

The gauge-invariant canonical energy-momentum tensor

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Abstract. The canonical energy-momentum tensor is often considered as a purely academic object because of its gauge dependence. However, it has recently been realized that canonical quantities can in fact be defined in a gauge-invariant way provided that strict locality is abandoned, the non-local aspect being dictacted in high-energy physics by the factorization theorems. Using the general techniques for the parametrization of non-local parton correlators, we provide for the first time a complete parametrization of the energy-momentum tensor (generalizing the purely local parametrizations of Ji and Bakker-Leader-Trueman used for the kinetic energy-momentum tensor) and identify explicitly the parts accessible from measurable two-parton distribution functions (TMDs and GPDs). As by-products, we confirm the absence of model-independent relations between TMDs and parton orbital angular momentum, recover in a much simpler way the Burkardt sum rule and derive three similar new sum rules expressing the conservation of transverse momentum.

1 Introduction

Following Noether's procedure, one obtains a canonical energy-momentum (EMT) $T^{\mu\nu}$ which is usually neither symmetric¹ nor gauge invariant. These properties are often considered as pathological and can be cured by adding to the EMT a superpotential term of the form $\partial_\alpha f^{[\alpha\mu]\nu}$ [2–4], where the square brackets stand for antisymmetrization. This amounts to a redefinition of the local density of energy and momentum [5, 6] while leaving the total linear and angular momenta unchanged.

Superpotential terms have also been used to decompose the angular momentum into spin and orbital contributions [7, 8]. According to standard textbooks like *e.g.* [9, 10], one cannot perform this decomposition for the gauge field in a gauge-invariant way. Nonetheless, it appears that the photon spin and orbital angular momentum (OAM) are routinely measured, see *e.g.* [11] and references therein. Similarly, a gauge-invariant quantity called ΔG , interpreted in the light-front gauge as the gluon spin [7], has been measured in polarized deep inelastic and proton-proton scatterings, see [12] for a recent analysis. Since physical observables are gauge invariant, Chen *et al.* [13] claimed that the textbooks were wrong and proposed a formal gauge-invariant decomposition of the photon and

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¹We stress that the symmetry requirement essentially comes from General Relativity where torsion is assumed to vanish. In more general theories of gravitation like *e.g.* Einstein-Cartan theory and metric-affine gauge theory, torsion is allowed to be nonzero leading to asymmetric EMTs and a natural coupling between gravitation and spin. These effects are however expected to be extremely small and to show up only under extreme conditions, see *e.g.* [1] and references therein.

gluon angular momentum. Their work received strong criticisms and triggered numerous theoretical papers on the subject, summarized in recent reviews [14, 15]. It is now understood that textbooks implicitly referred to local quantities only, whereas the formal construction of Chen *et al.* and the above-mentioned quantities extracted from experimental data are intrinsically non-local [16–19].

Typical examples of measurable non-local quantities in Quantum ChromoDynamics (QCD) are parton distribution functions (PDFs) whose gauge invariance is ensured by a Wilson line along a path determined by the factorization theorems [20]. Generalized Transverse-Momentum dependent Distributions (GTMDs) [21] are natural generalizations of the PDFs and provide a natural way to access the canonical OAM [22–24] and other angular momentum correlations [25]. Unfortunately, it is not known so far how to access them directly in experiments. They can however be accessed indirectly using *e.g.* realistic models [22, 26–29], or lattice QCD in the infinite-momentum limit [30–33].

We present here the first complete parametrization of the EMT obtained in [34] which can be applied to virtually any form of the EMT discussed in the literature, provided that the non-locality appears only along the light-front (LF) direction n .

2 A basis of gauge-invariant energy-momentum tensors

Most of the EMTs that appeared in the literature can be decomposed in a basis of five gauge-invariant tensors

$$\begin{aligned} T_1^{\mu\nu} &= \bar{\psi}\gamma^\mu \overleftrightarrow{D}^\nu \psi, & T_2^{\mu\nu} &= -2\text{Tr}[G^{\mu\alpha}G^\nu{}_\alpha] + g^{\mu\nu} \frac{1}{2}\text{Tr}[G^{\alpha\beta}G_{\alpha\beta}], \\ T_3^{\mu\nu} &= -\bar{\psi}\gamma^\mu gA_{\text{phys}}^\nu \psi, & T_4^{\mu\nu} &= \frac{1}{4}\epsilon^{\mu\nu\alpha\beta}\partial_\alpha[\bar{\psi}\gamma_\beta\gamma_5\psi], \\ T_5^{\mu\nu} &= -2\partial_\alpha\text{Tr}[G^{\mu\alpha}A_{\text{phys}}^\nu], \end{aligned} \quad (1)$$

