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Quasiparticle-phonon model and quadrupole mixed-symmetry states of ^{96}Ru

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Abstract. The structure of low-lying quadrupole states of ^{96}Ru was calculated within the Quasiparticle-Phonon Model. It is shown that symmetric and mixed-symmetry properties manifest themselves via the structure of the excited states. The first 2^+ state is collective and neutron and proton transition matrix elements M_n and M_p are in-phase, while the neutron and proton transition matrix elements M_n and M_p have opposite signs for the third 2^+ state. This property of the third 2^+ state leads to a large $M1$ transition between the first and third 2^+ states. It is an unambigous demonstration of the mixed-symmetry nature of the third 2^+ state. The structure of the first 1^+ state is calculated. The state is a member of the two-phonon multiplet generated by the coupling of the $[2_1^+]_{QRPA}$ and the $[2_2^+]_{QRPA}$ states.

1 Introduction

Recent experiments have discovered quite complex structures of low-lying states of nearly spherical nuclei. It originates from proton-neutron oscillations and generates mixed-symmetry states (MSS). This phenomenon was predicted within the proton-neutron version of the interacting boson model ($IBM-2$) [1]. Many years after the predictions unambiguous and comprehensive evidence of mixed-symmetry states based on absolute $M1$ transition rates was found in experiments on ^{94}Mo [2, 3]. A review of the experimental aspects on mixed-symmetry states in vibrational and weakly deformed transitional nuclei is given in [4]. A detailed explanation of the structure of mixed symmetry states was given within a microscopic approach based on the Quasiparticle-Phonon Model (QPM) [5]. The model treats the excitation as superposition of multiphonon states and was successfully applied in the domains around semi-magic numbers $N = 50$ and $N = 82$. These two domains reveal different behavior of mixed-symmetry excitations. The mixed-symmetry properties are concentrated predominantly in a single state [6, 7] of ^{94}Mo while around $N = 82$ there is fragmentation of mixed-symmetry strength [8–10].

The existence of the quadrupole mixed-symmetry state of ^{96}Ru has been announced in Ref.[11]. Its identification is based on the large $M1$ transition rate between the first and third 2^+ states. Later on two-phonon mixed-symmetry states, $3^+_{1,ms}$ and $2^+_{2,ms}$, of ^{96}Ru have been proposed [12].

Recently, new experimental information concerning mixed symmetry states of $N = 52$ isotones has been published. The mixed-symmetry excitations of ^{96}Ru have been studied via inelastic proton-scattering [13, 14].

Calculations of the structure of quadrupole mixed-symmetry states of ^{96}Ru within the QPM are presented in this paper.

2 The model

Following [15] the main building blocks of the QPM are $QRPA$ phonons. The phonon operator reads:

$$Q^+_{\lambda\mu i} = \frac{1}{2}\sum_{\tau}^{n,p}\sum_{jj'}\left\{\psi^{\lambda i}_{jj'}[\alpha^+_j\alpha^+_{j'}]_{\lambda\mu} - (-1)^{\lambda-\mu}\varphi^{\lambda i}_{jj'}[\alpha_{j'}\alpha_j]_{\lambda-\mu}\right\}, \tag{1}$$

where jm denote a single-particle level of the average field for neutrons (or protons) and the notation $[\cdots]_{\lambda\mu}$ means coupling to the total angular momentum λ with projection μ; $[\alpha^+_j\alpha^+_{j'}]_{\lambda\mu} = \sum_{mm'} C^{\lambda\mu}_{jmj'm'}\alpha^+_{jm}\alpha^+_{j'm'}$; the quantity $C^{\lambda\mu}_{jmj'm'}$ is the Clebsch-Gordon coefficient. Quasiparticles themselves are the result of the Bogoliubov transformation. In the QPM, quasiparticle energies and Bogoliubov's coefficients u_j and v_j are obtained by solving the BCS equations. A phonon basis is constructed by diagonalizing the QPM Hamiltonian on the set of one-phonon states [15]. The procedure yields the $QRPA$ equations, and solving these equations one obtains the phonon spectrum and the internal phonon structure, i.e., the coefficients $\psi^{\lambda i}_{jj'}$ and $\varphi^{\lambda i}_{jj'}$ of Eq. (1) for any multipolarity λ under consideration. The index i in the definition of the phonon operator (1) gets the meaning of the $QRPA$ root number. The phonons are of different degree of collectivity, from collective ones (e.g. $[2^+_1]_{QRPA}$) to pure two-quasiparticle configurations.

