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Neutrino-nucleus reactions in supernovae

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Abstract. We study thermal effects on neutrino-nucleus reactions occurring under supernova conditions. The approach we use is based on the QRPA extended to finite temperature by the thermo-field dynamics formalism. For the relevant supernova conditions we calculate inelastic neutrino scattering and neutrino absorption cross sections for two sample nuclei, ^{56}Fe and ^{82}Ge. In addition, we apply the approach to examine the rate of neutrino-antineutrino pair emission by hot nuclei.

1 Introduction

The significant role played by processes with neutrinos in core-collapse supernovae is well known [1]. In the collapsing core, energy emission via neutrinos helps to maintain the low entropy and, as a result, nucleons reside primarily in nuclei. At densities of $\rho \gtrsim 10^{11}\,\mathrm{g\,cm^{-3}}$ neutrino interactions with matter become important, leading to neutrino trapping and thermalization. Moreover, neutrino energy deposition behind the stalled shock may play an important role in successful explosion. Obviously collapse simulations should in principle include all potentially important neutrino reactions.

In the supernova environment neutrino-induced reactions with nuclei are dominated by allowed Gamow-Teller (GT) transitions. The extremely high temperatures of the environment cause thermal population of nuclear excited states which enormously increase the number of up- and down- GT transitions between them. It was first shown in [2] that such thermally unblocked transitions completely remove the reaction energy threshold and contribute to a significant enhancement of the cross section for low-energy neutrinos.

A currently accepted method to compute cross sections and rates for neutrino-nucleus reactions relevant for core-collapse simulations is based on a large-scale shell-model (LSSM) diagonalization approach [3–5]. For iron-group nuclei this approach provides a detailed GT strength distribution for the nuclear ground and lowest excited states. However, the LSSM partially employs the Brink hypothesis when treating GT transitions from high-lying excited states. In addition, present computer capabilities allow shell-model calculation only for nuclei with $A \leqslant 65$, whereas neutrino reactions with more massive and neutron-rich nuclei also may play important role in core-collapse supernovae.

In this paper we present results for neutrino-nucleus reactions obtained by an approach based on the thermal quasiparticle random phase approximation. This approach does not rely on Brink's hypothesis and can be applied to a nucleus with an arbitrary mass number. A more detailed discussion of the method and its application to study weak interaction processes with hot nuclei is given in [6–8].

2 Formalism

To compute cross sections and rates for neutrino processes with hot nuclei we apply a method which is based on a statistical formulation of nuclear many-body problem. In this method rather than compute strength distributions for individual states we determine an "averaged" thermal strength function for the GT transition operator

$$S_{\mathrm{GT}}(E,T) = Z^{-1}(T)\sum_{i,f}\exp\Big(-\frac{E_i}{T}\Big)|\langle i|\mathrm{GT}|f\rangle|^2 \times \delta(E-E_f+E_i), \quad (1)$$

where $Z(T)$ is the nuclear partition function, and $\mathrm{GT}_0 = \vec{\sigma} t_0$ for neutral-current reactions and $\mathrm{GT}_\mp = \vec{\sigma} t_\mp$ for charged-current reactions. The zero component of the isospin operator is denoted by t_0, while t_- and t_+ are the isospin-lowering and isospin-rising operators, respectively. The definition of $S_{\mathrm{GT}}(E,T)$ implies that the transition energy $E = E_f - E_i$ can be both positive and negative. The latter corresponds to transitions from higher to lower energy states.

To compute the thermal strength function we apply the formalism which is called the thermo-field dynamics (TFD). The concept of TFD is expounded in [9, 10], and here we just briefly outline the key

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points relevant for the present discussion. In TFD, the thermal average of an operator A is given by the expectation value with respect to a temperature-dependent state $|0(T)\rangle$ which is termed the thermal vacuum. The thermal vacuum is the zero-energy eigenstate of the thermal Hamiltonian, $\mathcal{H} = H - \widetilde{H}$, and it satisfies the thermal state condition

$$A|0(T)\rangle = \sigma_A \mathrm{e}^{\mathcal{H}/2T} \widetilde{A}^\dagger |0(T)\rangle. \tag{2}$$

Here H is the nuclear Hamiltonian and $\widetilde{H}$ is its tilde counterpart acting in the auxiliary Hilbert space; $\widetilde{A}$ is a tilde partner of a physical operator A; σ_A is a phase factor.

