

Collective states of even Xe isotopes in IBM+MQRPA

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Abstract. A modification of the Quasiparticle Random Phase Approximation (QRPA) with small ground state correlations is suggested. The lowest energy phonon is used as the image of d -boson of the Interacting Boson Model 1 (IBM1) and applied to microscopical calculations of the IBM1 parameters. Results are compared with experimental data for Xe isotopes.

The Quasiparticle Random Phase Approximation (QRPA) is one of the widely used methods for description of semimagic and deformed nuclei [1]. Application of this approximation is quite correct in those cases when the Ground State Correlations (GSC) i.e. the distinction between the quasiparticle vacuum (QV) and the RPA-phonon vacuum (PV), exist but are rather small. Quantitatively it means, *e.g.*, that $\Sigma\psi^2 \gg \Sigma\varphi^2$, ψ , φ being amplitudes of the RPA D_μ -phonons ($\mu=0, \pm 1, \pm 2$)

$$D_\mu^+ = \frac{1}{\sqrt{2}} \sum_{\tau=\{n,p\}} \sum_{i,k} [\psi_{ik}(\mu) a_i^+ a_k^+ + \varphi_{ik}(\mu) a_{\bar{i}}^- a_{\bar{k}}^-]. \quad (1)$$

Further, we'll work with the QRPA-phonons composed of the Bogoliubov quasiparticles. In Eq. (1), a_i^+ , a_k^- are creation and annihilation quasiparticle operators, i is a single-particle state in a spherical mean field, \bar{i} is a time-conjugated state.

Direct application of QRPA is not justified in the region where the transition takes place from spherical nuclei to deformed ones that is revealed in lowering the energies of the first 2^+ -states (*e.g.*, from 1.2 MeV in Sn isotopes up to 0.32 MeV in ^{120}Xe) and at the same time in increasing $B(E2, 2_1^+ \rightarrow 0_{g.s.}^+)$. Applying the standard QRPA yields the values $\Sigma\varphi^2$ smaller then but comparable with $\Sigma\psi^2$ that necessitates to modify QRPA conformably.

Several modification of RPA, included the particle-hole RPA and QRPA, have been suggested which take into account GSC more consistently [2–8]. The main idea of our modification of QRPA (MQRPA) was formulated many years ago [9, 10] and was developed further in [11, 12]. We construct special equations like RPA for amplitudes ψ and φ of the collective quadrupole phonon D_μ , Eq. (1), which give very small values of $\Sigma\varphi^2$, i.e. with small GSC in the fermion space. However, after mapping the D -phonons and PV onto the boson space the boson Hamiltonian proves to be such that its diagonalization

leads to a rather complicated ground state function comprising apart from the boson vacuum (BV) two, three and more quadrupole bosons. Thus, if one returned from the boson space to the fermion one the ground state in this space would turn out to be strongly correlated.

Transition to the boson description is implemented following the Interacting Boson Model (IBM) in its first variant (IBM1 or SU(6)-model) which was first introduced by Janssen *et al.* [13] and then in another formulation by Arima and Iachello [14]. The IBM1 Hamiltonian, H_{IBM} , contains 6 parameters ($\varepsilon_d, k_1, k_2, C_L$):

$$H_{\text{IBM}} = \varepsilon_d \hat{n}_d + k_1 \hat{p}_1 + k_2 \hat{p}_2 + \frac{1}{2} \sum_{L=0,2,4} C_L \hat{C}_L, \quad (2)$$

where

$$\begin{aligned} \hat{n}_d &= \sum_{\mu} d_{\mu}^+ d_{\mu}; & \hat{p}_1 &= d^+ \cdot d^+ s^+ s + \text{H.c.}; \\ \hat{p}_2 &= (d^+ d^+)^{(2)} \cdot ds + \text{H.c.}; & \hat{C}_L &= (d^+ d^+)^{(L)} \cdot (dd)^{(L)}. \end{aligned} \quad (3)$$

In Eqs. (2)-(3), d^+ , d , s^+ , s are the quadrupole and scalar boson operators. The seventh parameter is Ω – the maximum d -boson number in boson wave functions.

