
\Phi_N(E) &= \int\limits_0^1 \frac{dn_{N\to N}}{dx}(x)\,\Phi_N(Ex)\,dx + \\
&\int\limits_0^1 \frac{dn_{N\to\pi}}{dx}(x)\,\Phi_\pi(Ex)\,dx \\
&+ \int\limits_0^1 \frac{dn_{N\to 0}}{dx}(x)\,\Phi_0(Ex)\,dx + E\,\lambda_N(E) \\
\Phi_\pi(E) &= \int\limits_0^1 \frac{dn_{\pi\to\pi}}{dx}(x)\,\Phi_\pi(Ex)\,dx \\
&+ \int\limits_0^1 \frac{dn_{\pi\to 0}}{dx}(x)\,\Phi_0(Ex)\,dx + E\,\lambda_\pi(E) \\
\Phi_0(E) &= 2\int\limits_0^1 \frac{dx}{x}\Phi_\gamma(Ex)\,dx.
\end{aligned} \tag{2}$$

On the assumptions of logarithmically decreasing proton and pion interaction lengths an exact solution of that system of equations is possible. The obtained expression for the proton shower center of gravity looks:

$$\begin{aligned}
\overline{X_N(E)} &= X_0\left(\ln\frac{E}{E_c} + \delta - \frac{1}{2}\right) + \\
&\frac{1}{1-g_{NN}}\Big\{\lambda_N(E_N^{eff}) + X_0\cdot\mu_N + \\
&\frac{g_{N\pi}}{1-g_{\pi\pi}}\left[\lambda_\pi(E_\pi^{eff}) + X_0\cdot\mu_\pi\right]\Big\}
\end{aligned} \tag{3}$$

E is the energy of the proton, E_c is the critical energy and X_0 is the radiation depth in air, $\delta = 1.7$ and N_A is Avogadro number.

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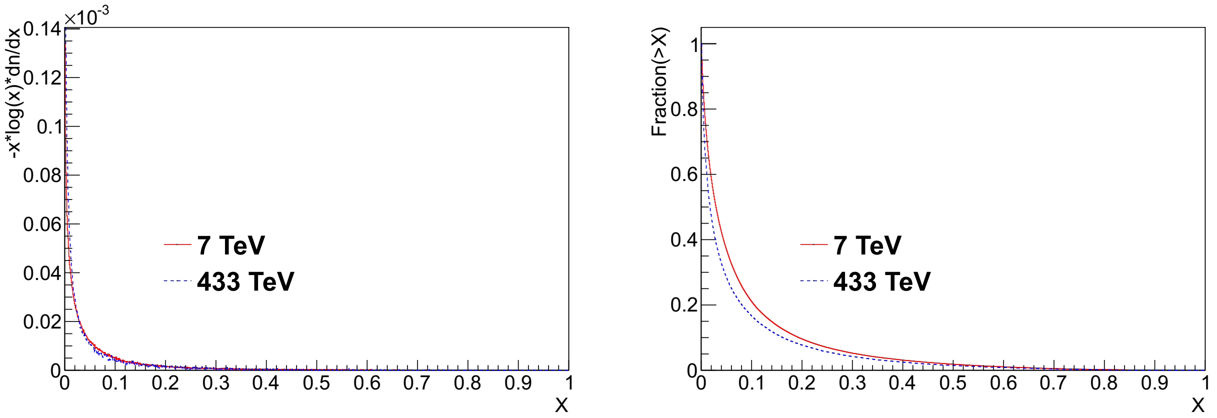

Figure 1. Inclusive distributions for all particles production in proton-air interactions at two energies from EPOS LHC generator: $x \cdot logx\ dn/dx$ (left) and the integral distibution, i.e. the fraction of energy contained above some x (right).

Interaction lengths have proven to be taken not at the primary energy but at some lower effective energies:

$$E_N^{eff} = E/\exp\left(\frac{\gamma_{NN}}{1-g_{NN}}\right) \tag{4}$$

and

$$E_\pi^{eff} = E_N^{eff}/\exp\left(\frac{\gamma_{N\pi}}{g_{N\pi}} + \frac{\gamma_{\pi\pi}}{1-g_{\pi\pi}}\right). \tag{5}$$

As one can see, the expression for the proton shower center of gravity explicitly splits into two terms: the center of gravity of the purely electromagnetic cascade at the proton primary energy and a modification of this by the hadronic cascade, $\overline{X_N(E)} = \overline{X_e(E)} + \Delta X_{had}$. The latter is determined by two competing oppositely directed processes: i) carrying through energy by hadrons, that elongates the shower, ii) energy dissipation in the hadronic interactions due to which electromagnetic subshowers start at smaller energies, and because of the logarithmic energy dependency of their center of gravity that results in shortening of the total shower. The first proccess is represented by λ terms in the final expression, the second one is represented by μ terms.