Let us assume that we can diagonalize the thermal Hamiltonian by finding its eigenvalues and eigenstates,

$$\mathcal{H} = \sum_k \omega_k(T)(Q_k^\dagger Q_k - \widetilde{Q}_k^\dagger \widetilde{Q}_k), \tag{3}$$

and the thermal vacuum $|0(T)\rangle$ is the vacuum state for the Q_k, $\widetilde{Q}_k$ operators. Then, the thermal GT strength function can be expressed through the matrix elements of the transition operator calculated between the thermal vacuum and 1^+ eigenstates of $\mathcal{H}$

$$S_{\mathrm{GT}}(E,T) = \sum_k \Big\{ |\langle Q_k|\mathrm{GT}|0(T)\rangle|^2 \delta(E_k - E) + |\langle \widetilde{Q}_k|\mathrm{GT}|0(T)\rangle|^2 \delta(E_k + E) \Big\}. \tag{4}$$

Upward transitions from the thermal vacuum to positive-energy (non-tilde) eigenstates correspond to excitation of the nucleus, while downward transitions to negative-energy (tilde) eigenstates describe the decay of thermally excited states.

To diagonalize the thermal Hamiltonian we apply the thermal quasiparticle random phase approximation (TQRPA) [11]. Within the TQRPA the thermal Hamiltonian is diagonalized in terms of thermal phonon operators. Transition strength for positive- and negative-energy phonon states are connected by the principle of detailed balance. For charge-neutral transitions we get

$$\widetilde{S}_k = \exp\Big(-\frac{\omega_k}{T}\Big) S_k. \tag{5}$$

Here, S_k and $\widetilde{S}_k$ are the transition strengths from the thermal vacuum to non-tilde and tilde phonon states

$$\begin{aligned} S_k &= |\langle Q_k \| \mathrm{GT}_0 \| 0(T)\rangle|^2, \\ \widetilde{S}_k &= |\langle \widetilde{Q}_k \| \mathrm{GT}_0 \| 0(T)\rangle|^2. \end{aligned} \tag{6}$$

For charge-changing transitions in a hot nucleus the detailed balance principle takes the form

$$\widetilde{S}_k^{\mp} = \exp\Big(-\frac{\omega_k}{T}\Big) S_k^{(\pm)}, \tag{7}$$

where

$$\begin{aligned} S_k^{(\mp)} &= |\langle Q_k \| \mathrm{GT}_\mp \| 0(T)\rangle|^2, \\ \widetilde{S}_k^{(\mp)} &= |\langle \widetilde{Q}_k \| \mathrm{GT}_\mp \| 0(T)\rangle|^2. \end{aligned} \tag{8}$$

We also note that for charge-neutral reactions, the transition energy E_k is given by the phonon energy, while for charge-changing transitions we have

$$\begin{aligned} E_k^{(\mp)} &= \omega_k \mp (\Delta\lambda_{np} + \Delta M_{np}), \\ \widetilde{E}_k^{(\mp)} &= -E_k^{(\pm)}, \end{aligned} \tag{9}$$

where $\Delta\lambda_{np}$ is the difference between neutron and proton chemical potentials in the parent nucleus and $\Delta M_{np} = 1.293$ MeV is the neutron-proton mass splitting. Thus, within the TQRPA for each $n\to p$ ($p\to n$) upward transition with energy E there is an inverse downward transition $p\to n$ ($n\to p$) with energy $-E$ and the respective transition probabilities are connected by (7).

In our study we use a phenomenological nuclear Hamiltonian of the quasiparticle-phonon nuclear model (QPM) [12]. It consists of a spherically symmetric Woods-Saxon mean-field potential for protons and neutrons, BCS pairing interaction and residual separable multipole and spin-multipole particle-hole interactions in the isoscalar and isovector channels. All parameters in the QPM Hamiltonian are fitted to reproduce experimental data at zero temperature.

3 Results

The TQRPA formalism is employed to study thermal effects for neutrino reactions with two sample nuclei, ^{56}Fe and ^{82}Ge. The iron isotope is among the most abundant nuclei at the early stage of collapse, while the neutron-rich germanium isotope can be considered as the average nucleus at later stages.

