

# Properties of isoscalar monopole particle-hole-type excitations in $^{208}\text{Pb}$

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**Abstract.** The particle-hole dispersive optical model developed recently is applied to describe properties of high-energy isoscalar monopole excitations in  $^{208}\text{Pb}$ . We consider, in particular, the double transition density averaged over the energy of the isoscalar monopole excitations in a wide energy interval, which includes the isoscalar giant monopole resonance and its overtone. The energy-averaged strength functions of these resonances are also analyzed. Some corrections to the calculation scheme are formulated to restore the model unitarity.

## 1 Introduction

The study of properties of collective states in nuclei provide information on the bulk properties of nuclear matter. In particular, the interest in experimental and theoretical studies of high-energy isoscalar monopole (ISM) (p-h)-type excitations in medium-heavy mass nuclei is explained by the possibility to get information about the compressibility modulus of nuclear matter, a fundamental physical quantity essential for cosmology and nuclear physics. The value of this quantity depends on the mean energy of the strength distribution, corresponding to the ISM external field  $r^2Y_{00}$  (in other words, on the energy of the isoscalar giant monopole resonance (ISGMR)) [1]. To deduce this strength from experimental  $(\alpha, \alpha')$ -reaction cross sections, it is usually assumed that the ISM strength is concentrated in a vicinity of the ISGMR and the properly normalized collective ISGMR transition density can be used in the analysis of experimental data (see, e.g. Ref. [2]). However, due to Landau damping the radial dependence of the (semi)microscopic transition density is changed with increasing of the excitation energy from the ISGMR to its overtone.

In the present work we use the particle-hole dispersive optical model (PHDOM) developed recently [3] to describe in a semi-microscopic way, and in average over the energy, some properties of high-energy isoscalar monopole (p-h)-type excitations in  $^{208}\text{Pb}$ . The other aim of this study is to examine from a microscopic point of view the applicability of the quasi-classical collective model transition densities of the ISGMR and its overtone (ISGMR2) to the description of these resonances. Some corrections to the PHDOM are also formulated to restore the model unitarity.

## 2 Calculation results

Within the model the main relaxation modes of high-energy (p-h)-type excitations (Landau-damping, coupling with the single-particle (s-p) continuum, the spreading effect) are commonly taken into account. Using this model, we evaluate the energy-averaged ISM radial double transition density (i.e., the product of transition densities taken at different points):

$$\rho(r, r', \omega) = -\frac{1}{\pi} \text{Im} A(r, r', \omega), \quad (1)$$

$$A(r, r', \omega) = A_0(r, r', \omega) + \int A_0(r, r_1, \omega) F(r_1) A(r_1, r', \omega) r_1^{-2} dr_1. \quad (2)$$

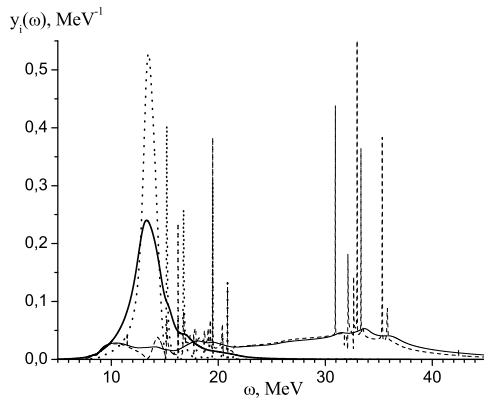
Here,  $A_0(r, r', \omega)$  is a "free" p-h propagator and  $F(r)$  is the intensity of the isoscalar part of Landau-Migdal interaction. Being doubly convoluted with an external ISM field  $V_0(\vec{r}) = V_0(r)Y_{00}(\vec{n})$ , the double transition density determines the corresponding energy-averaged strength function

$$S_{V_0}(\omega) = -\frac{1}{\pi} \text{Im} \int V_0(r) A(r, r', \omega) V_0(r) dr dr'. \quad (3)$$

To illustrate variations of the ISM double-transition-density radial dependence in a wide excitation-energy interval, we use the reduced quantity

$$R(r, r', \omega) = \frac{\rho(r, r', \omega)}{\int \rho(r, r' = r, \omega) dr}, \quad (4)$$

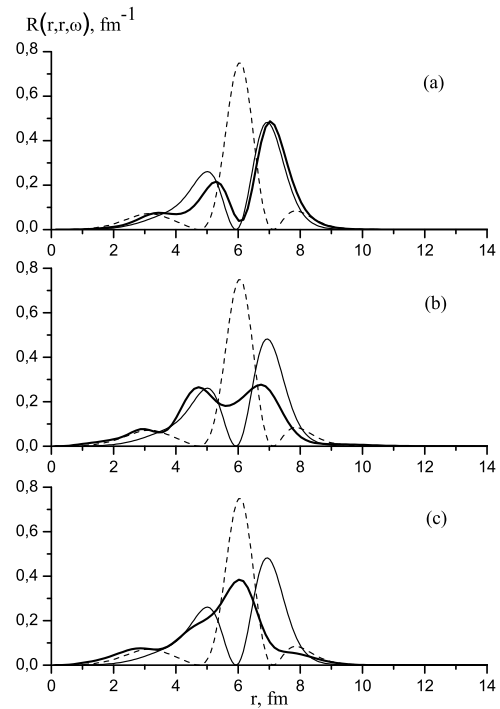
which is normalized by  $\int R(r, r' = r, \omega) dr = 1$ . These quantities are evaluated for  $^{208}\text{Pb}$  in a wide excitation energy interval, which includes the ISGMR



