Energetic properties of stellar pulsations across the Hertzsprung-Russell diagram

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Abstract. I will review types of modes which are observed in stars with different masses and on various evolutionary stages. In particular, I will focus on energetic aspects of oscillation modes, i.e., excitation, distribution of kinetic energy, mode amplitude and lifetime.

1 Introduction

The properties of stellar pulsations are determined by the internal structure of a star and excitation mechanism. In principal, two processes can cause excitation of stellar oscillation: thermal processes and dynamical processes. In the first case, the pulsations are self-excited and it includes the opacity and convective blocking mechanisms. In the second case, we deal with forced oscillations which include stochastic excitation by turbulent convection and excitation by tidal forces in binary systems. Because the topic is very broad I will only mention briefly basic properties of different types of stellar pulsations. Also, I will not discuss many important issues, e.g., the effects of magnetic field or mass loss on pulsations.

2 Pulsations driven by thermal processes

Considering the self-excitation mechanism, one has to calculate the work integral, \( W \), which measures the net energy gathered by an oscillation mode during one cycle. This quantity tells us whether excitation can overcome damping in order to get unstable pulsational mode. In general, \( W \) contains a perturbation of the rate of nuclear energy production, \( \varepsilon \), and a perturbation of the energy flux, \( \mathcal{F} \), which on the whole can be due to radiation and/or convection. Usually perturbations of \( \varepsilon \) are irrelevant and the opacity (\( \kappa \)) mechanism operates. This mechanism drives many types of pulsation across the HR diagram: classical Cepheids, RR Lyrae stars, B-type main sequence stars, \( \delta \) Scuti stars, roAp stars, white dwarf pulsators, hot subdwarfs.

Quite often the convective flux freezing approximation is applied in which one assumes that perturbation of the convective flux is zero. This is the case, e.g., for main sequence B-type pulsators, i.e., \( \beta \) Cephei and Slowly Pulsating B-type (SPB) stars, as well as for hotter \( \delta \) Scuti stars.

Here, I give examples of mode excitation for the three main sequence models with three different masses. In Fig. 1, with the red line (the right Y Axis), I show the differential work integral, \( d \log W / d \log T \), for the two modes excited in the \( \beta \) Cep model with the mass of \( 10 \, M_\odot \), \( \log T_{\text{eff}} = 4.370 \) and \( \log L/L_\odot = 3.93 \). The OPAL opacities were used for the initial hydrogen abundance of \( X_0 = 0.7 \), metallicity of \( Z = 0.015 \) and the element mixture of [1], hereafter AGSS09. In the left panel there is shown the \( p_{15} \) mode with the harmonic degree \( \ell = 1 \) and the frequency \( \nu = 7.276 \, \text{c/d} \) and in the right panel, the \( g_{15} \) mode with \( \ell = 6 \) and \( \nu = 1.948 \, \text{c/d} \). For both modes the positive contribution to the work integral, \( W \), comes from the \( Z \)-bump (log \( T \approx 5.3 \)). Both modes are globally excited because their normalized instability parameter, \( \eta = W / \int_0^R |dW/dr| \, dr \), is greater than zero, but the first one is almost

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Fig. 1. Properties of selected modes in the $10 M_\odot$ model with log $T_{\text{eff}} = 4.370$ and log $L/L_\odot = 3.93$, computed with the OPAL opacities for the initial hydrogen abundance of $X_0 = 0.7$, metallicity of $Z = 0.015$ and the AGSS09 element mixture. The values of $\eta$ and $E_{k,g}/E_k$ are given in the lower right corner of the the panels.

Fig. 2. The same as in Fig. 1 but for the $5 M_\odot$ model with log $T_{\text{eff}} = 4.221$ and log $L/L_\odot = 2.81$.

pure acoustic, with the ratio of the kinetic energy in the gravity propagation zone to the total kinetic energy, $E_{k,g}/E_k$, of about 0.01 whereas the $\ell = 6$, $g_{15}$ mode is almost pure gravity with $E_{k,g}/E_k = 0.99$.

With the black lines, the pressure eigenfunction, $\delta P/P$, is depicted (the left Y Axis). This function should be large and slowly varying in the driving zone in order to globally excite a given mode. The other condition for mode excitation concerns the thermal time scale which should be comparable or longer than the pulsational period in this zone [2].

The same quantities, but for modes of the model with $M = 5 M_\odot$, log $T_{\text{eff}} = 4.221$ and log $L/L_\odot = 2.81$, are presented in Fig. 2. The properties of the mode $\ell = 1$, $g_{12}$ and $\nu = 0.873$ c/d are depicted in the left panel and those of the mode $\ell = 2$, $p_1$ and $\nu = 13.030$ c/d in the right panel. The first, gravity mode is excited and the second, pressure one is damped. This is a typical feature of main-sequence models with intermediate masses.
The same as in Fig. 1 but for the two models with $M = 1.8 \, M_\odot$ and $\alpha_{\text{MLT}} = 1.5$. The parameters of the models are: $\log T_{\text{eff}} = 3.904$, $\log L/L_\odot = 1.14$ (the mid-MS model, the left panel) and $\log T_{\text{eff}} = 3.846$, $\log L/L_\odot = 1.18$ (the model near the TAMS, the right panel).