A microscopical way to calculate the IBM1 parameters was suggested in the first work on IBM [13] where quadrupole d -bosons were introduced as an image of the most collective two-quasiparticle phonons composed by both neutron and proton pairs (in the IBM2 there are separately neutron and proton bosons [15]). As the image of d -bosons we employ the quadrupole phonons of MQRPA with the lowest energy. As shown by IBM2 [16] and our [12] calculations, in order to obtain the realistic values of IBM parameters one has to renormalize them, taking into account the connection of the collective D -phonon or d -boson space with phonons of higher energies and angular momenta $L \geq 2$. So all calculations here for Xe isotopes and in Ref. [12] are performed with the renormalized IBM1 parameters. We consider all states inside of the yrast

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band and several ones outside with energies less than the doubled pairing gap, i.e. $\lesssim 3.0$ MeV, to be collective states which can be described in the D -phonon or d -boson space. The energy of such state E is formed by the following three constituents [12]:

$$E = E_{QV} + E_{PV} + E_{IBM}, \quad (4)$$

$E_{QV} = \langle QV|H_F|QV\rangle$, $E_{PV} = \langle PV|:H_F:|PV\rangle$, and $E_{IBM} = \langle I_\nu|H_{IBM}|I_\nu\rangle$. The first of them, E_{QV} , is the energy of QV and H_F is the fermion Hamiltonian. The second term, E_{PV} , is the energy of PV which is determined by the GSC, $:H_F:$ is the same Hamiltonian but the quasiparticle operators in it stand in the normal order. E_{IBM} is the expectation value of H_{IBM} , Eq. (2), $|I_\nu\rangle$ being a boson wave function of a state with spin I . Index ν shows which time this function appears, the larger ν the higher energy of the state $|I_\nu\rangle = \sum_n \alpha_{\nu n}|n\rangle$, where $|n\rangle$ is a function of the SU(5)-basis.

In Refs. [11, 12], we suggested a functional Φ . Minimization of Φ yields all the quantities determining E , Eq. (4), i.e. the Bogoliubov parameters (u, v) , D -phonon amplitudes (ψ, φ) and the set of amplitudes $\{\alpha_{\nu n}\}$ of boson wave functions. This functional Φ involves E and several additional conditions.

At first, such quantities as (u, v) , (ψ, φ) and $\{\alpha_{\nu n}\}$ are to be normalized. Secondly, it is supposed in IBM1 that D_μ^+ , D_ν and their commutators form SU(6)-algebra. Therefore, Ω characterizing a symmetric representation of the SU(6)-algebra is an integer. As the definition of Ω we take

$$\langle PV|\sum_\kappa [[D_\kappa, D_\kappa^+], D_\mu^+, D_\mu]|PV\rangle = 6/\Omega. \quad (5)$$

In the quasiparticle theory, the expectation values of the neutron and proton number operators have to be equal to the corresponding particle numbers in a given nucleus. Therefore, as usual, the particle number operators with the corresponding chemical potentials are introduced in Φ . Thus, the chemical potentials depend on state $|I_\nu\rangle$ and their alterations contribute to differences between theoretical level energies. To attain a small value of $\sum \varphi_{ik}^2$ we introduce into Φ a special regulator R of GSC.

The regulator R is proportional to the quasiparticle number operator and can fix the quasiparticle number in PV or the ratio r

$$R = \frac{\xi}{2} \sum_{i,m_i} a_i^+ a_i; \quad r = \sum_{i,k} \varphi_{i,k}^2(\mu) / \sum_{i,k} \psi_{i,k}^2(\mu). \quad (6)$$

The appearance of R in Φ distinguishes our MQRPA from the standard QRPA.

