Quasiparticle-phonon model and quadrupole mixed-symmetry states of $^{96}\text{Ru}$

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Abstract. The structure of low-lying quadrupole states of $^{96}\text{Ru}$ was calculated within the Quasiparticle-Phonon Model. It is shown that symmetric and mixed-symmetry properties manifest themselves via the structure of the excited states. The first $2^+$ state is collective and neutron and proton transition matrix elements $M_n$ and $M_p$ are in-phase, while the neutron and proton transition matrix elements $M_n$ and $M_p$ have opposite signs for the third $2^+$ state. This property of the third $2^+$ state leads to a large $M1$ transition between the first and third $2^+$ states. It is an unambiguous demonstration of the mixed-symmetry nature of the third $2^+$ state. The structure of the first $1^+$ state is calculated. The state is a member of the two-phonon multiplet generated by the coupling of the $[2^+]_{\text{QRPA}}$ and the $[2^+]_{\text{QRPA}}$ states.

1 Introduction

Recent experiments have discovered quite complex structures of low-lying states of nearly spherical nuclei. It originates from proton-neutron oscillations and generates mixed-symmetry states ($MSS$). This phenomenon was predicted within the proton-neutron version of the interacting boson model ($IBM-2$) [1]. Many years after the predictions unambiguous and comprehensive evidence of mixed-symmetry states based on absolute $M1$ transition rates was found in experiments on $^{94}\text{Mo}$ [2, 3]. A review of the experimental aspects on mixed-symmetry states in vibrational and weakly deformed transitional nuclei is given in [4]. A detailed explanation of the structure of mixed symmetry states was given within a microscopic approach based on the Quasiparticle-Phonon Model ($QPM$) [5]. The model treats the excitation as superposition of multiphonon states and was successfully applied in the domains around semi-magic numbers $N = 50$ and $N = 82$. These two domains reveal different behavior of mixed-symmetry excitations. The mixed-symmetry properties are concentrated predominantly in a single state [6, 7] of $^{94}\text{Mo}$ while around $N = 82$ there is fragmentation of mixed-symmetry strength [8–10].

The existence of the quadrupole mixed-symmetry state of $^{96}\text{Ru}$ has been announced in Ref.[11]. Its identification is based on the large $M1$ transition rate between the first and third $2^+$ states. Later on two-phonon mixed-symmetry states, $3^+_{1,\text{mix}}$ and $2^+_{2,\text{mix}}$, of $^{96}\text{Ru}$ have been proposed [12].

Recently, new experimental information concerning mixed symmetry states of $N = 52$ isotones has been published. The mixed-symmetry excitations of $^{96}\text{Ru}$ have been studied via inelastic-proton-scattering [13, 14].

Calculations of the structure of quadrupole mixed-symmetry states of $^{96}\text{Ru}$ within the $QPM$ are presented in this paper.

2 The model

Following [15] the main building blocks of the $QPM$ are $QRPA$ phonons. The phonon operator reads:

$$Q_{\lambda\mu} = \frac{1}{2} \sum_{jj'} \psi_{jj'}^{\mu} \left[ \alpha_j^* \alpha_{j'} \right]_{\lambda\mu} - \left[ -1 \right]^{\lambda \mu} \psi_{jj'}^{\mu} \left[ \alpha_j \alpha_{j'} \right]_{\lambda \mu},$$

(1)

where $j\mu$ denote a single-particle level of the average field for neutrons (or protons) and the notation $[\cdots]_{\lambda\mu}$ means coupling to the total angular momentum $\lambda$ with projection $\mu$: $[\alpha_j^* \alpha_{j'}^*]_{\lambda\mu} = \sum_{nm} c_{\alpha_j \alpha_{j'} \lambda \mu} \psi_{n\mu}^{\alpha_j \alpha_{j'}}$, the quantity $c_{\alpha_j \alpha_{j'} \lambda \mu}$ is the Clebsch-Gordon coefficient. Quasiparticles themselves are the result of the Bogoliubov transformation. In the $QPM$, quasiparticle energies and Bogoliubov’s coefficients $u_j$ and $v_j$ are obtained by solving the BCS equations. A phonon basis is constructed by diagonalizing the $QPM$ Hamiltonian on the set of one-phonon states [15]. The procedure yields the $QRPA$ equations, and solving these equations one obtains the phonon spectrum and the internal phonon structure, i.e., the coefficients $\psi_{jj'}^{\mu}$ and $\phi_{jj'}^{\mu}$ of Eq. (1) for any multipolarity $\lambda$ under consideration. The index $i$ in the definition of the phonon operator (1) gets the meaning of the $QRPA$ root number. The phonons are of different degree of collectivity, from collective ones (e.g. $[2^+]_{\text{QRPA}}$) to pure two-quasiparticle configurations.
In the case of even-even nuclei the QPM Hamiltonian is diagonalized in a basis of wave functions constructed as a superposition of one-, two-, and three-phonon components [16, 17].

