

Skyrme RPA description of γ -vibrational states in rare-earth nuclei

V.O. Nesterenko^{1,a}, V.G. Kartavenko¹, W. Kleinig^{1,2}, J. Kvasil³, A. Repko³, R.V. Jolos¹, and P.-G. Reinhard⁴

¹Bogoliubov Laboratory of Theoretical Physics, JINR, Dubna, 141980, Russia

²Technische Universität Dresden, Institut für Analysis, D-01062, Dresden, Germany

³Institute of Particle and Nuclear Physics, Charles University, CZ-18000, Praha 8, Czech Republic

⁴Institut für Theoretische Physik II, Universität Erlangen, D-91058, Erlangen, Germany

Abstract. The lowest γ -vibrational states with $K^\pi = 2_\gamma^+$ in well-deformed Dy, Er and Yb isotopes are investigated within the self-consistent separable quasiparticle random-phase-approximation (QRPA) approach based on the Skyrme functional. The energies E_γ and reduced transition probabilities $B(E2)_\gamma$ of the states are calculated with the Skyrme force SV-mas10. We demonstrate the strong effect of the pairing blocking on the energies of γ -vibrational states. It is also shown that collectivity of γ -vibrational states is strictly determined by keeping the Nilsson selection rules in the corresponding lowest 2qp configurations.

1 Introduction

The modern self-consistent mean-field methods have demonstrated a great success in description of multipole giant resonances [1–3] but so far got a modest progress in reproduction of lowest vibrational states in well deformed medium and heavy nuclei. To our knowledge, there are only a few studies of this kind. They are performed in the framework of the quasiparticle random-phase-approximation (QRPA): with Gogny interaction for multipole states in ^{238}U [4] and with Skyrme forces (SkM* [5] and SLy4 [6]) for $K^\pi = 2_\gamma^+$ and 0_β^+ states in rare-earth nuclei [7].

In the framework of this activity, we have recently performed a systematic Skyrme QRPA investigation of $K^\pi = 2_\gamma^+$ states in rare-earth and actinide nuclei [8]. Both early SkM* and recent SV-bas [9] Skyrme parameterizations were used. The main attention was paid to i) the impact of the pairing blocking effect (PBE) on the properties of $K^\pi = 2_\gamma^+$ states and ii) explanation of the nuclear regions with a low and high collectivity of these states. It was shown that PBE strongly affects the energies of 2_γ^+ states but almost does not change their $B(E2)_\gamma$ -values. The domains with a high collectivity were conditioned by keeping the Nilsson selection rules [10] in the lowest 2qp configurations with $K^\pi = 2^+$.

In this paper, we additionally check these results by using the Skyrme force SV-mas10 [9]. This force has a large isoscalar effective mass $m_0/m = 1$ and so should generate the mean field similar to the phenomenological Woods-Saxon one [11]. Numerous early studies of $K^\pi = 2_\gamma^+$ states were performed within schematic QRPA with the Woods-Saxon single-particle (s-p) scheme [12, 13]. In this connection, it would be interesting to see how well the simi-

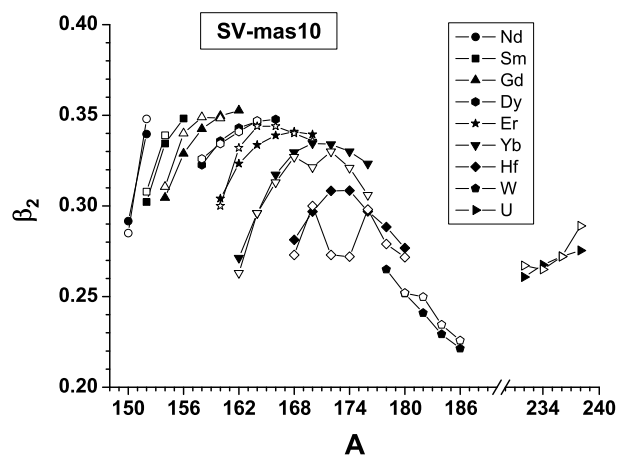


Figure 1. Parameter β_2 of the axial quadrupole deformation in rare-earth and actinide nuclei. The values calculated with the Skyrme force SV-mas10 (full symbols) are compared with the experimental values [14] (open symbols).

lar scheme works within the present self-consistent QRPA procedure.

