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CONJUGATE HEAT TRANSFER IN A CLOSED VOLUME WITH THE LOCAL HEAT SOURCES AND NON-UNIFORM HEAT DISSIPATION ON THE BOUNDARIES OF HEAT CONDUCTING WALLS

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Abstract. Is solved the problem of heat transfer in the closed volume, limited by heat-conducting walls, with the local source of heat emission and the heterogeneous conditions of heat sink on the outer boundaries of solution area. The problem of convective heat transfer is solved with using a system of differential Navier-Stokes equations in the Boussinesq approximation. The simulation of turbulent flow conditions of heated air is carried out within the framework to k-ε model. On the basis the analysis of the obtained temperature field and the contour lines of stream functions is made conclusion about the essential transiency of the process in question. The obtained values of temperatures and speeds in different sections of region illustrate turbulence of the process. Are investigated laws governing the formation of temperature fields in closed areas with a local heat emission source under the conditions of intensive local heat sink into environment and accumulation of heat in the enclosing constructions.

1 Introduction

The problems of conduction and convection in the closed volumes are solved for many versions of settings. Thus, for instance, the processes of free convection and thermal conductivity in the rectangular regions without the local sources of the heat emission and heat sink into the walls [1, 2], with the local heat emission sources [3-5], in the conjugate formulation [6-8], with radiation heat transfer [9, 10], different position of the heat sources [11-13], different boundary conditions on the solid walls [14], the radiation source of heating transparent air medium inside the closed cavity [15], cooling the microelectronics devices [16], turbulent regimes [17] of natural convection are investigated. The enumerated solutions compose only small part of the large number of publications according to the class of problems. But, in spite of intensive studies in this direction, until recently are not established laws governing the processes of heat transfer in the gas cavities with the solid heat-conducting walls under the conditions of local intensive heat transfer into the environment in relatively small section of region boundaries. At the same time such tasks are of obvious interest for microelectronics, aviation and space instrument manufacture.

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The purpose of this work is to solve the problem of heat transfer in a closed rectangular cavity with the solid heat-conducting walls and the local energy source at one of the boundaries under intense heat on the boundary sections into the environment.

2 Statement of the problem and the method of solution

The subject of a study is sufficiently typical rectangular region (Fig. 1).

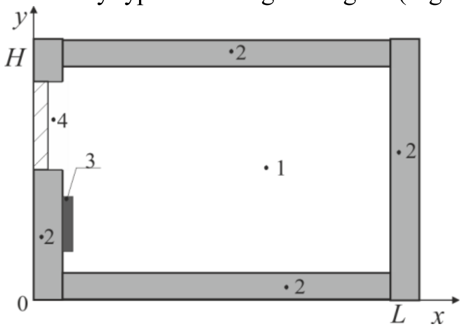

Figure 1. Area solution to the problem: 1 - air; 2 - solid thermal conductivity and heat storage walls; 3 - heat source; 4 - section of the outer wall of the heat sink; *L* - size of the area in the direction *X*; *H* - size of the solutions in the direction of *Y*.

In the longitudinal section the region of the solution is rectangle (Fig. 1). Air environment (1) on the contour is limited to heat-conducting walls of the materials with low (2) and high (4) thermal conductivity. On the left outer boundary (Fig. 1) is carried out heat to the environment by convection and radiation. The heat sink is absent on the remaining three boundaries. On all the internal borders of the pairing “thermally conductive wall – air” accept the terms of the equality of heat flows and temperatures (the boundary conditions of the fourth kind). On the surface of heat emission source (3) the temperature was assigned (boundary conditions of the first kind).

