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Fast Rotating Spiral Light Beams

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Abstract. Structurally stable rotating light fields (spiral beams) are investigated for various values of the rotation velocity. We prove that, in general, the fields may be described in terms of quasi-analytic functions. Some examples of spiral beams are presented.

Key words: paraxial rotating light fields, phase singularities.

It is known [1] that, in paraxial approximation, the evolution of a 2D coherent light field $F(x,y,\ell)$ propagating in free space along the ℓ axis is described by the equation

$$\frac{\partial^2 F}{\partial x^2}+\frac{\partial^2 F}{\partial y^2}+2ik\frac{\partial F}{\partial \ell}=0\,. \tag{1}$$

Here k is the wave number, ℓ is the propagation distance, and (x, y) are transverse coordinates.

A solution $F(x,y,\ell)$ whose transverse intensity distribution remains unchanged, up to scale, during propagation, is named a structurally stable beam. Well-known examples of such beams are Hermite-Gaussian and Laguerre-Gaussian beams.

In [2-4] the theory of coherent light beams keeping the shape of the transverse intensity, up to scale and rotation, during propagation in the Fresnel zone has been constructed. The intensity shape conservation condition is

$$\left|F(x,y,\ell)\right|^2=D(\ell)\left|F_0\left(\frac{x\cos\theta(\ell)-y\sin\theta(\ell)}{d(\ell)},\frac{x\sin\theta(\ell)+y\cos\theta(\ell)}{d(\ell)}\right)\right|^2, \tag{2}$$

where $\theta(\ell)$ is the dependence of the rotation angle of the intensity distribution during propagation of the field $F(x,y,\ell)$ and $d(\ell)>0$ is the scaling intensity variation.

It has been proven [2] that $D(\ell)=1/d^2(\ell)$ and the rotation and scaling functions may be written as follows:

$$\theta(\ell)=\theta_0\arg\sigma,\quad d(\ell)=\left|\sigma\right|, \tag{3}$$

where $\sigma=1+2i\ell/kw_0^2$ is an auxiliary complex parameter and w_0 is an arbitrary positive number (the Gaussian width of the beam). The value of θ_0 determines the beam rotation velocity during propagation and may be an arbitrary real number. The total field rotation angle during propagation is

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$$\theta(+\infty) - \theta(0) = \theta_0 \, \pi/2 \, . \tag{4}$$

If to fix some point (x_0 ,y_0) in the initial plane $\ell = 0$, then this point traces a spiral path $x + iy = (x_0 + iy_0) d(\ell) \exp\left(-i\theta(\ell)\right)$ during the field $F(x, y, \ell)$ propagation. For large values of θ_0 the point (x_0 ,y_0) executes $\frac{1}{4}\theta_0$ rotations around the ℓ axis in the clockwise or counterclockwise direction, depending on the sign of θ_0. Due to this reason, the fields $F(x, y, \ell)$ have been named spiral light beams.

In [3] the case $\theta_0 = -1$ and corresponding spiral beams

$$F(x, y, \ell) = \frac{1}{\sigma} \exp\left(-\frac{x^2 + y^2}{w_0^2 \sigma} \right) f\left(\frac{x + iy}{w_0 \sigma} \right), \tag{5}$$

where $f(z)$ is an arbitrary entire analytic function with the order of growth $\rho_f < 2$, have been investigated. In particular, it has been shown that for any planar curve $\zeta(t)$, $t \in [0, T]$ there are spiral beams whose transverse intensities are visually similar to the curve. For example, a circle $x + iy = \exp(it)$, $t \in [0, 2\pi]$, corresponds to the beams

$$F(x, y, \ell) = \frac{1}{\sigma} \exp\left(-\frac{x^2 + y^2}{w_0^2 \sigma} \right) \left(\frac{x + iy}{w_0 \sigma} \right)^n, \quad n = 1, 2, \ldots \tag{6}$$

Investigation of spiral beams for large values of $|\theta_0|$ is quite difficult and leads to cumbersome formulae. If $|\theta_0|$ is a negative integer and $\theta_0 \neq -1$, then, for the simplest case, the initial spiral beam complex amplitude is

$$F(x, y, 0) = \exp\left(z\bar{z}\right) \iint_{R^2} \exp\left(i\sqrt{2}[z\bar{w} + \bar{z}w] - w\bar{w} \right) f\left(w^n \bar{w}^m \right) ds dt \, , \tag{7}$$

where $z = (x + iy)/w_0$, $w = s + it$ are complex variables, $f(z)$ is an arbitrary entire analytic function such that $F(x, y, 0) \in L_2(\mathrm{R}^2)$, and integers n, m are defined by the rotation parameter value θ_0 only. For example, if $\theta_0 = -2$, then $(n, m) = (3,1)$; if $\theta_0 = -3$, then $(n, m) = (2,1)$; and if $\theta_0 = -8$, then $(n, m) = (9, 7)$.

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