\[ \Psi_{n}(JM) = \left( \sum_{i} R(Jv)Q_{JM}^{\dagger} + \sum_{\lambda_{1},\lambda_{2}} P_{\lambda_{1},\lambda_{2}}(Jv) \left[ Q_{\lambda_{1},\lambda_{2}}^{\dagger} \left( P_{\lambda_{1},\lambda_{2}} \right) \left( Q_{\lambda_{1},\lambda_{2}}^{\dagger} \right) \right]_{JM} + \sum_{\lambda_{1},\lambda_{2},\lambda_{3}} T_{\lambda_{1},\lambda_{2},\lambda_{3}}(Jv) \left[ Q_{\lambda_{1},\lambda_{2},\lambda_{3}}^{\dagger} \left( P_{\lambda_{1},\lambda_{2},\lambda_{3}} \right) \left( Q_{\lambda_{1},\lambda_{2},\lambda_{3}}^{\dagger} \right) \right]_{JM} \right) \Psi_{0}, \]

where \( \Psi_{0} \) represents the phonon vacuum state and \( R, P, T \) are unknown amplitudes. The index \( v \) specifies the particular excited state.

The foregoing formalism was applied to study the low-lying excited states of even-even nuclei having neutron numbers \( N = 80 \) and \( N = 84 \) and the domain around neutron number \( N = 50 \). The results are published in [6, 7, 9, 10].

### 3 Results

In the calculation the parameters of the QPM Hamiltonian are the same as used in [6, 7] for \( ^{94}\text{Mo} \). The corresponding single-particle spectra for the \( A = 90 \) region can be found in [18]. The strengths of the quadrupole-quadrupole and octupole-octupole interactions were fixed by a fit of the lowest \( 2^+ \) and \( 3^+ \) levels of \( ^{96}\text{Mo} \). The strengths of the other multipole terms are adjusted to keep unchanged the energy of the computed two-quasiparticle states [18]. This set of parameters was widely used and gave an overall description of the low-lying as well as the high-lying states of nuclei in this mass region [18].

The structure of the \( 2^+ \) states of \( ^{96}\text{Ru} \) calculated in QPM reveals that the \( [2^+]_{\text{QPM}} \) state is symmetric and the \( [2^+]_{\text{QPM}} \) state shows mixed symmetry. The total contribution of neutrons and protons to the structure of the \( 2^+ \) state is in-phase (the sum of neutron and proton transition matrix elements \( M_{\nu} \) and \( M_{\mu} \) is large and both have positive sign). The contribution of neutrons and protons in the structure of the \( [2^+]_{\text{QPM}} \) state is out-of-phase (the neutron and proton transition matrix elements \( M_{\nu} \) and \( M_{\mu} \) have opposite signs). It means that isoscalar correlations dominate in the first \( [2^+]_{\text{QPM}} \) state, while the structure of the second \( [2^+]_{\text{QPM}} \) state is dominated by isovector correlations. The main two-quasiparticle components contributing to the structure of the low-lying \( [2^+]_{\text{QPM}} \) states are given in Table 1. The first \( [2^+]_{\text{QPM}} \) state is collective. The state is connected with the second \( [2^+]_{\text{QPM}} \) with a large \( M1 \) transition. The \( B(M1, 2^+ \rightarrow 2^+) \) value is \( 1.37 \mu_{\text{nu}}^2 \).

The energies and the structure of the low-lying QPM states are given in Table 2. The first \( 2^+ \) state is dominated by the isoscalar \( [2^+]_{\text{QPM}} \) component and therefore it is a symmetric state. The second \( 2^+ \) state is mainly a two-phonon state dominated by isoscalar phonons and, therefore it is a two-phonon symmetric \( 2^+ \) state. The third \( 2^+ \) state is almost an isovector one-phonon state and therefore it is the one-phonon mixed-symmetry state.