The explicit form of components entering Φ depends on the method applied to treat the many-body problem. In particular, one can use the Hartree-Fock-Bogoliubov (HFB) method with a realistic effective interaction. In [11, 12], we exploited a rather simplified approach with the spherical Saxon-Woods mean field and residual effective forces. In our calculations, matrices of the monopole particle-particle forces are approximated by the well-known constants G_τ ($\tau = n, p$) that allow the standard Bogoliubov

transformation (u, v are numbers for each single-particle state) to be employed. The particle-hole and particle-particle forces are factorized in a traditional manner as products of matrix elements of the quadrupole single-particle operators. We do not include in calculations the spin-exchange and two-particle spin-orbit interactions. They can reduce the quadrupole collectivity and to compensate omitting those forces, our isoscalar strength of the particle-hole force is by 10% lower than the estimation by A. Bohr and B. Mottelson [17]. Since our approach is rather far from the universal self-consistency we admit some variations of the Saxon-Woods potential parameters to reproduce experimental data better.

Thus, the IBM1 parameters are functions of single-particle energies, matrix elements and strengths of the effective forces and also of quantities subject to determination, i.e., (u, v) , (ψ, φ) . To display the latter E_{IBM} we represent it as a sum of the IBM1 parameters multiplied by expectation values (EXPV) of the separate terms of H_{IBM} , namely, $\langle I_\nu|\hat{n}_d|I_\nu\rangle$, $\langle I_\nu|\hat{p}_1|I_\nu\rangle$ etc. At calculations of variations of E_{IBM} with respect to (u, v) , (ψ, φ) we allow for only terms with ε_d and k_1 , Eq. (2), where renormalizing factors are considered to be invariable. Our calculation show that renormalized parameters C_L , Eq. (2), practically do not change at small variations of (u, v) and (ψ, φ) . Variations of parameter k_2 with respect to (u, v) could influence single-particle energies as the main dependence of k_2 on (u, v) is contained in factor $(u_i u_k - v_i v_k)$ which is highly sensitive to a mutual arrangement of single-particle levels. However, a change of the spin-orbital potential depth, which we admit in transitional nuclei such as Xe isotopes, produces much stronger effect on level positions. Therefore, we calculate k_2 but do not consider its variations, i.e. its influence on the level positions.

At the end, the variations of Φ with respect to (u, v) , (ψ, φ) give a set of coupled equations. They include mentioned above EXPV which depends on boson amplitudes $\{\alpha_{\nu n}\}$. Thus, the self-consistence procedure is demanded: using (u, v) , (ψ, φ) one calculates the IBM1 parameters, diagonalizes H_{IBM} , finds $\{\alpha_{\nu n}\}$, calculates EXPV and returns to equations for (u, v) , (ψ, φ) and so on until the self-consistent values of all the unknown quantities are obtained.

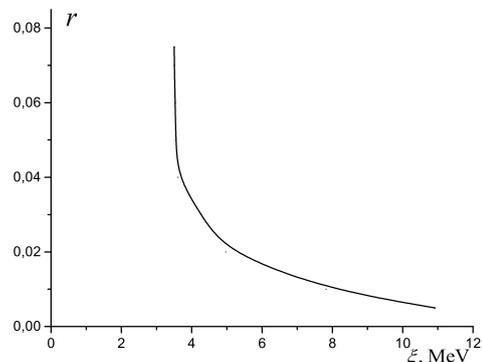


Figure 1. The ratio r , Eq. (6), as a function of ξ .

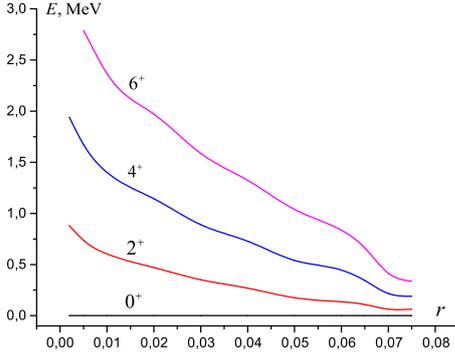


Figure 2. (Color online) The yrast band in ^{124}Xe as a function of r at fixed values of the quadrupole interaction strengths.