It was assumed that the thermophysical properties of enclosing constructions and air do not depend on temperature, air was considered as the viscous, heat-conducting, Newtonian liquid, satisfying the Boussinesq approximation. The process of heat transfer in this approximation is described by the system of the nonstationary two-dimensional equations of thermal conductivity for the solid walls, the turbulent free convection and the thermal conductivity in air [18] with the appropriate stated below boundary conditions. The mathematical formulation of the problem in dimensionless variables includes the following equations:

$$\frac{\partial\Omega}{\partial\tau}+U\frac{\partial\Omega}{\partial X}+V\frac{\partial\Omega}{\partial Y}=\frac{\partial^2}{\partial X^2}\left[\left(\frac{1}{\sqrt{Gr}}+\frac{1}{\mathrm{Re}_t}\right)\Omega\right]+\frac{\partial^2}{\partial Y^2}\left[\left(\frac{1}{\sqrt{Gr}}+\frac{1}{\mathrm{Re}_t}\right)\Omega\right]+\frac{\partial\Theta}{\partial Y}$$
$$+2\frac{\partial U}{\partial Y}\frac{\partial^2}{\partial X^2}\left(\frac{1}{\mathrm{Re}_t}\right)-2\frac{\partial V}{\partial X}\frac{\partial^2}{\partial Y^2}\left(\frac{1}{\mathrm{Re}_t}\right)+2\left(\frac{\partial V}{\partial Y}-\frac{\partial U}{\partial X}\right)\frac{\partial^2}{\partial X\partial Y}\left(\frac{1}{\mathrm{Re}_t}\right), \quad (1)$$

$$\frac{\partial\Theta}{\partial\tau}+U\frac{\partial\Theta}{\partial X}+V\frac{\partial\Theta}{\partial Y}=\frac{\partial}{\partial X}\left[\left(\frac{1}{\Pr\sqrt{Gr}}+\frac{1}{\Pr_t\mathrm{Re}_t}\right)\frac{\partial\Theta}{\partial X}\right]+\frac{\partial}{\partial Y}\left[\left(\frac{1}{\Pr\sqrt{Gr}}+\frac{1}{\Pr_t\mathrm{Re}_t}\right)\frac{\partial\Theta}{\partial Y}\right], \quad (2)$$

$$\frac{\partial^2\psi}{\partial X^2}+\frac{\partial^2\psi}{\partial Y^2}=-\Omega, \quad (3)$$

$$\frac{1}{\mathrm{Fo}}\frac{\partial\Theta}{\partial\tau}=\frac{\partial^2\Theta}{\partial X^2}+\frac{\partial^2\Theta}{\partial Y^2}, \quad (4)$$

$$\frac{\partial E}{\partial \tau}+U\frac{\partial E}{\partial X}+V\frac{\partial E}{\partial Y}=\frac{\partial}{\partial X}\left[\left(\frac{1}{\sqrt{Gr}}+\frac{1}{\sigma_{\varepsilon}\,\mathrm{Re}_t}\right)\frac{\partial E}{\partial X}\right]+\frac{\partial}{\partial Y}\left[\left(\frac{1}{\sqrt{Gr}}+\frac{1}{\sigma_{\varepsilon}\,\mathrm{Re}_t}\right)\frac{\partial E}{\partial Y}\right]+ \\ +c_{1\varepsilon}\left(\tilde{P}_k+c_{3\varepsilon}\tilde{G}_k\right)\frac{E}{K}-c_{2\varepsilon}\frac{E^2}{K}, \quad (5)$$

$$\frac{\partial K}{\partial \tau}+U\frac{\partial K}{\partial X}+V\frac{\partial K}{\partial Y}=\frac{\partial}{\partial X}\left[\left(\frac{1}{\sqrt{Gr}}+\frac{1}{\sigma_{k}\,\mathrm{Re}_t}\right)\frac{\partial K}{\partial X}\right]+\frac{\partial}{\partial Y}\left[\left(\frac{1}{\sqrt{Gr}}+\frac{1}{\sigma_{k}\,\mathrm{Re}_t}\right)\frac{\partial K}{\partial Y}\right]+ \\ +\tilde{P}_k+\tilde{G}_k-E, \quad (6)$$

where $\tilde{P}_k=\frac{1}{\mathrm{Re}_t}\left[2\left(\frac{\partial U}{\partial X}\right)^2+2\left(\frac{\partial V}{\partial X}\right)^2+\left(\frac{\partial U}{\partial X}+\frac{\partial V}{\partial X}\right)^2\right]$; $\tilde{G}_k=-\frac{1}{\mathrm{Re}_t\,\mathrm{Pr}_t}\frac{\partial \Theta}{\partial Y}$;

The model constants are given as: c_{μ} = 0,09; $c_{1\varepsilon}$ = 1,44; $c_{2\varepsilon}$ = 1,92; $c_{3\varepsilon}$ = 0,8; σ_{ε} = 1,3; σ_{κ} = 1.