Characteristics of multiple production enter two kinds of expressions.

The first kind reflects the energy transition between different sorts of hadrons, i.e. the from the baryon to the charged or neutral pions or from the charged pion to the neutral pions (i,j below denote sort). The obtained expressions are simply mean relative energies contained in the produced particles of some sort (like the inelasticity):

$$g_{ij} = \int_0^1 x\,\frac{dn_{i\to j}}{dx}(x)\,dx. \tag{6}$$

Energy transition governs the pace of the shower elongation over hadron cascading.

The second kind reflects the rate of energy dissipation:

$$\gamma_{ij} = \int_0^1 x\cdot\ln x\,\frac{dn_{i\to j}}{dx}(x)\,dx, \tag{7}$$

$$\mu_i = \int_0^1 x\cdot\ln x\,\frac{dn_{i\to X}}{dx}(x)\,dx. \tag{8}$$

The meaning of weight $x \cdot lnx$ is clear: each particle contributes to the total center of gravity proportionally to its energy and the longitudinal width of the produced at that energy electromagnetic shower which is proportional to the logarithm of energy. These factors are negative: the dissipation of energy in hadronic cascade results in electromagnetic subcascades starting at smaller and smaller energies and thus in reducing the total center of gravity relative to the absense of dissipation.

The total mutiplicity is usually considered as a shower property defining energy dissipation. Instead, energy dissipation here is represented by the above integrals which are in fact the forward multiplicities.

It should be noted that although equations are written for the case of Feynman scaling, integrals entering final formula would be obtained for each primary proton energy from MC simulations for that particular energy and accordingly with accounting for whatever physics these MC manages. including scaling violation. Thus scaling violation proves to be partially accounted for (may be considered as to first order).

3. Results

The shower center of gravity is calculated for five generators of hadronic interactions: Sibyll[5], QGSJETII-03[6,7], QGSJETII-04[8,9], EPOS 1.99[10,11] and EPOS LHC[12].

Figure 1 illustrates the statement that the integrals defining energy dissipation could be considered as the forward multiplicities. In the left, the distribution $x \cdot \log x \times dn/dx$ is shown for all produced particles in interactions of protons with air at two energies. From the right, where the distribution of the fraction of the energy contained above a certain value of x is shown, it can be extracted that the main energy is contained in the forward region, about 75% at $x > 0.01$.

Figure 2 compares the dependence on energy of the total and forward multiplicities for two generators,

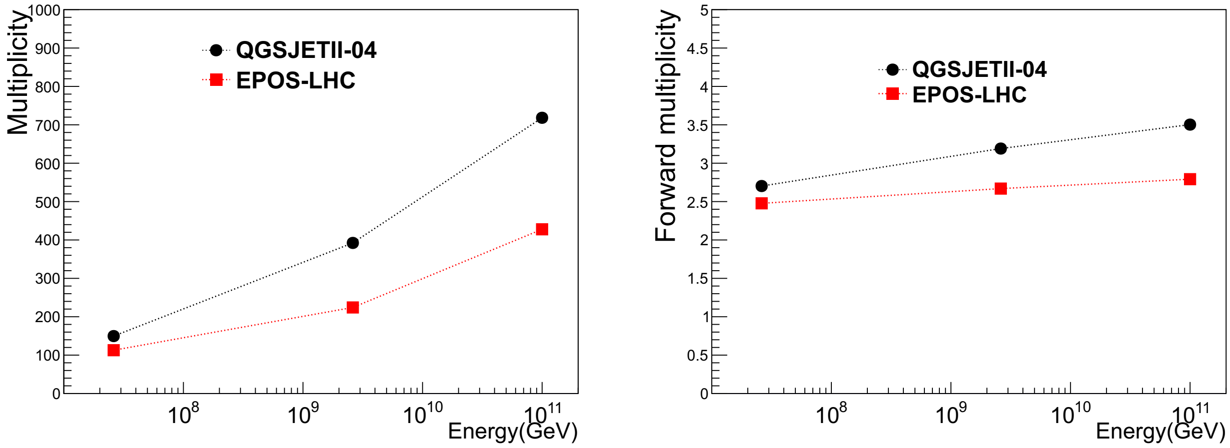

Figure 2. Dependence on energy of the total (left) and forward (right) multiplicities for QGSJETII-04 and EPOS LHC.

