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MULTI-CRITERIA OPTIMIZATION OF REFINERY

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Abstract. Vector optimization of refinery is discussed. As the target function, a multi-level hierarchy criterion convolution is used. This convolution is formed following the recurrent procedure based on a scalar invariant representing a linear combination of Hölder norms: the first order norm and the sup-norm. The issue of matrix representation of the suggested invariant has been analyzed; the procedure of building the respective matrix structure has been developed. The approach has been implemented in a respective package of application software programs.

Refinery in general may be depicted as an oriented net graph with node set $\mathbf{K} = \mathbf{U} \cup \mathbf{S}$, where $\mathbf{U}$ is the set of plants and $\mathbf{S}$ is the set of mix pools. The set of arches depicts the flows of petroleum products, $\mathbf{I}$ is the set of petroleum products. Each product can be represented by one or several flows, wherein x_j is the intensity of j-th flow measured as the mass of the respective product for the period of modeling. Let us make for each node the material balance equations, energy balance equations, for supplies of incoming raw materials and semi-finished products, for shipments of products, for node loading [1]:

$$\sum_{j\in J_i^+} x_j - \sum_{j\in J_i^-} x_j = 0,\ \ i\in\mathbf{I},\ P_i^- \le \sum_{j\in J_i^+} x_j \le P_i^+,\ \ i\in\mathbf{I}^+,\ V_i^- \le \sum_{j\in J_i^-} x_j \le V_i^+,\ \ i\in\mathbf{I}^-,$$
$$\sum_{j\in J_k^+} x_j - \sum_{j\in J_k^-} x_j = 0,\ k\in\mathbf{K},\ L_k^- \le \sum_{j\in J_k^+} x_j \le L_k^+,\ \ k\in\mathbf{K}, \tag{1}$$

where $\mathbf{J}_i^+(\mathbf{J}_i^-)$ is the set of flows making up (consuming) i-th product; $\mathbf{I}^+$ is the set of incoming products; $P_i^-(P_i^+)$ are the limits of supply; $\mathbf{I}^-$ is the set of commodity products; $V_i^-(V_i^+)$ are the limits of shipment; $\mathbf{J}_k^+(\mathbf{J}_k^-)$ is the set of incoming (outgoing) flows of k-th node; where $\mathbf{L}_k^-, \mathbf{L}_k^+$ are the limits of loading (potential) of k-th node. Besides, requirements for basic general economic indices (profit, coverage, internal rate of return) and technical-and-economic refinery-specific indices (yield of light petroleum products, oil conversion ratio) are introduced. One of them, for instance, profit, is considered as the maximized target function:

$$\sum_{i\in I_2} c_i^+ z_i - \left(\sum_{i\in I_0} c_i^- \sum_{j\in J_i} x_j + \sum_{n\in N} \beta_n \sum_{j\in J_n} x_j + C \right) = \sum_{j\in J} \rho_j x_j \to \max \tag{2}$$

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where z_i is the yield of i-th commodity product; c_i^+ is the price of i-th product; c_i^- is the price of incoming i-th semi-product; β_n is the factor of costs at n-th plant; $\mathbf{I}_2$ is the set of names of commodity products; ρ_j is the specific coverage of j-th flow.Thus, we receive a linear programming problem with a system of limitations (1) that is usually incompatible. Hence, it is expedient to define the optimization problem as a multi-criteria problem, which is substantiated by the industrial oil refinery practice [1, 2]. For convenience of further use, let us write it in a generalized canonical form:

$$\max\left\{(\mathbf{c},\mathbf{x}) \middle| \mathbf{A}\mathbf{x}=\mathbf{b}, \mathbf{x}\geq 0\right\}. \tag{3}$$

If vector $\mathbf{b}$ is considered a collection of target indices rather than the right part of limitations (1), then problem (3) should be modified as follows:

$$\max\left\{(\mathbf{c},\mathbf{x})-\mu\Phi(\boldsymbol{\delta}) \mid \boldsymbol{\delta}=\mathbf{W}^{-1}(\mathbf{A}\mathbf{x}-\mathbf{b}), \mathbf{x}\geq 0\right\}, \tag{4}$$

where $\boldsymbol{\delta}$ is the vector of weighted relative deviations of calculated values of indices from the prescribed values; $\Phi(\boldsymbol{\delta})$ is the fine for the deviation; μ is a number that is sufficiently large to ensure priority of achieving target indices over maximization of the initial target function; $\mathbf{W}=(\mathbf{E}\mathbf{b})\mathbf{G}^{-1}$, $\mathbf{G}$ is the scalar matrix, its elements being weights, e.g., expert assessments of the priority of respective indices.