2 Method

The calculations have been performed within the separable QRPA [15] based on the Skyrme functional [1]. The method is fully self-consistent since i) both the mean field and residual interaction are obtained from the same Skyrme functional, ii) the residual interaction includes all terms of the functional as well as the Coulomb direct and exchange terms. The self-consistent factorization of the residual interaction dramatically reduces the computa-

^ae-mail: nester@theor.jinr.ru

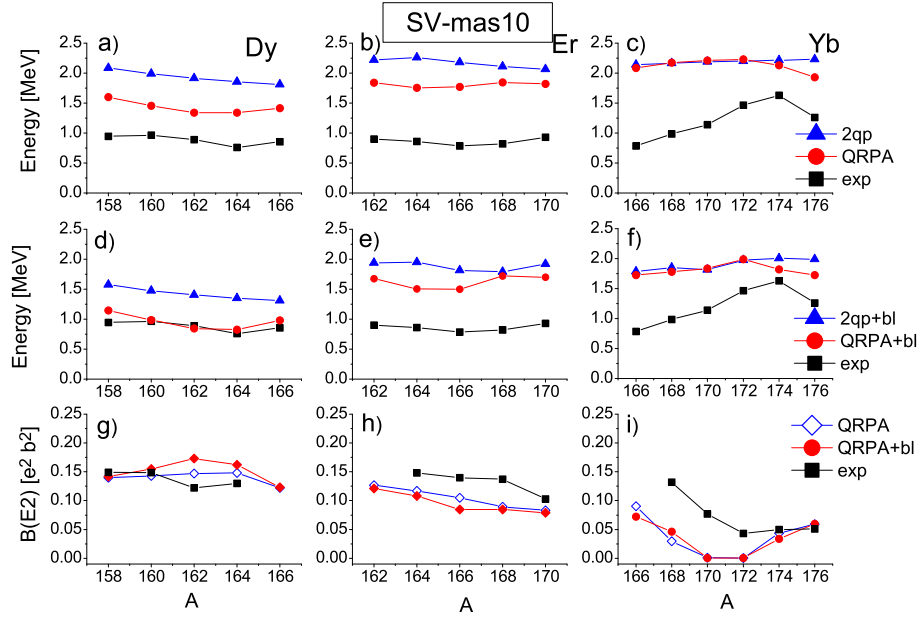


Figure 2. (Color online) The lowest 2qp and QRPA energies as well as $B(E2)$ values of 2^+_{γ} -states in Dy (left), Er (center) and Yb (right) isotopes, calculated with the force SV-mas10. The 2qp (filled blue triangles) and QRPA (filled red circles) energies obtained without (a-c) and with (d-f) PBE are compared with the experimental data (filled black squares)[14]. In the plots (g-i), the QRPA $B(E2)$ values calculated without (empty blue diamonds) and with (filled red diamonds) PBE are given versus the experimental data (filled black squares) [14].

tional effort for deformed nuclei while keeping high accuracy of the method.

The large configuration space is used. The 2qp excitations range up to 55-80 MeV and exhaust 95-98% of the energy-weighted sum rule. The equilibrium deformations are obtained by minimization of the total energy of the system. As seen from Fig. 1, the calculated deformation parameters β_2 are in a good agreement with the experimental data.

The volume δ -force pairing is treated within BCS [16]. The problem is solved separately for the ground state Ψ_0 and excited 2qp states

$$\Psi_{ij} = \hat{\alpha}_i^+ \hat{\alpha}_j^+ \Psi_0 = \hat{a}_i^+ \hat{a}_j^+ \prod_{k \neq i, j} (u_k(ij) + v_k(ij) \hat{a}_k^+ \hat{a}_k^+) \Psi_{\text{HF}}, \quad (1)$$

where \hat{a}_j^+ ($\hat{\alpha}_j^+$) creates the particle (quasiparticle) in the state j , Ψ_{HF} is the Hartree-Fock ground state, and $u_k(ij)$ and $v_k(ij)$ are Bogoliubov coefficients. In axial nuclei, the 2qp configurations with $K^{\pi} = 2^+$ are non-diagonal ($i \neq j$) and so composed by unpaired states. Since the pairing operates solely by nuclear pairs, the unpaired states i and j are blocked for the pairing process and come to the BCS equations as pure single-particle states. This is so called the pairing blocking effect (PBE) [12, 17–19].