Initial conditions:

$$\Psi(X,Y,0)=\Omega(X,Y,0)=\Theta(X,Y,0)=K(X,Y,0)=E(X,Y,0)=0\,. \quad (7)$$

Boundary conditions:

– on the external area contour except for the left border the boundary conditions of the second kind are set

$$\frac{\partial \Theta}{\partial \vec{n}}=0\,; \quad (8)$$

– on the left outer boundary:

$$\frac{\partial \Theta}{\partial X}=\mathrm{Bi}\cdot\Theta+\mathrm{Bi}\cdot(\Theta_e)+N\cdot\left[(\Theta)^4-(\Theta_e)^4\right], \quad (9)$$

– at the wall-gas interface the following conditions are set:

$$\Psi=0\,,\ \frac{\partial \Psi}{\partial \vec{n}}=0\,,\begin{cases}\Theta_1=\Theta_2\\ \dfrac{\partial \Theta_1}{\partial X}=\lambda_{1,2}\dfrac{\partial \Theta_2}{\partial X}\end{cases}, \quad (10)$$

– on the surface of the heater:

$$\Theta=1\,, \quad (11)$$

– for the κ-ε model near the solid surface the following is accepted:

$$\frac{\partial K}{\partial n}=0;\ E=\frac{c_{\mu}^{3/4}K^{3/2}}{k\Delta n}. \quad (12)$$

где X, Y – dimensionless Cartesian coordinates; Θ – dimensionless temperature; Ω – dimensionless analog vorticity; $\lambda_{1,2}$ – relative thermal conductivity coefficient; ν_t – coefficient of turbulent kinematic viscosity, m^2/s; $\mathrm{Fo}=at_0/L^2$ – Fourier number; $\mathrm{Gr}=g\beta L^3(T_{it}-T_0)/\nu_t^2$ – Grashof number; g – acceleration created by the mass forces, m/s^2; a – thermal diffusivity, m^2/s; β – thermal coefficient of volume expansion, K^{-1}; $\mathrm{Bi}=\alpha L/\lambda$ – Biot number; α – coefficient of heat exchange between the external environment and the area under consideration solutions; $\mathrm{N}=\varepsilon\sigma L(T-T_e)^3/\lambda$ – Starck number;

λ – thermal conductivity coefficient of solid wall, W/(m·K); ε – a reduced degree of blackness; σ – Stefan-Boltzmann constant; $\Pr = \nu_t / a$ – Prandtl number; T – temperature, K; T_0 – temperature at the initial time, K; T_{it} – scale of temperature, K; V_0 – velocity scale (convection velocity), m^2/s; U, V – dimensionless velocity; t – time, s; t_0 – time scale, S; τ – dimensionless time; Ψ – dimensionless analog stream function.

With the solution of problem (1) – (12) was used the algorithm [6,15], developed for the numerical solution of the tasks of free convection in the locked rectangular regions with the local energy sources.

3 The results of numerical modeling

In solving the problem (1) - (12), the following values of the dimensionless temperature: the source of heat emission – $\Theta_{it} = 1$, initial – $\Theta_0 = 0$, of the environment – $\Theta_e = -0.2$.

Fig. 2 shows the results of the numerical solution of problems for a relatively small time period (6200 seconds of physical time).