3.1 Inelastic neutrino scattering

As the first example we consider inelastic neutrino-nucleus scattering

$$\nu + A(Z,N) \to \nu' + A(Z,N). \tag{10}$$

For neutrino energies $E_\nu \leq 20$ MeV, which are typical for supernova neutrinos, this process is dominated by charge-neutral Gamow-Teller (GT_0) transitions and forbidden transitions contribute less than 10% [7].

The GT_0 distributions in ^{56}Fe and ^{82}Ge are displayed in Fig. 1 for the nuclear ground state and at three finite temperatures relevant in the supernova context. Since Brink's hypothesis is not valid within the TQRPA, the GT_0 strength for upward transitions exhibit a temperature dependence. Namely, due to vanishing of pairing correlations and thermal weakening of the residual particle-hole interaction, the GT_0 resonance in both nuclei move to lower energies. Moreover, the thermal smearing of the nuclear Fermi surfaces unblock low-energy GT_0 transitions which are Pauli blocked at $T = 0$. These thermal effects are not observed within shell-model calculations which are based on Brink's hypothesis [4, 5]. It is

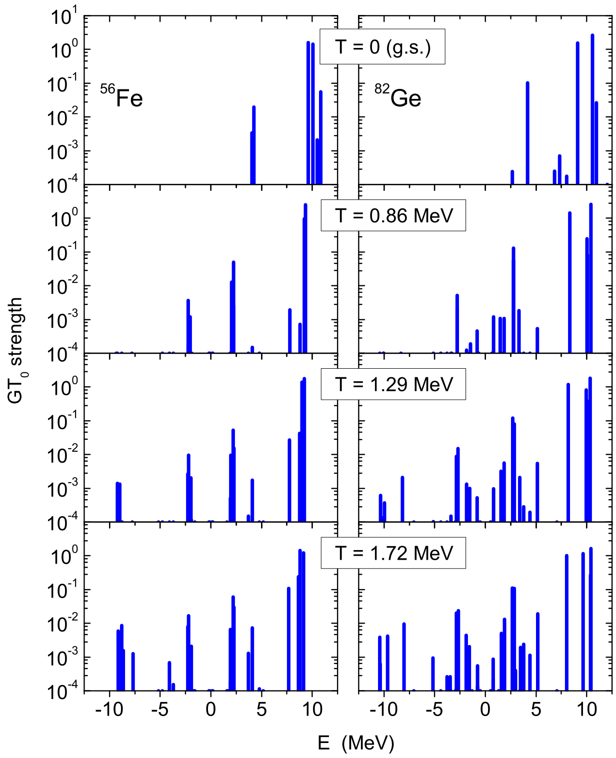

Figure 1. (Color oline) Temperature evolution of GT_0 distributions in ^{56}Fe (left panel) and ^{82}Ge (right panel).

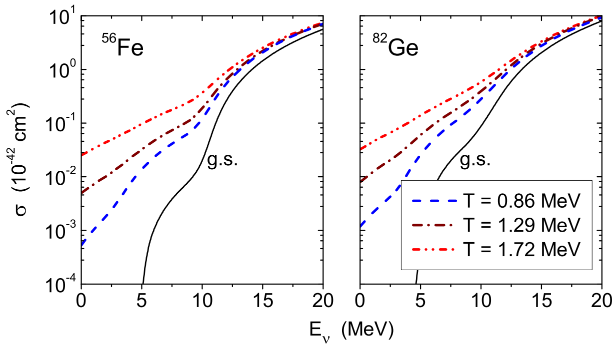

Figure 2. (Color online) Cross sections for inelastic neutrino scattering on ^{56}Fe (left panel)and ^{82}Ge (right panel).

clear that both these effects should favour neutrino inelastic scattering. Furthermore, temperature rise increases the population of excited states and magnifies the strength of downward transitions in accordance with the detailed balance relation (5). Note, that the downward strength at $E \approx -9$ MeV is attributed to the de-excitation of the GT_0 resonance. It is important that the temperature-induced shift of the upward GT_0 strength to lower energies makes it easier to thermally populate these states which, in turn, leads to an enhanced downward strength as well. In [7], this effect is demonstrated by considering the running sums for GT_0 downward strength obtained using and without using the Brink hypothesis.