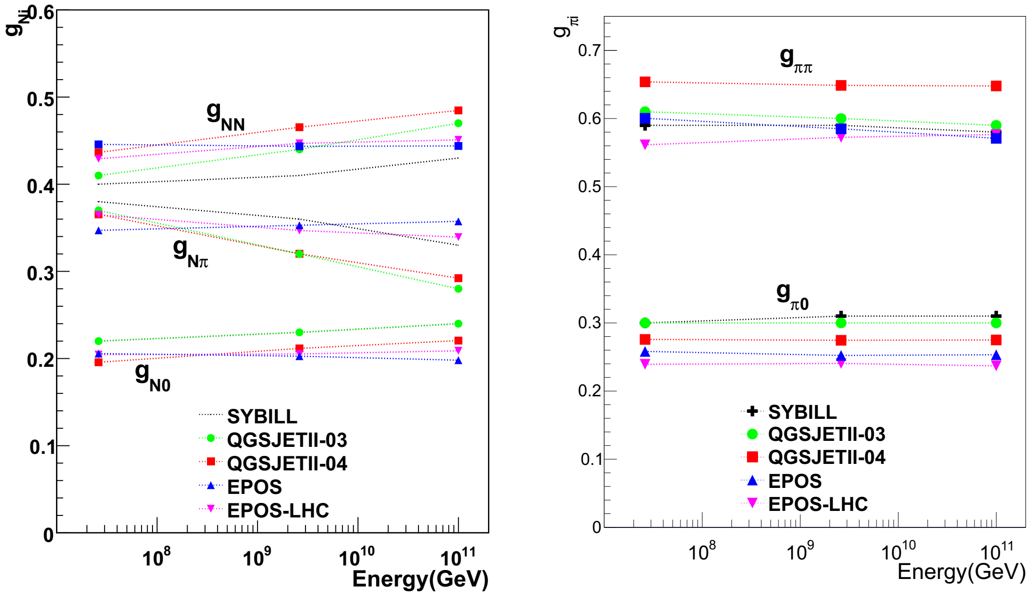

Figure 3. Dependence on the energy of the integrals defining the energy transition for proton-air (left) and pion-air (right) interactions.

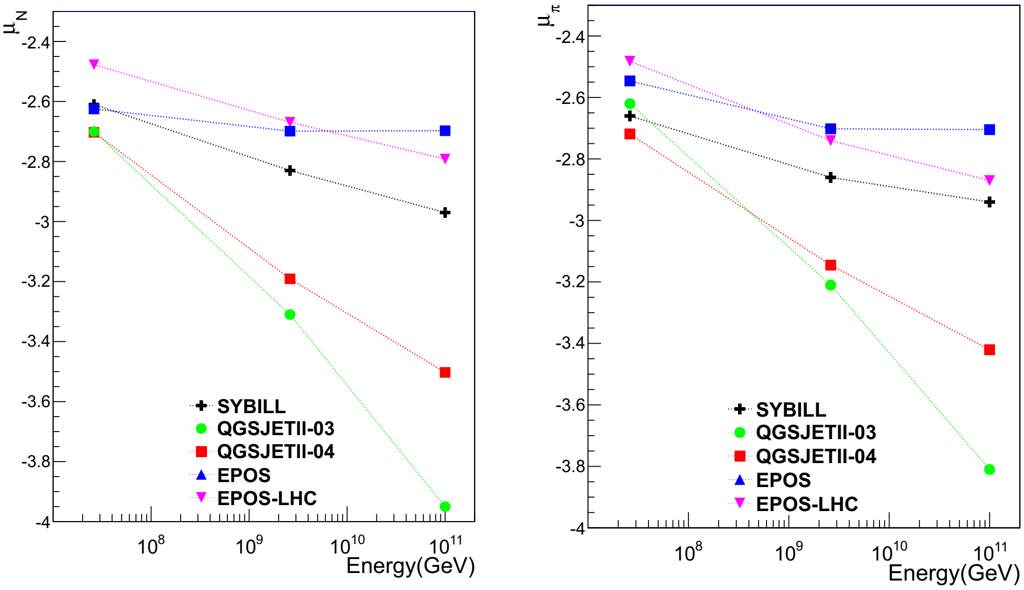

Figure 4. Dependence on the energy of the integrals defining the energy dissipation for proton-air (left) and pion-air (right) interactions.