**Figure 1.** Fractions of the energy-weighted strength functions,  $y_i(\omega) = \omega S_{V_{0,i}}(\omega)/(EWSR)_i$ , calculated within the PHDOM, ( $y_1(\omega)$  - the thick solid line,  $y_2(\omega)$  - the thin solid line) in comparison with  $y_i^{cRPA}(\omega)$ , calculated within the continuum-RPA ( $y_1^{cRPA}(\omega)$  - the thick dotted line,  $y_2^{cRPA}(\omega)$  - the thin dashed line).

and its overtone [4]. In Fig. 1, we show the calculated fractions of the energy-weighted strength functions  $y_i(\omega) = \omega S_{V_{0,i}}(\omega)/(EWSR)_i$ , obtained within the PHDOM for the external fields  $V_{0,1}(r) = r^2$  and  $V_{0,2}(r) = r^4 - \eta r^2$ . The choice of numerical value of the parameter  $\eta$ , as well as of other model parameters, is described in details in Ref. [4]. The centroid energy  $\omega_{GMR} = 13.8$  MeV and total width  $\Gamma_{GMR} = 2.9$  MeV calculated within our model for the excitation energy interval 10–35 MeV are in agreement with experimental quantities of  $13.96 \pm 0.2$  MeV and  $2.88 \pm 0.2$  MeV [5], respectively. In Fig. 2, we show the calculated reduced transition density  $R(r, r' = r, \omega)$  at the excitation energies  $\omega = 13$  MeV (a), 23 MeV (b), and 33 MeV (c) in comparison with transition densities of the quasi-classical collective model  $R_{qc,1}(r, r' = r)$  and  $R_{qc,2}(r, r' = r)$  [4, 6] for the ISGMR and ISGMR2, respectively, normalized by  $\int R_{qc,1}(r, r' = r) dr = 1$ . It is clearly seen that only near the maximum of the ISGMR the densities  $R(r, r, \omega)$  and  $R_{qc,1}(r, r)$  are close, especially near the nuclear surface. For other energies the difference is noticeable.

In connection with the description of the ISM excitations within the PHDOM (first results were obtained in Ref. [4]) the question of violation of the model unitarity arises. The signatures of violation are: (i) a non-zero value of the calculated strength function  $S_{sp}(\omega)$ , corresponding to the “spurious” external field  $V_{0,sp}(r) = 1$ ; (ii) negative values of the strength function  $S_{V_{0,1}}(\omega)$  at high excitation energies  $\omega$ , that leads to reduction of the total strength. The unitarity is slightly violated by the method used within PHDOM to describe the spreading effect in terms of the imaginary part  $W(\omega)$  of a specific energy-



**Figure 2.** The reduced ISM double transition density  $R(r, r' = r, \omega)$  calculated within the PHDOM for  $^{208}\text{Pb}$  at the excitation energies  $\omega = 13$  MeV (a), 23 MeV (b), and 33 MeV (c) (the solid thick lines) in comparison with the quasi-classical collective model transition densities  $R_{qc,1}(r, r)$  (the solid thin line) and  $R_{qc,2}(r, r)$  (the dashed line).

averaged p-h interaction responsible for a spreading effect (this part determines the corresponding real part  $P(\omega)$  via a proper dispersive relation) [3]. The other source of violation of the model unitarity is the use of the approximate spectral expansion for the optical-model Green’s function. This expansion allows one to take commonly into account the s-p continuum and spreading effect.

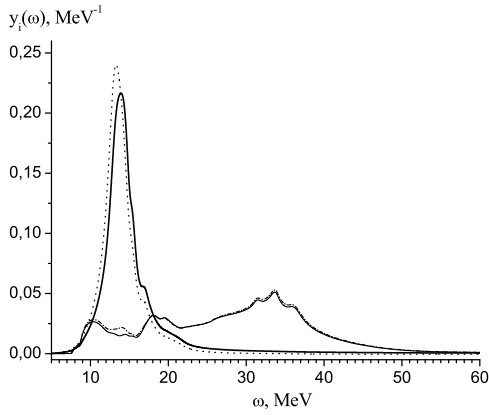
To restore the unitarity of the model, we properly modify the energy-averaged ISM double transition density by adding to it the terms involving the ground-state density  $\bar{n}(r)$ , normalized to unity

$$\begin{aligned} \rho^R(r, r', \omega) = & \rho(r, r', \omega) + S_{sp}(\omega) \cdot \bar{n}(r) \cdot \bar{n}(r') \\ & - \bar{n}(r) \int \rho(r'', r', \omega) dr'' \\ & - \bar{n}(r') \int \rho(r, r'', \omega) dr''. \end{aligned} \quad (5)$$