Given the GT_0 strength distributions, we calculate the cross section for inelastic neutrino scattering

$$\sigma(E_\nu, T) = \frac{(G_F g_A)^2}{\pi} {\sum_k}' (E_\nu - \omega_k)^2 S_k + \frac{(G_F g_A)^2}{\pi} \sum_k (E_\nu + \omega_k)^2 \exp\left(-\frac{\omega_k}{T}\right) S_k, \tag{11}$$

where G_F and g_A are the Fermi and axial weak coupling constants, respectively. The sum in the first term runs over 1^+ thermal phonon states with the positive energy $\omega_i < E_\nu$, while the second sum accounts for downward transition from excited states.

In Fig. 2, ground-state cross sections are compared with those calculated at three core-collapse temperatures. As seen from the figure, the ground-state gap in the cross section disappears at $T \neq 0$. This effect is caused by the fact that de-excitation of thermally excited states contributes to the cross section at all neutrino energies. For $E_\nu < 5$ MeV, this downward contribution dominates inelastic scattering and enhances the cross sections by up two orders of magnitude when the temperature rises from 0.86 MeV to 1.72 MeV. At neutrino energies $5 < E_\nu < 10$ MeV, both thermally unblock negative- and low-energy GT_0 transition contribute to the cross section enhancement. With increasing neutrino energies the cross sections at different temperatures converge and for $E_\nu > 10$ MeV the excitation of the GT_0 resonance dominate the cross section.

Comparing our finite-temperature cross sections for ^{56}Fe with those from [5] we find that at low neutrino energies they are several times larger than the shell model results. In [7], it is shown that the discrepancy stems from the violation of Brink's hypothesis within the TQRPA which, in turn, leads to a larger strength for negative- and low-energy thermally unblocked GT_0 transitions.

3.2 Neutrino-antineutrino pair emission

Charge-neutral GT_0 transitions also dominate in the decay of a thermally excited nucleus to a lower state via neutrino pair emission

$$(A, Z)^* \to (A, Z) + \nu_k + \overline{\nu}_k. \tag{12}$$

Here, the index $k = e,\ \mu,\ \tau$ corresponds to three neutrino flavours. It was first recognized by Pontecorvo, that this process may be a powerful mechanism for the energy loss by stars [13].

The decay rate includes summation over all possible GT_0 transitions from excited to lower energy states properly weighted by the Boltzmann factor [2]

$$\Lambda = 3\lambda_0 Z^{-1}(T) \sum_{i,f} \exp\left(-\frac{E_i}{T}\right) (\Delta E_{i,f})^5 B(GT_0)_{if}, \tag{13}$$

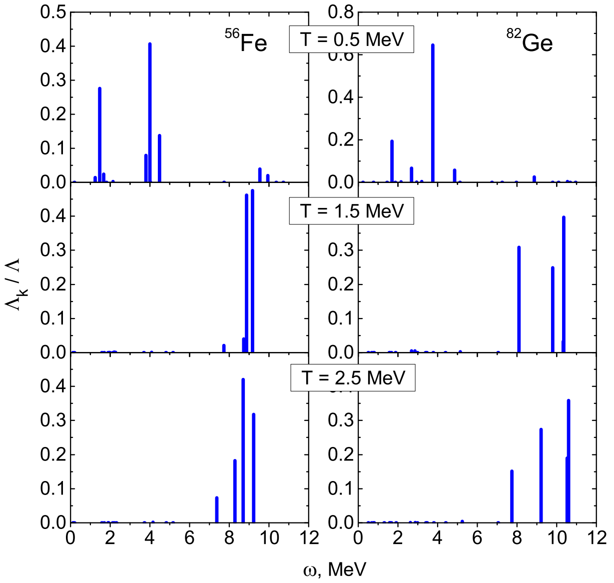

Figure 3. (Color online) Normalized partial decay rates as functions of the transition energy in ^{56}Fe (left panel)and ^{82}Ge (right panel).

where $\Delta E_{if} = E_i - E_f$ is the transition energy, $\lambda_0 = G_F^2 g_A^2/60\pi^3\hbar^7c^6 \approx 1.72 \times 10^{-4}\,\mathrm{s}^{-1}\mathrm{MeV}^{-5}$ and the factor of 3 accounts for the three possible neutrino flavours that can be produced in the decay.