As $\Phi(\boldsymbol{\delta})$, it is suggested to use the linear combination of Hölder norms L_p of orders $p=1$ and $p=\infty$:

$$\Phi(\boldsymbol{\delta},\alpha)=\alpha\|\boldsymbol{\delta}\|_1+(1-\alpha)\|\boldsymbol{\delta}\|_\infty, \tag{5}$$

where $\alpha\in[0,1]$ is the parameter; $\|\boldsymbol{\delta}\|_1$ is the sum of absolute values of vector coordinates; $\|\boldsymbol{\delta}\|_\infty$ is the maximal value among the absolute values of vector coordinates – the sup-norm.

Criterion (5) is easily introduced into the linear problem:

$$\begin{gathered}(\mathbf{c},\mathbf{x})-\mu\left\{\alpha\left[\mathbf{I},\left(\boldsymbol{\delta}^-+\boldsymbol{\delta}^+\right)\right]+(1-\alpha)\delta^*\right\}\rightarrow\max,\\ \begin{cases}\mathbf{A}\mathbf{x}+\mathbf{W}\boldsymbol{\delta}^--\mathbf{W}\boldsymbol{\delta}^+=\mathbf{b},\\ \mathbf{E}\boldsymbol{\delta}^--\mathbf{I}\delta^*\leq 0,\\ \mathbf{E}\boldsymbol{\delta}^+-\mathbf{I}\delta^*\leq 0,\\ \mathbf{x}\geq 0,\ \boldsymbol{\delta}^-\geq 0,\ \boldsymbol{\delta}^+\geq 0,\ \delta^*\geq 0\end{cases}\end{gathered} \tag{6}$$

where $\boldsymbol{\delta}^-(\boldsymbol{\delta}^+)$ is the vector of weighted relative deviations of calculated values of indices from the prescribed values; $\mathbf{I}$ is the vector of identity elements; $\delta^*=\|\boldsymbol{\delta}\|_\infty$.

The whole set of conditions, limitations, indices of an applied practical problem usually may be divided into groups, subgroups and so on, obtaining a hierarchy structure of a tree-graph, which leaves are the coordinates of vector $\boldsymbol{\delta}$, and nodes Δ are convolutions of low-level criteria. The convolution (5) may be considered as an invariant of a generalized hierarchic criterion, which node is:

$$\Delta_{v,n}^{\ell}=w_{v,n}^{\ell}\Phi\left(\alpha_{v,n}^{\ell},\ \Delta_{n,1}^{\ell-1},\ \Delta_{n,2}^{\ell-1},\ \ldots\right),\ \Delta^0=\delta, \tag{7}$$

where $\Delta_{v,n}^{\ell}$ is the n-th criterion of the ℓ-th level involved in formation of the v-th criterion of level $(l+1)$; $w_{v,n}^{\ell}$, $\alpha_{v,n}^{\ell}$ are the regulated parameters.

To formulate a problem with such a criterion, it is necessary to include into the model the additional variable groups Δ and respective equations and inequalities interrelating these variables and relating them with the basic variables. To form the index $\Delta_{l\nu}^{(\ell+1)}$, let us add the following block of conditions to the initial problem (3):

$$\left(\begin{array}{cc:c} -\mathbf{I}_\nu^\ell & \mathbf{E}_\nu^\ell & 0 \\ 0 & -\mathbf{W}_\nu^\ell & \mathbf{A}_\nu^\ell \\ \hdashline 0 & 0 & \mathbf{D}_\nu^\ell \end{array}\right)\left(\begin{array}{c} \mathbf{\Delta}_\nu^{*\ell} \\ \mathbf{\Delta}_\nu^\ell \\ \hdashline \mathbf{\Delta}_\nu^{\ell-1} \end{array}\right) \begin{array}{c} \leq \\ = \\ \end{array} \left(\begin{array}{c} 0 \\ 0 \\ \hdashline 0 \end{array}\right), \tag{8}$$