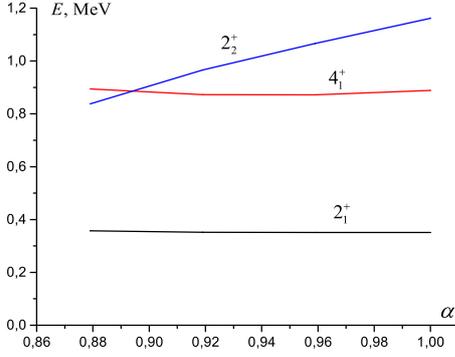


Figure 3. (Color online) Dependence of $E(2_2^+)$, $E(4_1^+)$, $E(2_1^+)$ on α for ^{124}Xe .

Results following from the equations for (u, v) show that the presence of bosons in the ground and excited states exerts blocking effect on the pairing, i.e. the pairing correlations are attenuated with the growth of spin and excitation energy. Thus, we confirm conclusions of calculations with Tamm-Dankoff phonons in fermion space [18]. However, inclusion the E_{IBM} value in (u, v) -variations at values of $\langle I_v | \hat{n}_d | I_v \rangle \geq 2$ (that takes place in Xe isotopes) results in delaying the attenuation caused by the quadrupole particle-hole forces. Thereby neutrons affect on proton pairing and vice versa. The effect of increasing neutron numbers on the proton pairing by means of the particle-hole forces is worth notice at analysis of the pairing energies in isotopic chains and, at replacement of protons by neutrons in isotonic ones as well.

Equations for (ψ, φ) arising mainly at variations of E_{IBM} involve the regulator R , Eq. (6). Since R joins to two-quasiparticle energies in ε_d and k_1 it is able to change the character of the phonon collectivity. The more is ξ the further from zero is the first pole in the secular equation for the phonon energy ω and the smaller are values of corresponding sums at $\omega = 0$. This leads to increasing ω

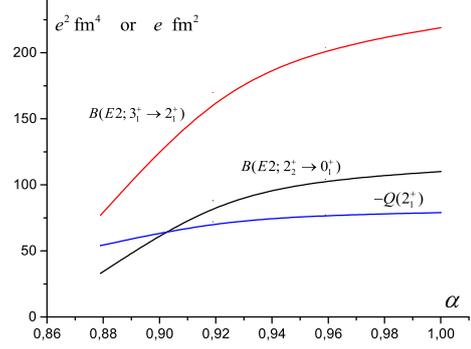


Figure 4. (Color online) Dependence of $Q(2_1^+)$ (in efm^2 units), $B(E2; 2_2^+ \rightarrow 0_1^+)$, and $B(E2; 3_1^+ \rightarrow 2_1^+)$ (in $e^2 fm^4$ units) on α for ^{124}Xe .

and, as the result, to a weakening of GSC. Note, that in our approach ω is always higher than the d -boson energy ε_d . Figure 1 shows r as a function of ξ . The quasiparticle number in PV, N_{qp} , is practically proportional to r : $N_{\text{qp}} \approx 12.1r + 0.1$, where $0.02 < r < 0.08$ ($N_{\text{qp}} \rightarrow 0$ at $r \rightarrow 0$). We did not find unique criterion that could give an undoubted value of N_{qp} or r . However, there are several arguments narrowing the region of their search. Firstly, it is the fulfilment of the exclusion principle. Addition of ξ to each two-quasiparticle energy draws together and equalizes D -phonon amplitudes. Thereby the quasiparticle number n_j in each single-particle j -state (given by one phonon) decreases. Therefore, even at large enough boson number in a state $|I_v\rangle$ the n_j value can remain $< j + 1/2$. However, it is possible only at the value of $\xi \geq 5$ MeV and $r < 0.075$ (Fig. 1). These numbers are found in calculations for Xe isotopes. Secondly, to achieve the self-consistency between (u, v) , (ψ, φ) and $\{\alpha_{vm}\}$, mentioned above, the values of ξ and r have to be larger than 5 MeV and less than 0.075, respectively. Thirdly, as shown in [5], a more correct boson mapping of the D -phonon operators, to get SU(6)-algebra, can be attained at small values of $\sum_{ik} \varphi_{ik}^2 = 0.05 \approx r$. If the effective interaction strengths were not subject to small alterations to fit them in with experimental spectra, then we could choose the value of ξ to reproduce better the experimental data as at fixed strengths the yrast spectrum is compressed with increasing r that is shown in Fig. 2.