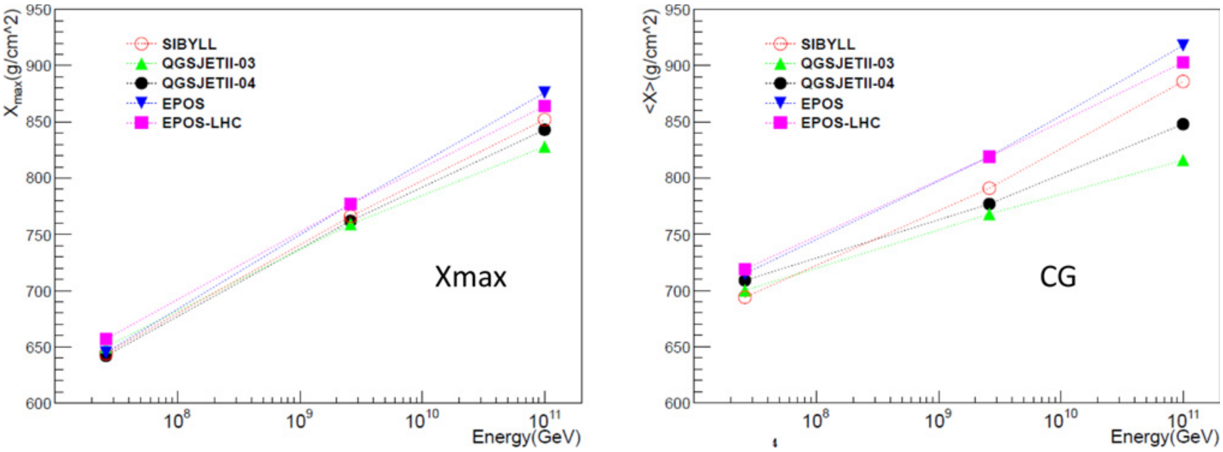

Figure 5. Energy dependence of the shower maximum (left) and center of gravity (right) of the the proton shower.

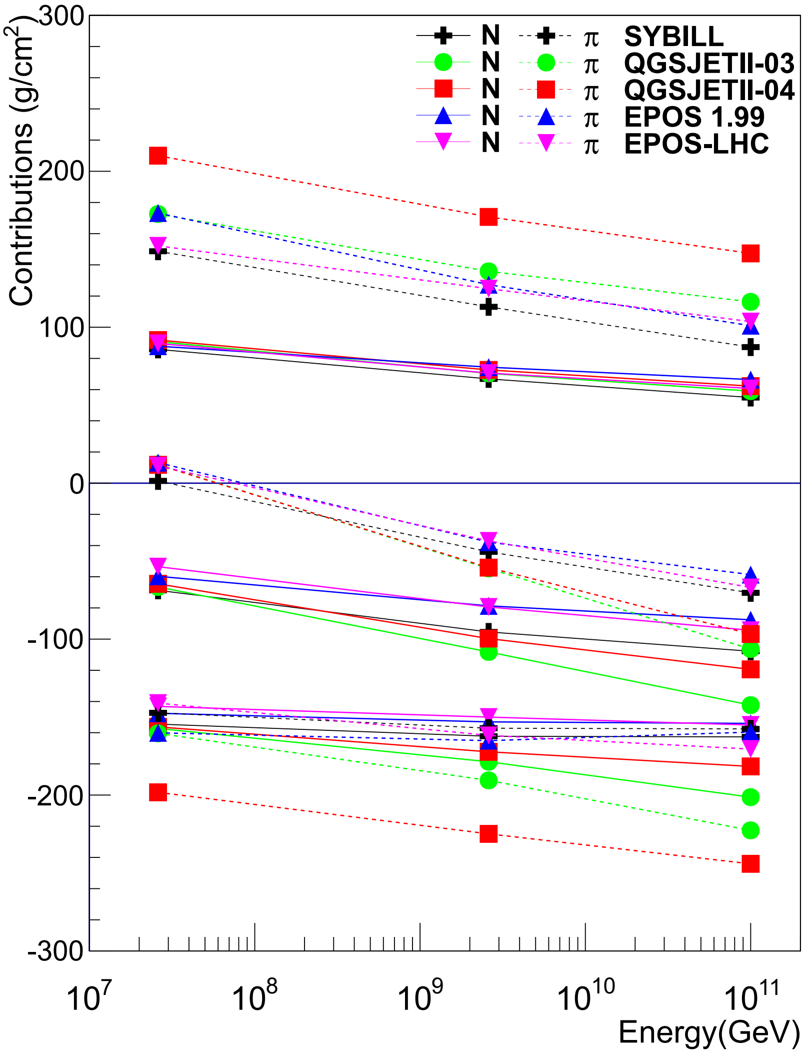

Figure 6. Energy dependence of two types of contributions. Full lines are for nucleon and dotted lines for pion contributions. Eight upper lines are ΔX_λ contributions, eight bottom lines are ΔX_μ contributions, eight middle lines are the sums $\Delta X_\mu + \Delta X_\lambda$.