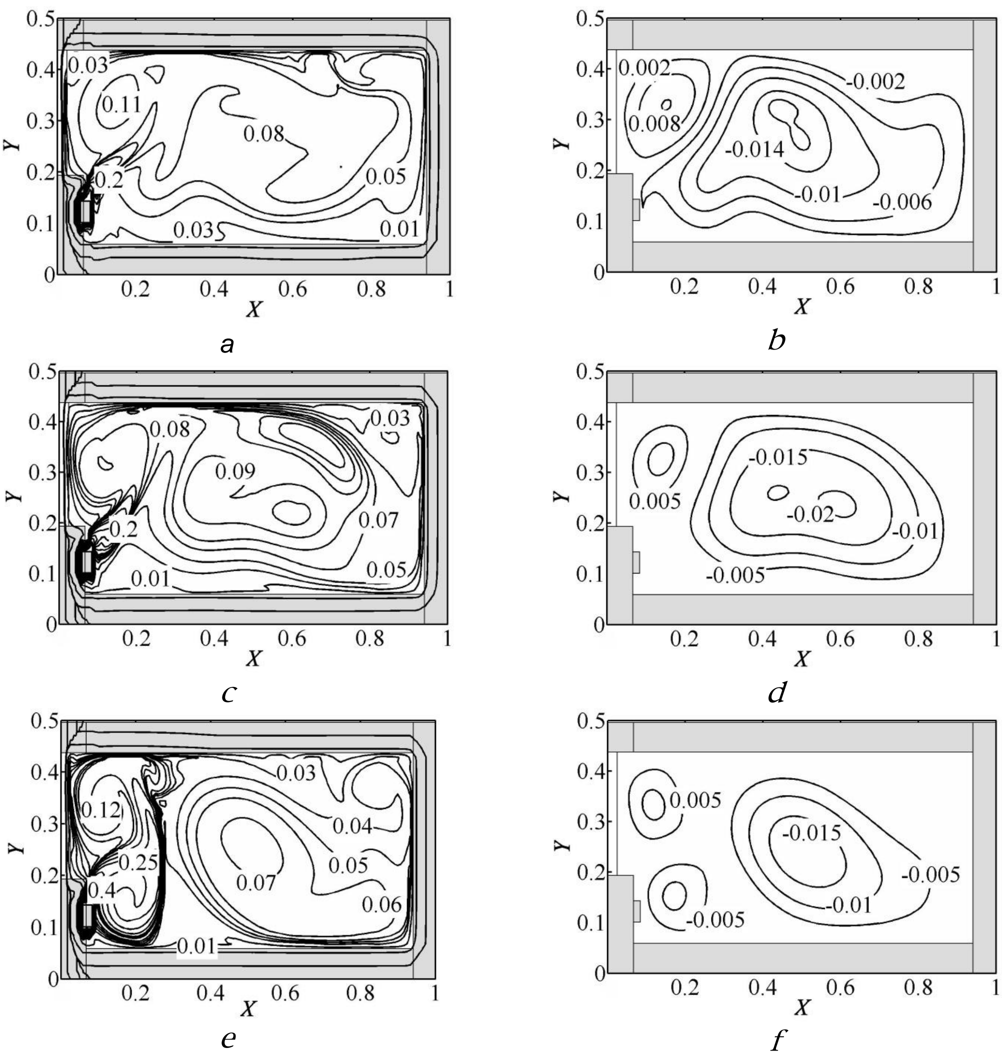

Figure 2. Fields temperature (*a*, *c*, *e*) and the contours of stream function (*b*, *d*, *f*) at different time: τ = 100 (*a*, *b*), τ = 150 (*c*, *d*), τ = 200 (*d*, *e*).

With the relatively short times (Fig. 2 *a–d*) are well identified steady circulation flow (one basic vortex and several small near the heat source). With the increase τ to 200 (Fig. 2 *e*, *f*) are intensified eddy formations in air heated from the heat emission source. Occurs the formation of separate large vortices near the section of heat sink into the environment and at the boundary $X = L$.

Fig. 2 well visible transformation of temperature distribution and hydrodynamic parameters with increasing τ. Generate sustainable over time vortex structure near the border area with intense heat sink. Generate sustainable over time vortex structure near the border area with intense heat sink.

In Fig. 3 the temperature distributions in the characteristic sections of region for several time moments are represented. The temperature gradients in the horizontal and vertical sections is significant when small τ. Over time, the temperature differential along the height of air region decreases, and in the section $Y = 0.25$ the extremum of temperatures near the source of heating remains constant. The temperature profile is equalized with the removal from it.