where $\mathbf{\Delta}_\nu^l = \left(\Delta_{\nu 1}^\ell,\ \Delta_{\nu 2}^\ell,\ \ldots,\ \Delta_{\nu|\nu|}^\ell\right)^T$; $\Delta_\nu^{*\ell} = \max\left\{\Delta_{\nu k}^\ell \mid k = 1, 2, \ldots |\nu|\right\}$; $\mathbf{I}_\nu^\ell$ is the vector of identity elements; $\mathbf{E}_\nu^\ell$ is the identity matrix; $\mathbf{W}_\nu^\ell$ is the matrix of weight factors of section indices; $\mathbf{\Delta}_\nu^{\ell-1} = \left(\widehat{\mathbf{\Delta}}_1^{\ell-1},\ \widehat{\mathbf{\Delta}}_2^{\ell-1},\ \ldots,\ \widehat{\mathbf{\Delta}}_{|\nu|}^{\ell-1}\right)^T$, $\widehat{\mathbf{\Delta}}_\nu^{\ell-1} = \left(\mathbf{\Delta}_\nu^{*\ell-1}, \mathbf{\Delta}_\nu^{\ell-1}\right)^T$ are the low-level vectors forming the current convolution; $\mathbf{A}_\nu^l$ is the matrix of factors with which the low-level indices are included in the linear combination of the convolution; $\mathbf{D}_\nu^l$ is the block-diagonal matrix.

$$\mathbf{A}_\nu^l = \begin{pmatrix} \mathbf{\Lambda}_\nu^l & 0 & \ldots & 0 \\ 0 & \mathbf{\Lambda}_\nu^l & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & \mathbf{\Lambda}_\nu^l \end{pmatrix}, \quad \mathbf{D}_\nu^l = \begin{pmatrix} \mathbf{M}_{\nu 1}^{l\text{-}1} & 0 & \ldots & 0 \\ 0 & \mathbf{M}_{\nu 2}^{l\text{-}1} & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & \mathbf{M}_{\nu|\nu|}^{l\text{-}1} \end{pmatrix},$$

where $\mathbf{\Lambda}_\nu^l = \left(1 - \alpha_\nu^\ell,\ \alpha_\nu^\ell,\ \ldots,\ \alpha_\nu^\ell\right)$; $\mathbf{M}_{\nu k}^{l\text{-}1}$ is the matrix corresponding to the k-th low-level index, $k = 1, 2, \ldots, |\nu|$.

Each of matrices $\mathbf{M}_{\nu k}^{l\text{-}1}$ has the same structure as the parent matrix under consideration:

$$\mathbf{M}_{\nu k}^{l\text{-}1} = \left(\begin{array}{cc:c} -\mathbf{I}_k^{\ell-1} & \mathbf{E}_k^{\ell-1} & 0 \\ 0 & -\mathbf{W}_k^{\ell-1} & \mathbf{A}_k^{\ell-1} \\ \hdashline 0 & 0 & \mathbf{D}_k^{\ell-1} \end{array}\right).$$

It may be considered as a matrix representation of invariant (7) which is used in implementation of the recurrent procedure of forming a matrix model with a generalized hierarchic criterion.

The described approach has been implemented in the author's software product RMOS (refinery modeling and optimization system) implemented at refineries in Russia [3]. It is used to determine:

- the optimal material balance, technical-and-economic indices corresponding thereto and directions of their improvement;
- the optimal commodity production plan expressed in kind and value;
- the loading and balance of each plant and assessment of each plant in terms of its role in achieving the maximal economic result;
- the material balance for each petroleum product and production facility in general, also the rate of return for production of particular types of petroleum products;
- the compounding schemes corresponding to the total material balance and quality requirements to the mix products;
- the demand in production resources and ingredients (total and broken down by plants);
- the generalized technical-and-economic indices (refining output, oil conversion ratio, yield of light petroleum products, coverage, etc.).

Thanks to the feature of parametric adjustment of the criterion, the software is used for a wide range of applications. Among them, the main most in-demand applications are: 1) investment analysis; 2) selection of the factory process flow diagram; 3) current planning and operational planning.

References

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