QGSJETII-04 and EPOS LHC. The forward multiplicity is less dependent on energy, the spread between generators is also smaller for the forward multiplicity.

Dependence on energy of the integrals of the two above kinds for five generators is presented in the next two plots.

Figure 3 compares the dependence on energy of the integrals defining energy transition $g_{ij} = \int_0^1 x \frac{dn_{i\to j}}{dx}(x)\,dx$ for proton and pion interactions with air. One can see a big spread between different generators which is present even at LHC energies.

Figure 4 compares the dependence on energy of the integrals defining energy dissipation. $\mu_i = \int_0^1 x \cdot \ln x \frac{dn_{i\to X}}{dx}(x)\,dx$. The behavior for pion and proton (nucleon) is very similar. The spread between generators significantly increases with energy.

Figure 5 compares the dependence on energy of the shower maximum and the center of gravity.

The disposition of generators relative to one another is the same for X_{max} and CG. However the spread for CG is significantly larger. The reason for that is that CG represents the whole shower while X_{max} represents the developing stage of the shower. Accordingly, the number of contributing generations of interactions is larger for CG. Since the effect of variation of interaction characteristics multiplicates with generations the spread in CG exceeds the spread in X_{max}.

Figure 6 shows contributions of four different terms to the hadronic part of CG:

$$\Delta X^N_\lambda = \frac{\lambda_N(E_N^{eff})}{1-g_{NN}}, \qquad \Delta X^N_\mu = \frac{X_0 \cdot \mu_N}{1-g_{NN}}$$

$$\Delta X^\pi_\lambda = \frac{\lambda_\pi(E_\pi^{eff})}{1-g_{NN}}\frac{g_{N\pi}}{1-g_{\pi\pi}}, \qquad \Delta X^\pi_\mu = \frac{X_0 \cdot \mu_\pi}{1-g_{NN}}\frac{g_{N\pi}}{1-g_{\pi\pi}},$$

such that $\Delta X^N_\lambda + \Delta X^N_\mu + \Delta X^\pi_\lambda + \Delta X^\pi_\mu = \Delta X_{had}$.

The spread of the pion contributions is greater than the spread of the nucleon contributions for both λ and μ terms. The elongation rate of the summed pion contribution is greater than the elongation rate of the summed proton contributions. The spread of the summed contributions significantly increases with energy, i.e. the elongation rate of CG is different for different generators.

4. Summary

Cascade equations for the proton shower center of gravity were solved under the assumptions of Feinman scaling, cascading only two types of hadrons, baryons (nucleons) and pions, neglecting pion decay and logarithmically decreasing proton and pion interaction lengths. The obtained expression for the proton shower center of gravity explicitely splits into the center of gravity of the purely electromagnetic cascade at the primary proton energy and a modification of that by hadronic cascading. The characteristics of interaction enter the expression through interaction lengths (cross-sections) of protons and pions taken at some effective, lower than primary, energies and integrals over inclusive distributions of two types: integrals defining energy transition between different sorts of hadrons which govern the cascade elongation and integrals defining energy dissipation which define cascade shortening. The latter are the forward multiplicities rather than the total multiplicities which are usually assummed to be responsible for energy dissipation.

Although the relations between interaction characteristics (integrals) and shower properties for X_{max} should be different from those for CG, presumably, more relevant for accounting for energy dissipation should be the forward multiplicities than the total multiplicities for X_{max} as well.

The center of gravity collects contributions from the whole shower, in particular essential contribution provides long tail, whereas for X_{max} matters the developing stage of shower. Accordingly, the number of contributing generations of particle production is larger for CG. Since the effect of the variation of interaction characteristics multiplicates with generations the difference between predictions of CG while using generators with different hadronic interaction models proves be larger than the difference between predictions of X_{max}.

Since the main contribution to the tail of the shower comes from pions CG exagerates the effect of the difference in characteristics of interactions of pions relative to X_{max}. On the other hand contribution of pions to the CG elongation rate is larger than that of nucleons. It could be that for X_{max} as well as the importance of the correct description of the pion interactions is not much less than of the proton interactions and tuning of the pion interactions deserves more attention than at present when the main attention in tuning is paid to the proton interactions.

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