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In the case of even-even nuclei the *QPM* Hamiltonian is diagonalized in a basis of wave functions constructed as a superposition of one-, two-, and three-phonon components [16, 17].

$$\Psi_\nu(JM) = \Bigg\{ \sum_i R_i(J\nu) Q^+_{JMi} + \sum_{\substack{\lambda_1 i_1 \\ \lambda_2 i_2}} P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) \left[Q^+_{\lambda_1\mu_1 i_1} \times Q^+_{\lambda_2\mu_2 i_2} \right]_{JM} + \sum_{\substack{\lambda_1 i_1 \lambda_2 i_2 \\ \lambda_3 i_3 I}} T^{\lambda_1 i_1 \lambda_2 i_2 I}_{\lambda_3 i_3}(J\nu) 2 \left[\left[Q^+_{\lambda_1\mu_1 i_1} \times Q^+_{\lambda_2\mu_2 i_2} \right]_{IK} \times Q^+_{\lambda_3\mu_3 i_3} \right]_{JM} \Bigg\} \Psi_0, \quad (2)$$

where Ψ_0 represents the phonon vacuum state and R, P, and T are unknown amplitudes. The index ν specifies the particular excited state.

The foregoing formalism was applied to study the low-lying excited states of even-even nuclei having neutron numbers $N = 80$ and $N = 84$ and the domain around neutron number $N = 50$. The results are published in [6, 7, 9, 10].

3 Results

In the calculation the parameters of the *QPM* Hamiltonian are the same as used in [6, 7] for ^{94}Mo. The corresponding single-particle spectra for the $A = 90$ region can be found in [18]. The strengths of the quadrupole-quadrupole and octupole-octupole interactions were fixed by a fit of the lowest 2^+_1 and 3^-_1 levels of ^{96}Ru. The strengths of the other multipole terms are adjusted to keep unchanged the energy of the computed two-quasiparticle states [18].This set of parameters was widely used and gave an overall description of the low-lying as well as the high-lying states of nuclei in this mass region [18].

The structure of the 2^+ states of ^{96}Ru calculated in *QRPA* reveals that the $[2^+_1]_{QRPA}$ state is symmetric and the $[2^+_2]_{QRPA}$ state shows mixed symmetry. The total contribution of neutrons and protons to the structure of the 2^+_1 state is in-phase (the sum of neutron and proton transition matrix elements M_n and M_p is large and both have positive sign). The contribution of neutrons and protons in the structure of the $[2^+_2]_{QRPA}$ state is out-of-phase (the neutron and proton transition matrix elements M_n and M_p have opposite signs). It means that isoscalar correlations dominate in the first $[2^+]_{QRPA}$ state, while the structure of the second $[2^+]_{QRPA}$ state is dominated by isovector correlations. The main two-quasiparticle components contributing to the structure of the low-lying $[2^+]_{QRPA}$ states are given in Table 1. The first $[2^+]_{QRPA}$ state is collective. The state is connected with the second $[2^+]_{QRPA}$ with a large $M1$ transition. The $B(M1, 2^+_2 \to 2^+_1)$ value is 1.37 μ^2_N.

The energies and the structure of the low-lying *QPM* states are given in Table 2. The first 2^+ state is dominated by the isoscalar $[2^+_1]_{QRPA}$ component and therefore it is a symmetric state. The second 2^+ state is mainly a two-phonon state dominated by isoscalar phonons and, therefore it is a two-phonon symmetric 2^+ state. The third 2^+ state is almost an isovector one-phonon state and therefore it is the one-phonon mixed-symmetry state.

