

A New Model for the Mechanochemical Corrosion of a Thin Spherical Shell

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Abstract. A pressurized thin-walled spherical shell under the conditions of double-sided uniform corrosion is considered. The rates of corrosion are supposed to depend linearly on the effective stress. Previous solutions for thin shells obtained by other authors reflect only the effect of differential pressure (i.e., the difference between internal and external pressure). The model proposed here allows to include also the effect of hydrostatic pressure on the durability of the shell under the corrosion conditions.

1 Introduction

Corrosion is a process of material destruction that occurs as a result of chemical interaction between a material and its environment. The most common type of corrosion damages the entire surface of the material at about the same rate, and, as a consequence, results in a failure. If a structural element, immersed in an aggressive environment, is subject to mechanical stresses, then the so-called mechanochemical effect may occur (i.e. corrosion accelerated by stress) [1, 2].

Many scholars have studied the stress state of structural members under mechanochemical corrosion conditions. In the case when the corrosion is understood as a uniform dissolution, there are a few works reporting closed-formed expressions, see, e.g., [3–9]. Stress-assist chemical reactions sustained by diffusion in deformable solids were modeled in [10–13]. Initial boundary value problems with evolving boundaries have to be solved for other types of wear [14–16] and in other areas [17]. Because of the complexity of such problems, most of the research was done using numerical methods.

In this paper an analytical solution is presented for a thin-walled spherical shell subject to internal and external pressure under the conditions of double-sided uniform corrosion. The rates of corrosion are supposed to depend linearly on the effective stresses. Previous solutions for thin shells [4] do not depend on the internal and external pressures themselves, but only on their difference (see [18]). However, the hydrostatic pressure can affect the corrosion rate [1]. Accurate solutions for the double-sided corrosion of shells under internal and external pressure based on the solutions of Lamé problems for thick cylinder and sphere are rather cumbersome [19–25]. The model proposed here is simpler but, it allows to take into account the effect of the hydrostatic pressure on the durability of the thin shells under the mechanochemical corrosion.

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2 Problem formulation

Consider a linearly elastic thin-walled sphere with the inner r and outer R radii subjected to dead internal p_r and external p_R pressure (figure 1). The sphere is uniformly corroding from the inside and the outside with the dissolution rates v_r and v_R , respectively. The corrosion process affects the shell sizes: the inner radius increases while the outer radius decreases from the initial time t_0 values denoted respectively by r_0 and R_0 .

According to [1, 3], the rates of corrosion on the inner and outer surfaces are:

$$v_r = \frac{dr}{dt} = a_r + m_r \sigma(r, R, p_r, p_R), \quad v_R = -\frac{dR}{dt} = a_R + m_R \sigma(r, R, p_r, p_R), \quad (1)$$

where the constants a_r , a_R , m_r , and m_R are determined experimentally [1]. In general, they are different under tension and compression; $\sigma(r, R, p_r, p_R)$ is the maximum principal stress assumed to be identical through the entire thickness.

We suppose that at any time t the maximum principal stress σ (which is a circumferential stress) depends on the instantaneous values of the inner $r(t)$ and outer $R(t)$ radii of the shell and on the given constant pressures p_r and p_R . The problem is to find the time dependence of the maximum stress at any $t > t_0$.

Consider a spherical coordinate system (ρ, θ, ϕ) with the origin identical to the center of the hollow sphere. In previous solutions obtained by other authors (for example, [4]), σ was chosen as

$$\sigma(r, R, p_r, p_R) = \sigma_{\phi\phi} = \sigma_{\theta\theta} = \frac{(p_r - p_R)r_c}{2h}, \quad (2)$$

where $r_c = (R+r)/2$ is the radius of the middle surface of the spherical shell, $h = R-r$ is the thickness of the shell. Naturally, the solution derived using (2) does not depend on the pressure values p_r and p_R , but only on the difference $p_r - p_R$. However, it is known that the hydrostatic pressure $p = \min\{p_r, p_R\}$ could affect the corrosion kinetics [1]. To take into account the effect of the hydrostatic pressure p we suppose that the effective stress is defined by the more accurate formula [26]:

$$\sigma = \sigma(r, R, p_r, p_R) = \sigma_{\phi\phi} = \sigma_{\theta\theta} = \frac{(p_r - p_R)r_c}{2h} - q, \quad (3)$$

where $q = (p_r + p_R)/2$.