This equation allows to determine the modified strength function corresponding to the field  $V_0(r)$

$$S_{V_0}^R(\omega) = \int V_0(r) \rho^R(r, r', \omega) V_0(r') dr dr'. \quad (6)$$

As a result, we get from (6): (i) the zero value for the modified “spurious” strength function  $S_{sp}^R(\omega)$ ; (ii)



**Figure 3.** The comparison of  $y_i(\omega)$  calculated with ( $y_i^R(\omega)$ ) and without ( $y_i(\omega)$ ) account of the unitarity restoration:  $y_1^R(\omega)$  - thick solid line,  $y_2^R(\omega)$  - thin solid line ;  $y_1(\omega)$  - dotted line,  $y_2(\omega)$  - dot-dashed line.

the modified ISM strength functions for the external field  $V_0(r)$  have now no negative values in the considered wide energy interval. It is also seen that the same results may be obtained using  $\rho(r, r', \omega)$  and the modified external fields  $V_0(r) - \bar{V}_0$ , where  $\bar{V}_0$  means averaging of  $V_0(r)$  over the ground-state density:

$$\begin{aligned} S_{V_0}^R(\omega) &= \int (V_0(r) - \bar{V}_0) \rho(r, r', \omega) (V_0(r') - \bar{V}_0) dr dr' \\ &= S_{V_0 - \bar{V}_0}(\omega). \end{aligned} \quad (7)$$

For the restoration of the model unitarity it is also necessary to introduce the factor  $(1 - dP(\omega)/d\omega)$  in the expression for the energy-averaged “free” p-h propagator.

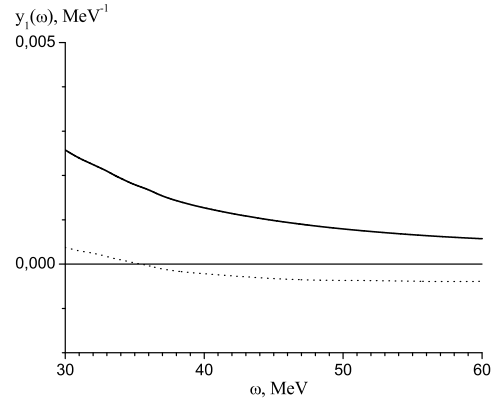
The results for the strength functions may be also obtained within the concept of the effective field  $V(r, \omega)$ , which satisfies to the following equation:

$$V(r, \omega) = V_0(r) + r^{-2} F(r) \int A_0(r, r', \omega) V(r', \omega) dr'. \quad (8)$$

The modified strength function can be obtained with using the modified external fields  $V_0(r) - \bar{V}_0$  instead external fields  $V_0(r)$  in (8).

In Fig. 3, we compare the functions  $y_i(\omega)$ , calculated with ( $y_i^R(\omega)$ ) and without ( $y_i(\omega)$ ) restoration of the model unitarity.

In Table 1, the corresponding results obtained for the total strength,  $x_i = \int y_i(\omega) d\omega$ , are presented. These data are obtained for the energy interval 3-83 MeV (with account of the negative values of  $y_1(\omega)$ ). The quantities  $y_1(\omega)$  and  $y_1^R(\omega)$  calculated in the energy region  $\omega > 30$  MeV are presented in Fig. 4.



**Figure 4.** Comparison of the  $y_1(\omega)$  values calculated in the energy region  $\omega > 30$  MeV with the help of  $\rho^R(r, r', \omega)$  (solid line) and  $\rho(r, r', \omega)$  (dotted line).

**Table 1.** The calculated values of  $x_i$  obtained with ( $x_i^R$ ) and without ( $x_i$ ) account of the unitarity restoration.

ISGMR		ISGMR2	
$x_1^R$	$x_1$	$x_2^R$	$x_2$
1.01	0.95	1.00	1.04

### 3 Conclusion

In this work, we have demonstrated applicability of the PHDOM in calculations of the ISM double transition densities and strength functions. In particular, it has been shown that in the intermediate excitation energy region (between the energies of the ISGMR and ISGMR2) the double transition density is quite different from that obtained from the quasi-classical collective model transition densities, which are commonly used in the analysis of hadron inelastic scattering cross sections for the ISM excitations. The improved version of the PHDOM taking into account the restoration of the model unitarity will be considered in details elsewhere and applied to the description of the energy-averaged ISM double transition density, which determines the energy-averaged  $(\alpha, \alpha')$ -reaction cross sections.

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