Fig. 4. The instability parameter, $\eta$, as a function of frequency for the dipole mode for models at different ages from ZAMS to TAMS. Three masses were considered: $M = 10 \, M_\odot$ (left), $M = 5 \, M_\odot$ (middle) and $M = 1.8 \, M_\odot$ (right). The OPAL opacities for $X_0 = 0.7$, $Z = 0.02$ and the AGSS09 mixture were used.

The left panel of Fig. 3 shows the properties of the high order radial mode with $\nu = 38.716 \, \text{c/d}$ which is excited in the $M = 1.8 \, M_\odot$ model with $\log T_{\text{eff}} = 3.904$ and $\log L/L_\odot = 1.14$ (the mid-MS model), the computations were done assuming the convective flux freezing approximation with the mixing length parameter of $\alpha_{\text{MLT}} = 1.5$. The excitation take place in the HeII opacity bump ($\log T \approx 4.7$) with a small contribution from the H opacity bump ($\log T \approx 4.1$). In the right panel of Fig. 3, I put a low frequency mode $g_{57}$ with $\ell = 1$ and $\nu = 0.54 \, \text{c/d}$ which is excited in the near-TAMS model with $\log T_{\text{eff}} = 3.846$ and $\log L/L_\odot = 1.18$. It seems that driving of this mode is activated in the H ionization zone. The excitation of low frequency modes in δ Scuti stars will still be discussed at the end of this section.

Fig. 4 shows how the $\eta(\nu)$ dependence changes with the effective temperature (age) for the dipole mode for models with masses of $M = 10 \, M_\odot$ (the left panel), $M = 5 \, M_\odot$ (the middle panel) and $M = 1.8 \, M_\odot$ (the right panel). Again the OPAL opacities were adopted and models from the zero–to terminal age main sequence were considered. For each mass, one can see the two local maxima of $\eta$, which increase with the age (decreasing effective temperature) and are shifted towards the lower frequencies.

It is also worth to mention that for low frequency modes, the $\eta$ parameter is $\ell$–dependent whereas for the high frequency modes it is independent of $\ell$ (not shown). For B-type pulsators, the low fre-
quency maximum of $\eta$ is higher for the higher mode degree, $\ell$, and its maximum is shifted to the higher frequencies. In the case of $\beta$ Cep model and with the OPAL data, the low frequency modes become unstable from $\ell = 5$. With the OP data, this maximum of $\eta$ is higher by about 0.2 and modes become unstable already from $\ell = 2$. In the whole range of frequency I found instability for the model of 10 $M_\odot$ for modes up to $\ell \approx 30$. For the SPB stars, the $\eta(\nu)$ dependence has a pronounced maximum at low frequencies and the high frequency modes never reach instability, neither with the OPAL nor OP data, although for some models $\eta$ is not so far from zero and reaches $\eta = -0.06$. For the 5 $M_\odot$ model the instability extends up to $\ell \approx 17$.

In the case of $\delta$ Scuti pulsators, the high frequency modes are excited and their instability continues up to $\ell \approx 60$. However, for cooler models (close to the TAMS) one can see that for frequencies of the dipole mode around 0.5 c/d the values of $\eta$ are positive (cf. the right panel of Fig. 3). The interesting fact is that recently [3] found, on the basis of the Kepler data, that all $\delta$ Scuti stars have low frequency peaks. However, these few unstable modes cannot account for the whole range of the observed low frequency peaks as well as for the amount of stars in which the peaks were found. Moreover, for the $M = 1.8$ $M_\odot$ model the shift of the maximum of $\eta$ for low frequency modes with the mode degree, $\ell$, is in the same direction (i.e., towards higher frequencies) but these maxima do not increase. So the higher $\ell$ modes cannot help either. [3] suggested that the low frequencies could be explained by the convective blocking mechanism ([4]) if it can be active even in the hottest A-type stars. On the other hand our preliminary results shows that increasing opacities in the $Z$–bump increases the low frequency maximum of $\eta$ very close to $\eta = 0.0$. These results will be published in a separate paper.

It should be mentioned here that convective blocking mechanism, which involves the modulation of the radiative flux in the convective envelope, can account for oscillations of $\gamma$ Doradus stars ([4]).

2.1 $\epsilon$–mechanism

The perturbation of $\epsilon$ depends on the linear combination of the derivatives of $\epsilon$ over temperature ($\epsilon_T$) and density ($\epsilon_\rho$). Because the values of these derivatives are always positive, the perturbation of $\epsilon$ has a positive contribution to the work integral $W$. This is called the $\epsilon$ mechanism and has been firstly suggested by [5] for very massive stars. Because of small pulsation amplitudes at such high temperatures this mechanism is usually inefficient and pulsational modes cannot be globally excited in such a way. However, there are some theoretical predictions that in some stars the $\epsilon$ mechanism can be effective. This mechanism have been suggested for pulsations in the Population III stars ([6]), M dwarf stars ([7]), DAO white dwarfs originating from post-extreme horizontal branch stars ([8]) and Wolf-Rayet stars ([9]). By this mechanism, [10] have tried also to explain the multi-periodic variability of Rigel.