All our calculations for Xe isotopes are carried out with $r \sim 0.02 \div 0.03$ ($\xi \approx 5$ MeV) (see Fig. 1). These values of r and ξ ensure the fast convergence of the self-consistency process and fulfilment of the exclusion principle. This value of ξ gives rise to the extension of the D -phonon space, i.e. to increasing the maximum d -boson number Ω because of increasing a relative role of the high energy two-quasiparticle states in the D -phonon structure.

Apart from the interaction strengths in the fermion Hamiltonian we vary the spin-orbital potential depth V_{ls}

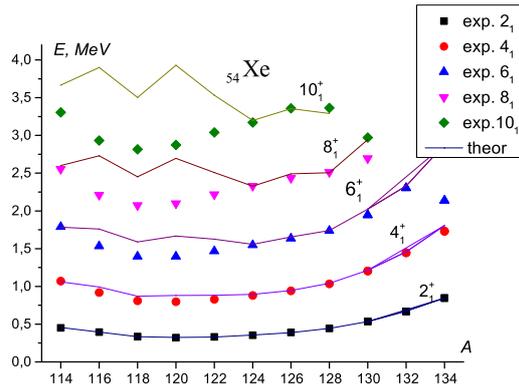


Figure 5. (Color online) Theoretical and experimental energies of the yrast states in Xe-isotopes.

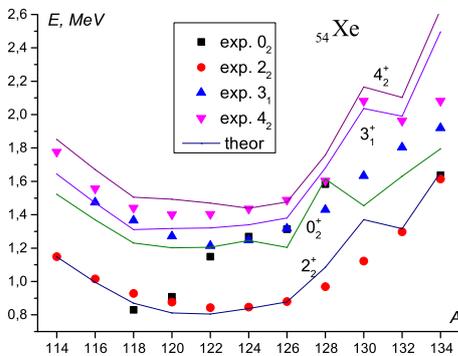


Figure 6. (Color online) Theoretical and experimental energies of non-yrast states in Xe-isotopes.

which strongly affects the value of parameter k_2 in H_{IBM} , Eq. (2). We keep invariable all parameters of the spin-orbital potential taken from Ref. [19] (Table 4, set 1) with the only exception of the depth. We denote the depth as αV_{ls} (V_{ls} is given in [19]) and slightly change parameter α ($0.8 < \alpha < 1$). At a fixed IBM parameter ε_d the growth of k_2 (it almost linearly follows α) is accompanied by convergence of the yrast band states and rising the energies of 2_2^+ -, 3_1^+ -collective states simultaneously with probabilities of inter-band $E2$ -transitions (e.g., $2_2^+ \rightarrow 0_1^+$). To compensate the impact of α (through k_2) on the yrast state energies we alter linearly with α the quadrupole strengths but not more than by 3%. The dependencies on α of state energies, $E2$ -probabilities and the quadrupole moment of the 2_1^+ -state are given in Figs. 3 and 4. Thus, the value of α can be fixed for each of Xe isotopes and more or less reasonable agreement with experimental data can be obtained. We did not investigate in detail the influence of the effective forces entering into parameters of H_{IBM} on single-particle energies and so we cannot assert that the weakening of the spin-orbital potential ($\alpha_{\text{average}} \approx 0.9$) cannot be imitated by this influence. However, it is undoubted that the presence of the collective quadrupole phonons (bosons) could change the spatial density on the nuclear edge and, as $V_{\text{ls}} \sim \partial\rho/\partial r$, affects the spin-orbital potential.

Table 1. $B(E2)$ (in $e^2 \cdot \text{fm}^4$ units) inside the yrast bands in Xe isotopes.