Within the TQRPA the initial equilibrium state of hot nucleus is described by the thermal vacuum and neutrino-pair emission causes downward transitions to negative-energy phonon states. Therefore, for the partial decay rate we can write

$$\Lambda_k = 3\lambda_0\omega_k^5\widetilde{S}_k. \tag{14}$$

or, taking into account the principle of detailed balance (5),

$$\Lambda_k = 3\lambda_0\omega_k^5 \exp\Big(-\frac{\omega_k}{T}\Big)S_k. \tag{15}$$

The partial decay rate Λ_k determines the spectrum of emitted neutrino pairs $\omega_k = E_\nu + E_{\overline{\nu}}$. The Boltzmann factor in (15) suppresses high-energy pairs, while the factor ω_k^5 and increasing temperature favor them.

In Fig. 3, we show normalized partial decay rates computed for ^{56}Fe and ^{82}Ge at temperatures T = 0.5, 1.5, and 2.5 MeV. Although the details vary, two common features are observed for both nuclei. Namely, low-energy neutrino pairs dominate the spectrum at $T = 0.5$ MeV. These pairs are produced due to thermally unblocked GT$_0$ transitions between single-particle levels close to Fermi surfaces in the nucleus. At higher temperatures, however, the de-excitation of the GT$_0$ resonance completely dominate the process. As a result, in both nuclei the spectrum of emitted pairs shifts to higher energies.

Performing summation over all negative-energy states we obtain the decay rate

$$\Lambda = \sum_k \Lambda_k \tag{16}$$

and the energy emission rate

$$P = \sum_k \omega_k\Lambda_k. \tag{17}$$

In the left and middle panels of Fig. 4 we show the decay rates Λ and the energy emission rates P as functions of temperature. As expected, both the rates rapidly increases with the temperature. From the above discussion it is clear that the main reason for that is the thermal population and the subsequent decay of the GT$_0$ resonance. We also note that the computed energy emission rates for ^{56}Fe are very close to those obtained in [2] within the independent single-particle shell-model.

Given the energy emission and the total decay rates we compute a mean energy of emitted neutrino pairs, $\langle E\rangle = P/\Lambda$. The results are shown on the right panel of Fig. 4. As seen from the plot, $\langle E\rangle$ rises rapidly with temperature till $T \approx 1.0$ MeV. After that, the de-excitation of the GT$_0$ resonance dominates the process and $\langle E\rangle$ becomes nearly independent of temperature, $\langle E\rangle \approx 9 - 10$ MeV.

3.3 Neutrino absorption

We also consider neutrino absorption reaction

$$\nu_e + A(Z,N) \to A(Z+1,N-1) + e^- \tag{18}$$

which is dominated by charge-changing Gamow-Teller (GT$_-$) transitions at $E_\nu \leq 20$ MeV. In Fig. 5, the GT$_-$ strength distributions for ^{56}Fe and ^{82}Ge are shown for the ground-state and at three values of temperature. For the GT$_-$ distribution in ^{56}Fe we observe the same thermal effects as for the strength of charge-neutral GT$_0$ transitions in Fig. 1. Namely, temperature-induced weakening of the pairing and of the residual interaction shifts the resonance energy to lower energies by about 1.5 MeV. As temperature increases, GT$_-$ transitions, which are blocked at low temperatures due to closed proton subshells, become thermally unblocked due to thermal smearing of the nuclear Fermi surface. Similarly, neutrons which are thermally excited to higher orbitals can undergo GT$_-$ transitions. Because of thermally unblocked transitions, some GT$_-$ strength appears below the zero-temperature reaction threshold, including negative energies. For the neutrino absorption this threshold is given by $Q = M_f + m_ec^2 - M_i$, where $M_{i,f}$ are the masses of the parent and daughter nuclei. In ^{56}Fe, the most part of the downward strength is located around $E \approx -4.1$ MeV. According to the principle of detailed balance (7), this strength originates from a transition inverse to the GT$_+$ resonance. Referring