The structure of the first 1^+ state is mainly a two-phonon mixed-symmetry state. The main component is of two-phonon character, coupling the symmetric $[2^+_1]_{QRPA}$ state with the mixed symmetry $[2^+_2]_{QRPA}$ state.

The corresponding transition probabilities between the low-lying quadrupole states of ^{96}Ru are given in Table 3. The comparison with *IBM* results is presented in Table 4.

Table 1. Contributions of the main components in the structure of the low-lying *QRPA* 2^+ states of ^{96}Ru. The corresponding $E2$ transitions are shown.

E(MeV)	Structure	$B(E2, g.s. \to 2^+)$ (e^2 fm^4)
0.999	$0.8(2d_{5/2})^2_n + 1.1(1g_{9/2})^2_p$	0.25E+04
2.276	$-1.2(2d_{5/2})^2_n + 0.7(1g_{9/2})^2_p$	0.99E+02

Table 2. Structure of the low-lying quadrupole *QPM* states of ^{96}Ru.

State	E (MeV)		Structure,%
J^π	EXP.	*QPM*	
2^+_1	0.832	0.775	$88\%[2^+_1]_{QRPA}$
2^+_2	1.932	1.824	80 % $([2^+_1]_{QRPA} \otimes [2^+_2]_{QRPA})$
2^+_3	2.283	2.164	90 $\%[2^+_2]_{QRPA}$
1^+_1	3.154	3.192	93 % $([2^+_1]_{QRPA} \otimes [2^+_2]_{QRPA})$

Table 3. Results of the *QPM* calculations for ^{96}Ru in comparison to the experimental data [13, 14]. $M1$ strengths are in units of μ^2_N, $E2$ strengths are given in units of *W.u.*

State	Transition		Transition strength	
J^π	$J^\pi_i \to J^\pi_f$	$\sigma\lambda$	$B(\sigma\lambda)_{exp}$	$B(\sigma\lambda)_{QPM}$
2^+_1	$2^+_1 \to 0^+_1$	E2	18.1(5)	16
2^+_2	$2^+_2 \to 2^+_1$	M1	0.05(2)	0.11
	$2^+_2 \to 0^+_1$	E2	....	0.77
	$2^+_2 \to 2^+_1$	E2	28(9)	30
2^+_3	$2^+_3 \to 2^+_1$	M1	0.69(14)	0.63
	$2^+_3 \to 0^+_1$	E2	1.36(19)	0.75
1^+_1	$1^+_1 \to 0^+_1$	M1	0.17(5)	0.13

Table 4. Results of the *QPM* calculations for ^{96}Ru in comparison to *IBM-2* results [13, 14]. $M1$ strengths are in units of μ^2_N, $E2$ strengths are given in units of *W.u.*

State	Transition		Transition strength	
J^π	$J^\pi_i \to J^\pi_f$	$\sigma\lambda$	$B(\sigma\lambda)_{IBM}$	$B(\sigma\lambda)_{QPM}$
2^+_1	$2^+_1 \to 0^+_1$	E2	18.4	16
2^+_2	$2^+_2 \to 2^+_1$	M1	0.0	0.11
	$2^+_2 \to 2^+_1$	E2	24	30
2^+_3	$2^+_3 \to 2^+_1$	M1	0.69	0.63
	$2^+_3 \to 0^+_1$	E2	2.53	0.75
1^+_1	$1^+_1 \to 0^+_1$	M1	0.13	0.13

Table 5. Structure of the low-lying quadrupole *QPM* states of ^{94}Mo. Values of the largest components are given.