$I_i^+ \rightarrow I_f^+$	^{116}Xe		^{120}Xe	
	exp.	theor.	exp.	theor.
$2_1^+ \rightarrow 0_1^+$	2470 ± 92	2460	3560 ± 160	3600
$4_1^+ \rightarrow 2_1^+$	4320 ± 190	3680	4120 ± 300	5290
$6_1^+ \rightarrow 4_1^+$	3890^{+360}_{-300}	4330	5100^{+1240}_{-1230}	6100
$8_1^+ \rightarrow 6_1^+$	3370^{+450}_{-360}	4770	3650^{+850}_{-580}	6590
$10_1^+ \rightarrow 8_1^+$	3830^{+870}_{-600}	5070	3230^{+630}_{-460}	6900

$I_i^+ \rightarrow I_f^+$	^{122}Xe		^{128}Xe	
	exp.	theor.	exp.	theor.
$2_1^+ \rightarrow 0_1^+$	2890 ± 120	2870	1560 ± 85	1580
$4_1^+ \rightarrow 2_1^+$	4150 ± 190	4250	2270 ± 200	2390
$6_1^+ \rightarrow 4_1^+$	3820^{+2120}_{-1010}	4970	2990^{+300}_{-250}	2860
$8_1^+ \rightarrow 6_1^+$	2980^{+3000}_{-1000}	5450	3680^{+450}_{-360}	3190
$10_1^+ \rightarrow 8_1^+$	4450^{+3110}_{-1300}	5820	–	–

Table 2. $B(E2)$ (in $e^2 \cdot \text{fm}^4$ units) and $Q(2_1^+)$ (in $e \cdot \text{fm}^2$ units) in ^{124}Xe .

$I_i^+ \rightarrow I_f^+$	exp.	theor.	$I_i^+ \rightarrow I_f^+$	exp.	theor.
$2_1^+ \rightarrow 0_1^+$	2120 ± 54	2150	$3_1^+ \rightarrow 2_1^+$	84 ± 15	77
$4_1^+ \rightarrow 2_1^+$	2490 ± 70	3130	$3_1^+ \rightarrow 4_1^+$	925 ± 200	870
$6_1^+ \rightarrow 4_1^+$	3250^{+310}_{-260}	3630	$5_1^+ \rightarrow 3_1^+$	1380 ± 190	2180
$2_2^+ \rightarrow 2_1^+$	1140^{+2150}_{-450}	2210	$5_1^+ \rightarrow 4_2^+$	1310 ± 250	1020
$2_2^+ \rightarrow 0_1^+$	28^{+9}_{-7}	33	$5_1^+ \rightarrow 4_1^+$	27 ± 5	27
$3_1^+ \rightarrow 2_2^+$	3020 ± 540	2640	$Q(2_1^+)$	–	–54

Figures 5 and 6 compare the theoretical energies of the yrast band states and of states 2_2^+ , 0_2^+ , 3_1^+ , 4_2^+ with the experimental data in Xe isotopes. Calculations were performed with Ω which are about two times higher than the half of the number of valence particles or holes, that is caused by the choice of ξ . The levels 2_1^+ , 4_1^+ in all considered isotopes are in the best agreement with experiment. For $^{124,126}\text{Xe}$ the very good agreement is observed even up to $I = 12$. However, as a rule, the theoretical yrast spectra are stretched. Perhaps that explicit consideration of non-collective two-quasiparticle high-spin states ($I \geq 6$), as it was done in [10], improve the situation. Reasonable description is obtained for 2_2^+ -, 4_2^+ -states. Apparently for adequate description of the low-lying 0_2^+ -states in $^{118,120}\text{Xe}$ one has to consider explicitly monopole pair-vibrational states. The better description of 3_1^+ -states can be obtained by means of the terms of H_{IBM} containing 6 d -boson operators (i.e. with taking into account “nonaxiality”). Tables 1 and 2 show that $B(E2)$ values inside the yrast band and those for inter-band transitions in ^{124}Xe are in reasonable agreement with experimental data.

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