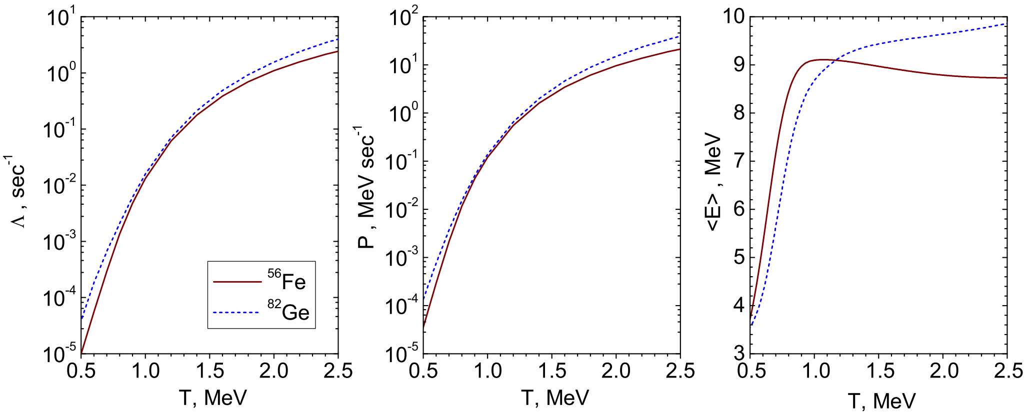

Figure 4. (Color online) Total decay rates (left panel) and energy emission rates (middle panel) as functions of temperature. The mean energy of $\nu\bar{\nu}$ pairs is given on the right panel.

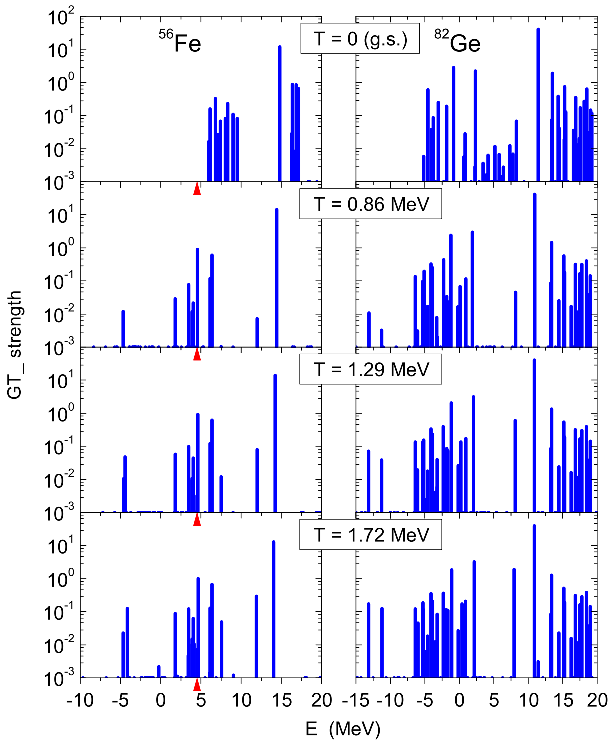

Figure 5. (Color online) Temperature evolution of GT$_-$ distributions in ^{56}Fe (left panel) and ^{82}Ge (right panel). The arrow indicates the ground-state reaction threshold for neutrino absorption on ^{56}Fe ($Q = 4.56$ MeV).

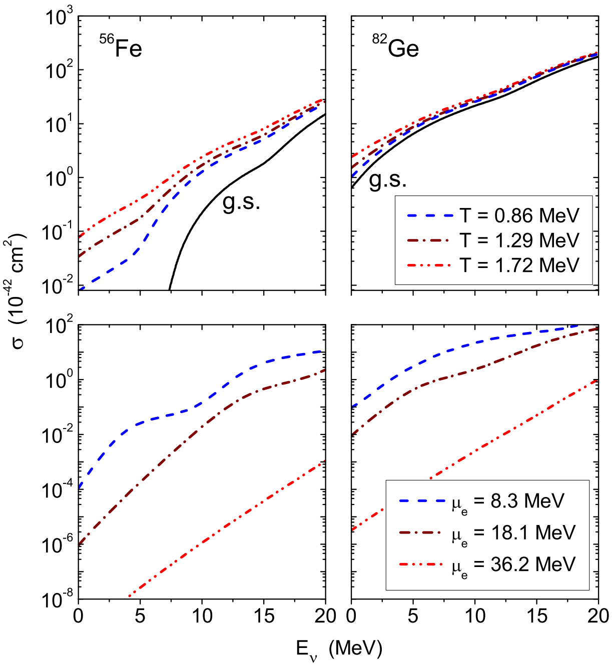

Figure 6. (Color online) Cross sections for neutrino absorption on ^{56}Fe (left panels) and ^{82}Ge (right panel). The cross sections are computed without considering (upper panels) and considering (lower panels) electron blocking in the final state.

to the figure, for the GT$_-$ strength in ^{82}Ge thermal effects are less important. Due to negative Q-value ($Q = -4.71$ MeV), the GT$_-$ strength in ^{82}Ge contains a downward component even at $T = 0$. Temperature rise yields some new strength below -10 MeV, but its contribution is relatively small. Moreover, since there is no neutron pairing correlation in ^{82}Ge, the GT$_-$ resonance is weakly sensitive to thermal effects and with increasing the temperature up to $T = 1.72$ MeV its excitations energy is only lowered by ~ 0.5 MeV.