In both equations (2) and (3), the radius r_c is assumed to remain constant, while the thickness h is changing with the time.

3 Problem solution

Adding Eqs. (1)₁ and (1)₂ (with negative signs), we obtain

$$\frac{d(R-r)}{dt} = \frac{dh}{dt} = -(a + m\sigma), \quad (4)$$

where $a = a_r + a_R$, $m = m_r + m_R$.

Differentiating (3) with respect to t and using (4) yields the ordinary differential equation:

$$\frac{d\sigma}{dt} = \frac{2(a + m\sigma)(\sigma + q)^2}{(p_r - p_R)r_c}. \quad (5)$$

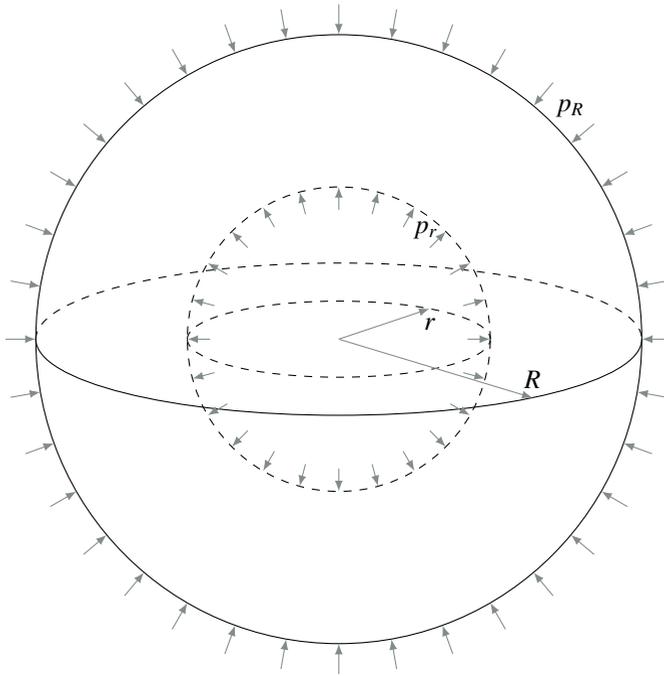


Figure 1. The model of a pressurised hollow sphere

The initial condition to be satisfied at $t = t_0$ is

$$\sigma|_{t=t_0} = \sigma_0 = \frac{(p_r - p_R)r_c}{2h_0} - q,$$

where $h_0 = R_0 - r_0$. The solution of (5), with the above initial condition is

$$t = t_0 + \frac{(p_r - p_R)r_c}{2(a - mq)} \left(\frac{m}{a - qm} \ln \frac{(a + m\sigma)(\sigma_0 + q)}{(a + m\sigma_0)(\sigma + q)} + \frac{\sigma - \sigma_0}{(\sigma + q)(\sigma_0 + q)} \right).$$

It is known that a higher hydrostatic pressure may lead to an increase in the corrosion pitting growth probability. Note that for spherically and cylindrically symmetric problems, analytical solutions can often be obtained [27–29, etc.]. In the case of a more complex geometry of the solid (especially, made of nonlinear materials), it is necessary to use numerical methods [30–32]. Some results concerning the stress distribution near corrosion pits in spherical shells are presented in [33, 34].

4 Comparison with other solutions

Let us compare calculation results obtained by the proposed model (using Eq. (3) as the effective stress) with the results based on both the classical thin shell theory formula (Eq. (2) is used as the effective stress) and the Lamé’s formula for a thick pressurised spherical shell [23, 24]. In figure 2 the dependencies of $|\sigma|$ on t are demonstrated for different corrosion models of the spherical shell with the initial radii $R_0 = 100[l_c]$ and $r_0 = 96[l_c]$; $t_0 = 0$. Here, l_c is a certain length unit. Other corrosion