Following early schematic QRPA studies [12, 13], the PBE noticeably affects $K^{\pi} = 2^+_{\gamma}$ -energies in even-even axial nuclei. Indeed, 2qp states form the QRPA configuration space. Moreover, first 2qp states are main contributors to the lowest QRPA excitations. So the accurate (with PBE) treatment of the first 2qp states is important for a correct description of the lowest QRPA solutions. Since PBE

mainly affects the 2qp energies [12, 13], one may correct only them and keep unchanged the g.s. set of Bogoliubov coefficients. Being economical, this prescription takes into account the major PBE impact. Besides it leaves the 2qp basis ortho-normalized and allows to avoid the problems with conservation of the proton and neutron numbers.

Of course this approximate scheme makes QRPA calculations somewhat inconsistent. However, since a fully consistent QRPA scheme with blocked pairs is still a demanding and unsolved problem, we prefer to use an approximate scheme rather than neglect the PBE at all.

The PBE formalism for the volume δ -force pairing can be found elsewhere [8, 20]. In the present calculations, we replace first five 2qp energies

$$\epsilon_{ij} = \epsilon_i + \epsilon_j \quad (2)$$

by the PBE-corrected values

$$\mathcal{E}_{\text{bl}}(ij) = \mathcal{E}(ij) - \mathcal{E}_0, \quad (3)$$

where \mathcal{E}_0 and $\mathcal{E}(ij)$ are energies of the system in the ground and 2qp (ij)-states.

3 Results and discussion

In Fig. 2, results of our calculations for Dy, Er and Yb isotopes are exhibited. In the upper (middle) panels, the 2qp energies calculated with (without) the PBE are shown. The QRPA energies of $K^{\pi} = 2^+_{\gamma}$ -states are compared with the experimental data. In the bottom panels, the $B(E2)$ -values obtained in QRPA with and without PBE are presented.

Table 1. Quadrupole 2qp matrix elements $f_{ij}^{22} = \langle ij|r^2 Y_{22}|0\rangle$ for the lowest 2qp states $qq[Ln_z\Lambda]_i[Ln_z\Lambda]_j$ in ^{164}Dy and ^{172}Yb .

Nucleus	$qq[Ln_z\Lambda]_i[Ln_z\Lambda]_j$	f_{ij}^{22} [fm ⁴]
^{164}Dy	pp[411] \downarrow [411] \uparrow	6.49
^{172}Yb	pp[402] \downarrow [411] \downarrow	0.22

Figure 2 shows that collectivity of calculated 2_γ^+ states decreases from Dy to Yb isotopes. In Dy, we have the largest collective shifts (the difference between QRPA and first 2qp energies) and B(E2) values. In all the nuclei, the PBE considerably decreases the 2qp and QRPA energies. In Dy isotopes, this leads to a nice agreement with the experimental energies. The B(E2) is also well described. In Er and Yb, the PBE also noticeably improves description of 2_γ^+ energies. However here E_{QRPA} still remain considerably higher than E_{exp} , calculated B(E2) are accordingly underestimated. Altogether we see that PBE significantly changes QRPA energies E_{QRPA} but slightly affects B(E2) values.

The next point to be clarified is a different collectivity of 2_γ^+ states, which is maximal in Dy and minimal in Yb isotopes. This difference can be explained in terms of the Nilsson selection rules for E2(K=2) transitions in axial nuclei [10, 12]. The rules read

$$\Delta K = 2, \quad \Delta N = 0, \pm 2, \quad \Delta n_z = 0, \quad \Delta \Lambda = 2, \quad (4)$$

where N is the principle quantum shell number, n_z is its fraction along the z-axis, Λ is the orbital momentum projection onto z-axis. As seen from Table 1, the selection rule $\Delta n_z = 0$ is most important. It is kept in ^{164}Dy but not in ^{172}Yb . As a result, the quadrupole matrix element f_{ij}^{22} in ^{164}Dy is much large than in ^{172}Yb . Following our analysis, just the quadrupole matrix for the lowest 2qp state is decisive for the collectivity of the corresponding QRPA state (see more analytical and numerical arguments in [8, 20]). The large f_{ij}^{22} leads to a high collectivity and vice versa. This finding is valid for all considered nuclei. Thus we get a simple recipe for prediction of the collectivity (weak or large) of the first QRPA state: it suffices to check for the Nilsson selection rule $\Delta n_z = 0$ for the lowest 2qp state.