2.2 Effects of convection on pulsational instability

Convection itself is one of the most difficult phenomena in astrophysics to understand and describe. Therefore, the pulsation-convection interaction is one of the most unsolved problem in stellar pulsation theory. This interaction is especially important for cooler pulsating variables, e.g., classical Cepheids, RR Lyrae, Mira and $\gamma$ Dor stars.

Due to an increase of opacities and/or reduction of the adiabatic gradient, in each opacity bump some convection occurs. In the case of the B-type main sequence pulsators this convection does not carry the energy and can have only a mixing effect. Therefore, for these pulsators the convection-pulsation interaction does not play any role. In the cooler models as for some $\delta$ Scuti stars or ZZ Ceti white dwarfs, the transport by convection in these layers can be effective and the pulsation-convection coupling should be taken appropriately into account.

Time dependent convection treatment has been applied, e.g., to the delta Sct models by [11] who determined the red edge of the instability strip for these variables. In the case of DAV white dwarfs the task is made easier by the fact that the convective turnover timescale is about 2 to 3 orders of magnitude smaller than the pulsation periods (e.g., [12]). Thus, it is possible to make nonadiabatic calculations under the assumption that the outer convection zone reacts instantaneously to the perturbations.
2.3 Effects of rotation on pulsational instability

The effects of rotation on pulsational instability and mode properties have been studied by many authors and it is impossible to name all of them here. In rotating stars the instability parameter $\eta$ depends not only on frequency and the shape of eigenfunction but also the rotation rate. These effects are particularly important in two cases: when the angular rotational velocity reaches the half of its critical value and when the pulsational frequencies are of the order of the rotational frequency. In the first case, one has to include the effects of the centrifugal force and it is mostly important for the higher order p-modes (e.g., [13], [14]). The second case needs to take into account the effect of the Coriolis force which cannot be neglected for the high order g-modes (e.g., [15], [16], [17]).

Also in the presence of rotation completely new type of modes can be generated (e.g., [18])

3 Pulsations driven by dynamical processes

3.1 Solar-like oscillations

Solar-like oscillations are driven by a motion of a huge number of convective bubbles (turbulent convection) and thus their character is stochastic. Therefore, the observed amplitude of a pulsational mode changes with time. Having oscillation spectrum from enough long observing time, the height of a single peak can be expressed by the damping rate and the energy supply rate (e.g., [19], [20]). From asteroseismic point of view, the CoRoT and Kepler mission have been by far most successful for the solar-like pulsators. Thanks to these extensive data it was possible to determine, with a reasonable accuracy, many scaling relations, e.g., for the global maximum amplitude, $A_{\text{max}}$ and its frequency, $\nu_{\text{max}}$, the damping rate, $\eta$, and the mode linewidth, $\Gamma$ (e.g., [21], [22], [23], [24]). The most recent and comprehensive review of solar-like oscillation can be found in [25].

3.2 Tidal effects in binary systems

Considering tidal forces in binary system one can think about two effects: perturbation of free oscillations and tidally induced oscillations.

The influence of tidal forces on pulsation of some some $\delta$ Sct and $\beta$ Cep stars were already mentioned by [26]. There are for example many objects in which ellipsoidal variations have been detected, e.g., XX Pys ([27], [28]), DG Leo ([29]).

Tidally forced oscillations were first predicted by [30] and then studied by many authors. For example, stability of tidally forced oscillations in resonance with high order quadruple g-modes of the 5 $M_\odot$ model was studied by [31]. The idea is that if the resonance between eigenfrequencies of free oscillations and the orbital frequency occurs then the mode may be resonantly excited by the companion. A suitable resonance depends on the mode properties, the eccentricity of the orbit, the component masses and radii. This tidal excitation may become particularly effective for g-mode pulsations, because their frequencies are of the order of the orbital frequencies.

The examples of stars in which tidally induced pulsations were found are: $\delta$ Sct/γ Dor star HD209295 ([32]), the SPB star HD177863 ([33], [34]. There are also such pulsations detected from the CoRoT and/or Kepler photometry. For example, in the SPB star by [35] and in several $\delta$ Sct stars by [36] and [37].

The very interesting recent finding by [38] is that g-mode pulsations in some sdB star are most probably tidally excited by a planetary companion.

4 Summary and conclusions

The diversity of stellar pulsations originates from different internal structure and excitation mechanism. Moreover, some other effects, like rotation convection, tidal forces, can strongly modify properties of stellar oscillations. Therefore, construction of seismic models is a very complicated task and should take into account all effects which can matter and affect mode properties.
The analysis of the unprecedented amount of data, in particular from space, brings us definitely closer to a better understanding and leads to new solutions. On the other hand, interpretation of such data poses new problems and challenges. Nevertheless, many recent results show that it is right time for precision asteroseismology.

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References