The structure of the first \( 1^+ \) state is mainly a two-phonon mixed-symmetry state. The main component is of two-phonon character, coupling the symmetric \( [2^+]_{\text{QPM}} \) state with the mixed symmetry \( [2^+]_{\text{QPM}} \) state.

The corresponding transition probabilities between the low-lying quadrupole states of \( ^{96}\text{Ru} \) are given in Table 3. The comparison with IBM results is presented in Table 4.
The agreement with experimental data is quite well. The phonon mixed-symmetry nature of the first $1^+$ state appears to be quite large and confirms the two-ground state is less than the experimental value but nevertheless remains in good agreement with the data. Its relatively large strength distribution confirms the suggestion that the structure of the $2^+_2$ state is dominated by the two-phonon ($Q_{\text{QRPA}} \otimes Q_{\text{QRPA}}$) component. The state is a member of the two-phonon multiplet generated by symmetric and mixed-symmetry quadrupole $Q_{\text{QRPA}}$ states. The corresponding $M1$ transition connecting the $1^+$ state with the ground state is less than the experimental value but nevertheless appears to be quite large and confirms the two-phonon mixed-symmetry nature of the first $1^+$ state qualitatively.

The experimental data as well as the IBM results are taken from [13, 14].

It is seen from Table 3 that the QPM reproduces the main features of the low-lying quadrupole states in $^{96}$Ru. The agreement with experimental data is quite well. The $M1$ transition strength between the $2^+_1$ and the $2^+_2$ state is in good agreement with the data. Its relatively large value confirms the suggestion that the structure of the $2^+_1$ state corresponds to a mixed-symmetric state. The structure of the first $1^+$ state is dominated by the two-phonon ($Q_{\text{QRPA}} \otimes Q_{\text{QRPA}}$) component. The state is a member of the two-phonon multiplet generated by symmetric and mixed-symmetry quadrupole $Q_{\text{QRPA}}$ states. The corresponding $M1$ transition connecting the $1^+$ state with the ground state is less than the experimental value but nevertheless appears to be quite large and confirms the two-phonon mixed-symmetry nature of the first $1^+$ state qualitatively.

It is interesting to compare the structure of low-lying quadrupole states of $^{94}$Mo with those of $^{96}$Ru. The structure of the states of $^{94}$Mo is published in [6, 7] and is shown in Table 5. The contribution of the main components in the structure of the lowest $2^+$ states is very similar to those of $^{96}$Ru. The calculated values of $B(M1; 2^+_1 \to 2^+_2)$ for both the nuclei are shown in Fig. 1. The behaviour of both distributions is very similar. The $M1$ strength is concentrated predominantly in a single state. In both nuclei the $2^+_2$ state is the quadrupole one-phonon mixed-symmetry state. In $^{96}$Ru its excitation is higher than in $^{94}$Mo. This trend is also well reproduced in the QPM calculations.

### Table 5. Structure of the low-lying quadrupole QPM states of $^{94}$Mo. Values of the largest components are given.

<table>
<thead>
<tr>
<th>State $J^+$</th>
<th>E (MeV)</th>
<th>Structure,%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1^+$</td>
<td>0.871</td>
<td>93% $[2^+<em>1]</em>{\text{QRPA}}$</td>
</tr>
<tr>
<td>$2^+_1$</td>
<td>1.864</td>
<td>82% $([2^+<em>1]</em>{\text{QRPA}} \otimes [2^+<em>1]</em>{\text{QRPA}})$</td>
</tr>
<tr>
<td>$2^+_2$</td>
<td>2.067</td>
<td>95% $[2^+<em>2]</em>{\text{QRPA}}$</td>
</tr>
<tr>
<td>$1^+_3$</td>
<td>3.129</td>
<td>90% $([2^+<em>1]</em>{\text{QRPA}} \otimes [1^+<em>3]</em>{\text{QRPA}})$</td>
</tr>
</tbody>
</table>

The presented results on the structure of low-lying quadrupole states within the QPM confirm the complicated nature of low-lying quadrupole states of $^{96}$Ru. The spectrum consists of symmetric as well as mixed symmetry states. The regularities of $E2$ and $M1$ transitions correspond to the idea about the existing of both symmetries in the low-energy sector of excited states.

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### References