Change in the distributions of Θ (X) and Θ (Y) (Fig. 3) in the characteristic sections of the region ($X = 0.5$ and $Y = 0.25$) is interconnected with a change in the numerical values of the speeds in the same characteristic sections (Fig. 4).

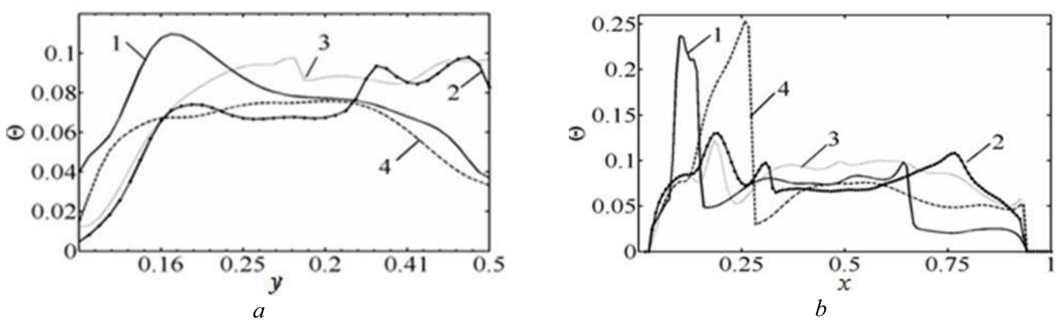

Figure 3. The temperature distribution in the cross section $X = 0.5$ (*a*) $Y = 0.25$ (*b*) at different τ: 1 – τ=50, 2 – τ=100, 3 – τ=150, 4 – τ=200.

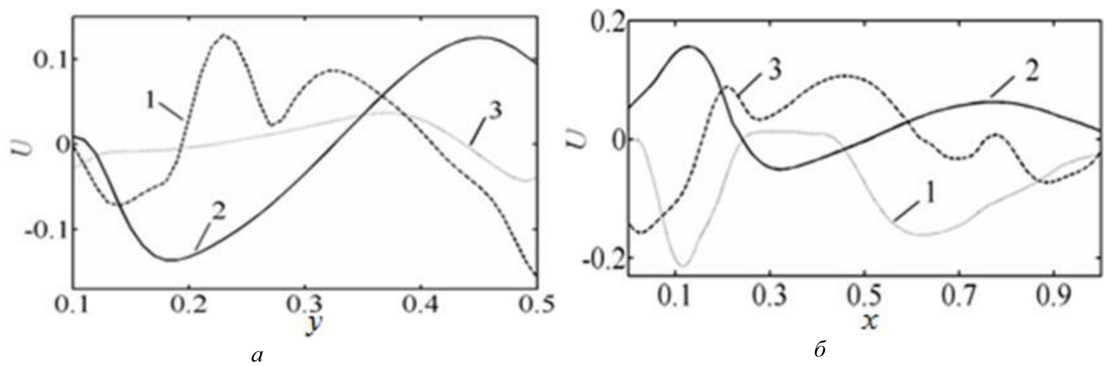

Figure 4. The distribution of velocities in the directions: *a*) – Y air cavity when $\tau = 200$ for different sections: 1 – $X = 0.1$; 2 – $X = 0.5$, 3 – $X = 0.9$; *б*) – X air cavity when $\tau = 200$ for different sections: 1 – $Y = 0.1$; 2 – $Y = 0.25$, 3 – $Y = 0.4$

But also during the levelling off of the temperature profile in the air region with $\tau = 200$ remains the significant velocity gradient both in the vertical and in the horizontal sections. This connected with the fact that air region has already been warmed thoroughly, and on the internal interfaces of media occurs the intensive draining of heat both through the left enclosing wall into the environment and into the internal heat-storing walls.

4 Conclusion

The numerical study of convective-conductive heat transfer in a closed area with a heat source, a limited heat-conducting walls in the inhomogeneous boundary conditions leads to the conclusion about the impact heat-retaining walls on amplifying transients, turbulence flow, temperature distribution and velocity field.

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