Taking into account the detailed balance principle (7) and neglecting the electron rest mass in the final state, the cross section for neutrino absorption can be written as

$$\sigma(E_\nu, T) = \frac{(G_F g_A \cos\theta_C)^2}{\pi} \times \Big\{ \sum_k (E_\nu - E_k^{(-)})^2 S_k^{(-)} F(Z+1, E_e) + \sum_k (E_\nu + E_k^{(+)})^2 \exp\Big(-\frac{\omega_k}{T}\Big) S_i^{(+)} F(Z+1, E_e) \Big\} \tag{19}$$

Here, θ_C is the Cabbibo angle and the function $F(Z, E_e)$ corrects for the Coulomb distortion of the final electron wave function.

In the upper panels of Fig. 6, the ground-state cross sections are compared with thermal ones. For ^{56}Fe, the enhancement due to thermal excitations is notable for $E_\nu \leq 15$ MeV. For energies $E_\nu < 5$, the enhancement is mostly caused by downward GT_- transitions from excited states. The role of downward transitions decreases with increasing neutrino energy and for energies $5 < E_\nu < 15$ MeV, thermal effects on the GT_- resonance and its low-energy tail become important for the cross section enhancement. In contrast, thermal effects on the $^{82}\mathrm{Ge}(\nu_e, e^-)$ reaction are noticeably milder than in the previous case and the temperature rise from $T = 0$ to 1.72 MeV enhances the low-energy cross section only by a factor of about four.

In the supernova environment nuclei are surrounded by degenerate electron gas and, therefore, neutrino absorption can be hindered by Pauli blocking of the electron in the final state. We investigate this effect by introducing a blocking factor $(1-f(E_e))$ into Eq. (11), where $f(E_e)$ is the Fermi-Dirac distribution at temperature T and chemical potential μ_e. Following [3], we calculate cross sections with the blocking factor defined at three different sets of temperature and chemical potential (μ_e in MeV): $(T, \mu_e) = (0.86, 8.3), (1.29, 18.1)$, and $(1.72, 36.2)$. The calculated cross sections are shown in the lower panels of Fig. 6. Due to electron blocking, neutrino cross sections are drastically reduced and their values become significantly smaller than those for the inelastic neutrino scattering in Fig. 2. Moreover, as the chemical potential increases faster than the temperature, the cross sections decreases with temperature.

The calculated neutrino absorption cross sections for ^{56}Fe by two to three orders of magnitude exceed the shell-model values [3] at low neutrino energies. This discrepancy is caused by the violation of Brink's hypothesis within the TQRPA and due to the fact that the shell-model GT_- is more strongly fragmented owing to multinucleon correlations.

4 Conclusion

We have studied the effects of finite temperature on the neutrino-nucleus reactions in supernova environment. We found that neutrino inelastic scattering and absorption cross sections are enhanced at finite temperature due to thermally unblocked negative- and low-energy GT transitions from excited states. However, the absorption cross sections are drastically reduced if the electron blocking in the final state is taken into account. All these observations are in line with the shell-model studies [3–5]. However, within the TQRPA the thermal enhancement of cross sections appears to be more significant than those observed in the LSSM calculations. One of the possible reason of this discrepancy is that shell-model calculations are partially based on Brink's hypothesis when treating GT transitions from excited states. The other possible reason is that the TQRPA underestimate multinucleon correlations which are responsible for the GT strength fragmentation. This requires to go beyond the TQRPA and take into account the coupling of thermal phonons with more complex (e.g., two-phonon) configurations.

We also studied the emission of neutrino pairs from the de-excitation of thermally excited states. It was found that temperature increase leads to a considerable enhancement of the emission rates. According to our calculations, this enhancement is mainly due to thermal population of the GT_0 resonance.

Recently, the influence of the external magnetic field on nuclear properties in neutron star was studied in [14, 15]. In particular, it was shown that under supernova conditions the strong magnetic field may affect the single-particle levels in nuclei. It would be interesting to study how the interference between finite temperature and magnetic field influence cross sections and rates for supernova weak interaction processes.

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