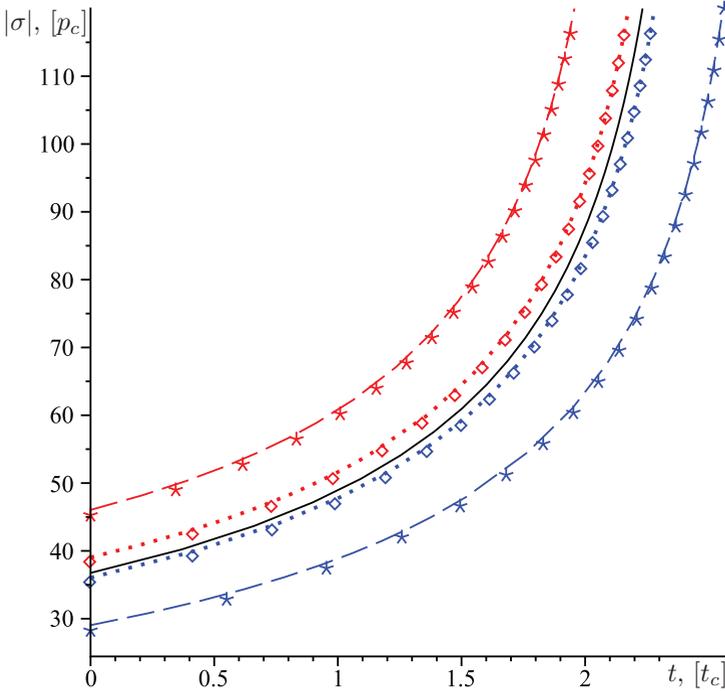


Figure 2. Dependencies of $|\sigma|$ on the time t for various pressure values

rate parameters are $a_r = a_R = 0.16[l_c/t_c]$ and $m_r = m_R = 0.008[l_c/(t_c p_c)]$. Here and below, t_c is an appropriate unit of time; p_c is a given unit of stress.

In order to observe the effect of hydrostatic pressure $p = \min\{p_r, p_R\}$, four sets of the internal and external pressure values have been used:

- $p_r = 3[p_c]$, $p_R = 0$ (blue dotted line and diamonds);
- $p_r = 9[p_c]$, $p_R = 6[p_c]$ (blue dashed line and asterisks);
- $p_r = 0$, $p_R = 3[p_c]$ (red dotted line and diamonds);
- $p_r = 6[p_c]$, $p_R = 9[p_c]$ (red dashed line and asterisks).

These sets are chosen such that, despite various values of the pressures, the absolute value of the pressure difference remains constant: $|p_r - p_R| = 3[p_c]$.

It is obvious that, under an effective stress given by (2), the dependencies of $|\sigma|$ on the time t for all the above sets of pressure values are the same. In other words, for the given parameters, the classical thin shell theory formula provides only one curve; it is the black curve in figure 2.

For the other two models we have different curves for each of the considered sets of pressure values. Red curves in figure 2 correspond to the case when $p_r - p_R < 0$, while blue ones correspond to positive differences, $p_r - p_R > 0$. Dotted and dashed curves indicate the dependencies $|\sigma(t)|$ obtained in [23, 24] by the means of the Lamé's formula for hydrostatic pressures $p = 0$ and $p = 6$, respectively. Diamonds and asterisks are used for the model proposed here at the corresponding pressure values.

The figure shows that the curves for all three models are close to each other at zero hydrostatic pressure, $p = \min\{p_r, p_R\} = 0$ (compare solid line, dotted lines and diamonds). However, the difference between the classical thin shell model (solid line) and two other models (dashed lines and stars) increases with the growing p . At the same time, the curves corresponding to the accurate formulas [23] and to the proposed thin shell model are hardly different from each other at any pressure values. A good agreement between these two models is also observed even for a thick shell (with thickness relation $h/r_c = 1/5$). In the framework of our accurate models it was shown that the durability of the sphere increases as p grows if $p_r - p_R > 0$, and decreases if $p_r - p_R < 0$.

5 Conclusion

A new mathematical model for the mechanochemical corrosion of thin spherical shells was defined. A linearly elastic sphere subject to internal p_r and external p_R pressure, and uniformly corroding from the inside and outside has been considered. The rates of corrosion are supposed to be linearly dependent on the effective stress. An analytical solution of the problem has been obtained.

The computational results for the classical thin shell model and for the proposed here refined model are very close to each other for zero hydrostatic pressure $p = \min\{p_r, p_R\} = 0$, but the difference between them grows as p increases. It has been shown that the durability of the sphere increases as p grows if $p_r - p_R > 0$, and decreases if $p_r - p_R < 0$. The solutions based on the Lamé's formula and that proposed here are hardly different from each other at any pressure values.

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