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Influence of Deterministic Attenuation and Amplification of Optical Signals on Entanglement and Distillation of Gaussian and Non-Gaussian Quantum States

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Abstract. We study general rules of entanglement dynamics for two-mode continuous variable states subjected to deterministic phase-insensitive attenuation and amplification. Since those processes inevitably involve quantum noises, we propose and solve the problem of finding noise levels destroying entanglement of Gaussian and non-Gaussian states. The amplification with power gain greater or equal than 2 necessarily leads to the total loss of entanglement. We consider non-Gaussian states whose entanglement is robust to arbitrary signal attenuation and amplification provided the noise is restricted by some value close to the quantum limited operation. We calculate the lower bound on distillation rate of Bell states.

Keywords: attenuator, amplifier, quantum entanglement, distillation, non-Gaussian states.

The processes of deterministic attenuation and amplification of light modes in quantum optics must preserve canonical commutation relations for creation and annihilation photon operators, hence, those processes cannot be noiseless [1]. The resulting dynamics is Markovian and can be described by a Gaussian quantum channel [2]. A single mode transformation is easily described in terms of the characteristic function $\varphi(\mathbf{z}) = Tr\,\rho W(\mathbf{z})$, where $W(\mathbf{z}) = \exp[i(qx+py)]$ is the Weyl operator and operators q, p satisfy the relation $[q,p]=i$. The single-mode attenuation and amplification are given by the formula $\varphi_{\text{out}}(\mathbf{z}) = \varphi_{\text{in}}(\mathbf{Kz})\exp(-\frac{1}{2}\mathbf{z}^{\mathrm{T}}\mathbf{Mz})$, where the 2×2 matrices $\mathbf{K}=\sqrt{\kappa}\mathbf{I}$ and $\mathbf{M}=\mu\mathbf{I}$ are proportional to the 2×2 unity matrix $\mathbf{I}$. Parameter μ corresponds to the noise, κ is the power gain: the signal is attenuated if $\kappa<1$, extra classical noise is added if $\kappa=1$, and the signal is amplified if $\kappa>1$. The transformation above is a valid physical process, i.e. a completely positive trace preserving map, if $\mu \ge \frac{1}{2}|\kappa-1|$. Single-mode channels with parameters κ and μ will be denoted by $\Phi(\kappa,\mu)$. We find Kraus operators of the channel $\Phi(\kappa,\mu)$ in the representation of coherent states and use them to prove the following results [3,4]:

1. An N-mode channel $\Phi(\kappa,\mu)^{\otimes N}$, N = 2, 3, ... totally destroys entanglement of any Gaussian input if and only if $\mu \ge \frac{1}{2}$.

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2. The two-mode channel $\Phi(\kappa_1,\mu_1)\otimes\Phi(\kappa_2,\mu_2)$ totally destroys entanglement of any Gaussian input if and only if $\kappa_1\mu_2+\kappa_2\mu_1 \geq (\kappa_1+\kappa_2)/2$.

3. The two-mode channel $\Phi(\kappa,\mu)\otimes\Phi(\kappa,\mu)$ preserves entanglement of the non-Gaussian state $|\psi\rangle \propto |\gamma\rangle|0\rangle - |0\rangle|\gamma\rangle$ if $\mu \leq \frac{1}{2}\sqrt{\kappa^2+1}$.

The obtained results are illustrated in Fig. 1. A practical conclusion is that the entanglement of high energy Gaussian states is robust to very asymmetric attenuations, whereas that of specially prepared non-Gaussian states is at an advantage in the case of symmetric attenuation and general amplification.

Since maximally entangled pairs of qubits (Bell states) are of particular importance in quantum information applications, we further investigate the problem of distillable entanglement. Using the hashing inequality [5], the lower bound on entanglement distillation is derived for Gaussian and non-Gaussian input states.

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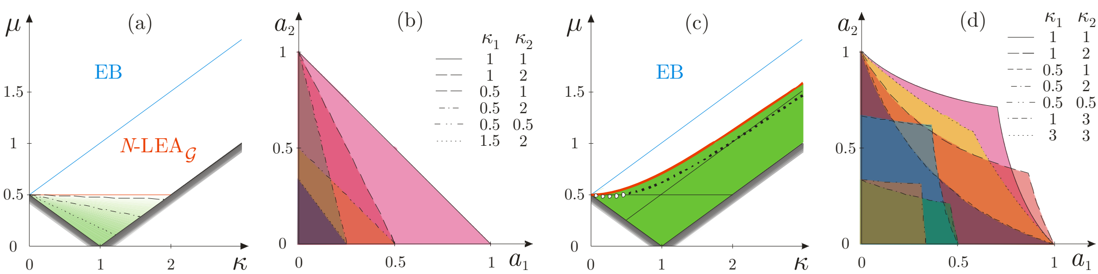

Figure 1. Entanlgement robustness of Gaussian (a, b) and non-Gaussian states (c, d). Power gain κ and total noise μ correspond to a physical map Φ if $\mu \geq \frac{1}{2}|\kappa-1|$ (quantum limited operation). Entanglement of Gaussian states may survive noises $\mu \leq \frac{1}{2}$ only (a), whereas non-Gaussian states of the form $|\psi\rangle \propto |\gamma\rangle|0\rangle - |0\rangle|\gamma\rangle$ exhibit more robust entanglement (c). Regions of extra noises $a_i = \mu_i - \frac{1}{2}|\kappa_i - 1|$ for a channel $\Phi(\kappa_1,\mu_1)\otimes\Phi(\kappa_2,\mu_2)$ preserving entanglement (b, d).

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