State	E (MeV)		Structure,%
J^{π}	EXP.	*QPM*	
2^+_1	0.871	0.860	93 %$[2^+_1]_{QRPA}$
2^+_2	1.864	1.750	82 % $([2^+_1]_{QRPA} \otimes [2^+_2]_{QRPA})$
2^+_3	2.067	1.940	95 %$[2^+_2]_{QRPA}$
1^+_1	3.129	2.880	90 % $([2^+_1]_{QRPA} \otimes [1^+_1]_{QRPA})$

The experimental data as well as the *IBM* results are taken from [13, 14].

It is seen from Table 3 that the *QPM* reproduces the main features of the low-lying quadrupole states in ^{96}Ru. The agreement with experimental data is quite well. The $M1$ transition strength between the 2^+_3 state and the 2^+_1 state is in good agreement with the data. Its relatively large value confirms the suggestion that the structure of the 2^+_3 state corresponds to a mixed-symmetric state. The structure of the first 1^+ state is dominated by the two-phonon $([2^+_1]_{QRPA} \otimes [2^+_2]_{QRPA})$ component. The state is a member of the two-phonon multiplet generated by symmetric and mixed-symmetry quadrupole *QRPA* states. The corresponding $M1$ transition connecting the 1^+ state with the ground state is less than the experimental value but nevertheless appears to be quite large that confirms the two-phonon mixed-symmetry nature of the first 1^+ state qualitatively.

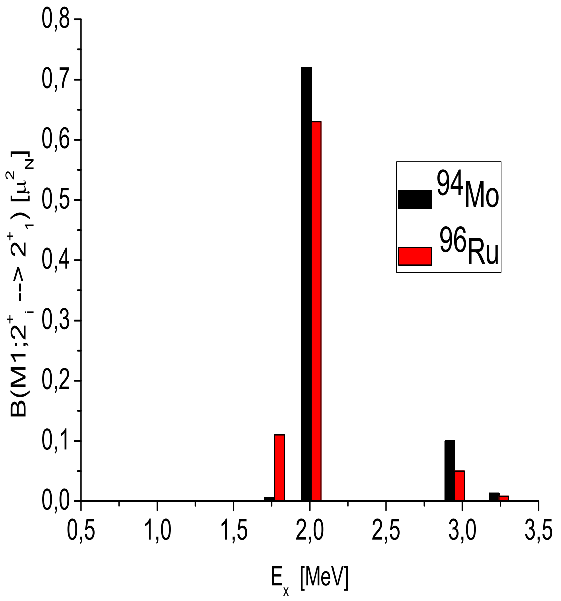

Figure 1. (Color online) Comparison of calculated *QPM* strength distribution $B(M1; 2^+_i \rightarrow 2^+_1)$ for ^{94}Mo and ^{96}Ru.

It is seen from Table 4 that the calculated values within the *QPM* are very similar to those within the *IBM-2*. The results of both the models on the $M1$ transition rate between the 2^+_3 state and the 2^+_1 state are in good agreement. This confirms the mixed-symmetry nature of the 2^+_3 state.

It is interesting to compare the structure of low-lying quadrupole states of ^{94}Mo with those of ^{96}Ru. The structure of the states of ^{94}Mo is published in [6, 7] and is shown in Table 5. The contribution of the main components in the structure of the lowest 2^+ states is very similar to those of ^{96}Ru. The calculated values of $B(M1; 2^+_i \rightarrow 2^+_1)$ for both the nuclei are shown in Fig. 1. The behaviour of both distributions is very similar. The $M1$ strength is concentrated predominantly in a single state. In both nuclei the 2^+_3 state is the quadrupole one-phonon mixed-symmetry state. In ^{96}Ru its excitation is higher than in ^{94}Mo. This trend is also well reproduced in the *QPM* calculations.

4 Conclusion

The presented results on the structure of low-lying quadrupole states within the *QPM* confirm the complicated nature of low-lying quadrupole states of ^{96}Ru. The spectrum consists of symmetric as well as mixed symmetry states. The regularities of $E2$ and $M1$ transitions correspond to the idea about the existing of both symmetries in the low-energy sector of excited states.

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