In conclusion, the present QRPA calculations for 2_γ^+ states with the Skyrme force SV-mas10 confirm the main results [8] obtained with the forces SkM* and SV-bas. Namely, a robust description of the deformation in rare-earth nuclei is certified. A strong PBE impact on the 2_γ^+ -energies is demonstrated. A simple explanation of nuclear domains with a high and low collectivity is verified. From comparison of present SV-mas10 and previous SV-bas [8] calculations, it is seen that these two Skyrme parameterizations give very close results.

Acknowledgements

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References

- [1] M. Bender, P.-H. Heenen, and P.-G. Reinhard, *Rev. Mod. Phys.* **75**, 121 (2003)
- [2] N. Paar, D. Vretenar, E. Khan, and G. Colo, *Rep. Prog. Phys.* **70**, 691 (2007)
- [3] J. Erler, W. Kleinig, P. Klüpfel, J. Kvasil, V.O. Nesterenko, and P.-G. Reinhard, *Proceedings of the Intern. Bogoliubov Conf. Problems of Theoretical and Mathematical Physics* (Dubna, Russia, August 21-27, 2009); *Phys. Part. Nucl.* **41**, 851 (2010)
- [4] S. Péru, G. Gosselin, M. Martini, M. Dupuis, S. Hilaire, and J.-C. Devaux, *Phys. Rev. C* **83**, 014314 (2011)
- [5] J. Bartel, P. Quentin, M. Brack, C. Guet, and H.-B. Haakansson, *Nucl. Phys. A* **386**, 79 (1982)
- [6] E. Chabanat, P. Bonche, P. Haensel, J. Meyer, and R. Schaeffer, *Nucl. Phys. A* **635**, 231 (1998).
- [7] J. Terasaki and J. Engel, *Phys. Rev. C* **84**, 014332 (2011)
- [8] V.O. Nesterenko, V.G. Kartavenko, W. Kleinig, R.V. Jolos, J. Kvasil, and P.-G. Reinhard, submitted to *Phys. Rev. C*
- [9] P. Klüpfel, P.-G. Reinhard, T.J. Burvenich, and J.A. Maruhn, *Phys. Rev. C* **79**, 034310 (2009)
- [10] S.G. Nilsson, *Mat. Fys. Medd. Dan. Vid. Selsk.* **29**, 16 (1965)
- [11] V.O. Nesterenko, V.P. Likhachev, P.-G. Reinhard, V.V. Pashkevich, W. Kleinig, and J. Mesa, *Phys. Rev. C* **70**, 057304 (2004)
- [12] V.G. Soloviev, *Theory of Complex Nuclei* (Oxford, Pergamon Press, 1976)
- [13] V.G. Soloviev and N.Yu. Shirikova, *Z. Physik A* **334**, 149 (1989)
- [14] <http://www.nndc.bnl.gov>, Evaluated Nuclear Structure Data File
- [15] V.O. Nesterenko, W. Kleinig, J. Kvasil, P. Vesely, P.-G. Reinhard, and D.S. Dolci, *Phys. Rev. C* **74**, 064306 (2006)
- [16] M. Bender, K. Rutz, P.-G. Reinhard, and J.A. Maruhn, *Eur. Phys. J. A* **8**, 59 (2000)
- [17] J. M. Eisenberg, and W. Greiner, *Nuclear Theory: Microscopic theory of the nucleus*, Vol. III (North-Holland, Amsterdam, London, 1972)
- [18] P. Ring and P. Schuck, *Nuclear Many Body Problem* (Springer-Verlag, New York, 1980)
- [19] S.G. Nilsson and I. Ragnarsson, *Shapes and Shells in Nuclear Structure* (Cambridge University Press, Cambridge, 1995)
- [20] V.O. Nesterenko, V.G. Kartavenko, W. Kleinig, R.V. Jolos, J. Kvasil, and P.-G. Reinhard, arXiv:1504.06492[nucl-th]