On the excitation energy of deep-hole states in medium-heavy-mass spherical nuclei

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Abstract. Within the particle-hole dispersive optical model it is shown that the spreading effect determines a significant part of the anomalously large excitation energy of deep-hole states in the $^{90}$Zr and $^{208}$Pb parent nuclei.

1 Introduction

An anomalously large excitation energy of deep hole states in the $^{90}$Zr and $^{208}$Pb parent nuclei has been experimentally found by Vorobyov et al. [1]. In most Hartree-Fock calculations exploiting different versions of Skyrme-type forces, the energy of deep-hole states in the $^{208}$Pb parent nucleus is markedly underestimated (Fig. 1). The same conclusion follows from calculations exploiting a realistic partially self-consistent phenomenological mean field provided that the parameters of this mean field adjusted to describe the observable single-quasiparticle spectra near the Fermi energy [3].

In the present work we attempt to show that the spreading effect due to coupling of deep-hole states to many-quasiparticle configurations contributes significantly to the deep-hole state excitation energy. To reach this aim, we use the single-quasiparticle dispersive optical model (SQDOM) formulated in a rather formal way long ago [4]. Microscopically-based transition to the SQDOM was performed recently [3].

Table 1. Variants of Skyrme forces provided by S. Shlomo [2] and used in calculations presented in Fig. 1.

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Figure 2. (Color online) The quantities $\Delta(\varepsilon_{\lambda})$ (squares) and $W(\varepsilon_{\lambda})$ (circles) deduced from the experimental data of Ref. [1] with the use of Eqs. (1) and (2), respectively for the neutron (a) and proton (b) one-hole states in the $^{90}$Zr parent nucleus. The adopted function $W_{\text{fit}}(\varepsilon)$ of Eq. (5) and the calculated function of Eq. (2) are shown by the dot-dashed and solid lines, respectively.

Figure 3. (Color online) The same as in Fig. 2, but for the neutron (a) and proton (b) one-hole states in the $^{208}$Pb parent nucleus.

2 Basic relations

In the analysis we use, as the starting point, the phenomenological partially self-consistent mean field (described in details in Ref. [3]), which determines the single-particle (s-p) Hamiltonian $H_0(x)$. Then the optical-model Hamiltonian is (the isobaric index, on which all considered quantities are diagonal, is omitted)

$$H(x) = H_0(x) + [\mp iW(\varepsilon) + \Delta(\varepsilon)]f(x).$$ (1)

Here, the signs “–” and “+” are related to single-particle (s-p, $\varepsilon > \mu$) and single-hole (s-h, $\varepsilon < \mu$) excitations, respectively; $\mu$ is the mean (for particle and holes) chemical potential; $f(x)$ is the Woods-Saxon function used also in the definition of the mean field.

In the further use of Eq. (1) it is supposed the following: (i) the energy dependence of $W$ is similar for particles and holes, i.e. $W(\varepsilon) = W(|E|)$, where $E = \varepsilon - \mu$, and $|E|$ is the single-quasiparticle excitation energy; (ii) the real quantity $\Delta$ can be presented as the sum $\Delta^d + \Delta^p$, where the first (“dispersive”) term is due to the spreading effect while the second (“potential”) term simulates the mean-field energy dependence. Using the above supposition for $W(|E|)$, one gets the dispersive relationship which determines the dispersive part of $\Delta$ via $W$ [3, 4]:

$$\Delta^d(E) = \frac{2E}{\pi} \text{P.V.} \int_0^\infty \frac{W(E')}{E'^2 - E^2} dE'.$$ (2)

To find the empirical function $W(|E|)$, we use, firstly, the experimental energies s-h states, $\varepsilon_{\lambda}$. Within the model, these energies can be calculated together with the s-h wave functions according to the equation

$$(H_0(x) + \Delta(\varepsilon_{\lambda}))f(x) = 0.$$ (3)

The wave functions, $\phi_\lambda(x)$, are supposed to be normalized to unity. Thus, we can find a set of the empirical quantities $\Delta(\varepsilon_{\lambda})$. As shown in Ref. [3], the function $W(\varepsilon)$ determines the spreading (in fact, total) width of s-h states according to the relationship

$$\Gamma_{\lambda} = 2W(\varepsilon_{\lambda}) \int f(x)|\phi_\lambda(x)|^2 dx.$$ (4)

Using Eq.(4) together with the experimental values of s-h total widths, we can find a set of quantities $W(\varepsilon_{\lambda})$.  

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Such a set can be adopted by an empirical function \( W^{fit}(|E|) \) which is proportional to \( E^2 \) for small \(|E|\), as it takes place in infinite Fermi-systems. In accordance with Eq. (2), the adopted function \( W^{fit}(|E|) \) determines the dispersive part of the optical-model potential, \( \Delta^d(\varepsilon) \). Being compared with the values \( \Delta(\varepsilon) \), this quantity determines contribution of the spreading effect to the s-h excitation energies.

### 3 Calculation results

The sets of quantities \( \Delta(\varepsilon_\lambda) \) and \( W(\varepsilon_\lambda) \) found by the above-described way are shown in Figs. 2, 3 for neutron and proton subsystems of \(^{90}\text{Zr}\) and \(^{208}\text{Pb}\). The adopted functions \( W^{fit}(|E|) \) are parameterized as follows:

\[
W^{fit}(|E|) = W_0 \tanh^2(E/B)
\]

(5)

with the adjusted parameters \( W_0^p = 25, W_0^n = 20 \) and \( B^p = 60, B^n = 70 \) (in MeV). The functions \( W^{fit}(|E|) \) are also shown in Figs. 2, 3. Finally, the dispersive part of the optical-model potential, \( \Delta^d(\varepsilon) \), is evaluated by means of Eq. (2). For \( \varepsilon < \mu \) these functions are shown in Figs. 2, 3 and compared with the sets of quantities \( \Delta(\varepsilon_\lambda) \). From these comparison it follows that the spreading effect gives significant contribution to the excitation energy of deep-hole states.

In conclusion, we demonstrate abilities of the single-quasiparticle dispersive optical model to describe the spreading effect contribution to the excitation energy of deep-hole states in medium-heavy-mass spherical nuclei.

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### References