where $\overleftrightarrow{D}^\mu = \overrightarrow{D}^\mu + gA^\mu$ is the hermitian covariant derivative with $\overleftrightarrow{\partial}^\mu = \overrightarrow{\partial}^\mu - \overleftarrow{\partial}^\mu$, and $\epsilon_{0123} = +1$. For example, the kinetic form of the quark and gluon EMTs are given by $T_1^{\mu\nu}$ and $T_2^{\mu\nu}$, respectively, and the corresponding canonical forms are given by $T_1^{\mu\nu} + T_3^{\mu\nu}$ and $T_2^{\mu\nu} - T_3^{\mu\nu} + T_5^{\mu\nu}$, respectively. The superpotentials $T_4^{\mu\nu}$ and $T_5^{\mu\nu}$ together with the QCD equations of motion

$$\begin{aligned} \bar{\psi}(r)\gamma^{[\mu}\overleftrightarrow{D}^{\nu]}\psi &= -\epsilon^{\mu\nu\alpha\beta}\partial_\alpha[\bar{\psi}\gamma_\beta\gamma_5\psi], \\ 2[\mathcal{D}_\alpha G^{\alpha\beta}]^c{}_{c'} &= -g\bar{\psi}_{c'}\gamma^\beta\psi^c, \end{aligned} \quad (2)$$

where c, c' are color indices in the fundamental representation and $\mathcal{D}_\mu = \partial_\mu - ig[A_\mu, \]$ is the adjoint covariant derivative, can be used to relate the various EMTs [14, 34]. Because of the first identity in Eq. (2), we have $T_4^{\mu\nu} = -\frac{1}{2}T_1^{[\mu\nu]}$ which can then be discarded in the following discussions.

In order to obtain gauge-invariant canonical EMTs, one needs to decompose the gauge potential A_μ into pure-gauge and “physical” (or covariant) contributions

$$A_\mu^{\text{pure}} \equiv \frac{i}{g}\mathcal{W}\partial_\mu\mathcal{W}^{-1}, \quad A_\mu^{\text{phys}} \equiv A_\mu - A_\mu^{\text{pure}}, \quad (3)$$

where \mathcal{W} is a non-integrable phase factor transforming as $\mathcal{W} \mapsto U\mathcal{W}$ under gauge transformations. In the gauge where $\mathcal{W}(r) = 1$, the gauge-invariant canonical decomposition formally reduces to the Jaffe-Manohar decomposition [7], and can therefore be considered as a gauge-invariant extension of the latter [14, 17, 35, 36].

3 General parametrization

Since our aim is to relate the matrix elements of the EMT to measurable parton distributions, we identify the non-local phase factor \mathcal{W} with a straight Wilson line running along the LF direction n to $r_n = r + \infty \eta n$, and then in the transverse direction to ∞_\perp . It is then clear that, beside the average target momentum $P = (p' + p)/2$ and the momentum transfer $\Delta = p' - p$, the matrix elements of the generic LF EMT also depend in principle on $N = \frac{M^2 n}{P \cdot n}$ for a target of mass M , and on the direction of the Wilson line $\eta = \pm 1$. The scalar functions parametrizing the generic LF EMT are complex-valued functions of $\xi = -(\Delta \cdot N)/2(P \cdot N)$ and $t = \Delta^2$, which are the only two independent scalars that can be formed with P , Δ and N .

In Ref. [34], we used the techniques presented in the Appendix A of Ref. [21] and found that the generic LF EMT for a spin-1/2 target can be parametrized as $\langle p', S' | T_a^{\mu\nu}(0) | p, S \rangle = \bar{u}(p', S') \Gamma_a^{\mu\nu}(P, \Delta, N; \eta) u(p, S)$ with $a = 1, \dots, 5$ and the matrix $\Gamma_a^{\mu\nu}$ given by

$$\begin{aligned}
 \Gamma_a^{\mu\nu} = & M g^{\mu\nu} A_1^a + \frac{P^\mu P^\nu}{M} A_2^a + \frac{\Delta^\mu \Delta^\nu}{M} A_3^a + \frac{P^\mu i\sigma^{\nu\Delta}}{2M} A_4^a + \frac{P^\nu i\sigma^{\mu\Delta}}{2M} A_5^a \\
 & + \frac{N^\mu N^\nu}{M} B_1^a + \frac{P^\mu N^\nu}{M} B_2^a + \frac{P^\nu N^\mu}{M} B_3^a + \frac{N^\mu i\sigma^{\nu\Delta}}{2M} B_4^a + \frac{N^\nu i\sigma^{\mu\Delta}}{2M} B_5^a + \frac{\Delta^\mu i\sigma^{\nu N}}{2M} B_6^a + \frac{\Delta^\nu i\sigma^{\mu N}}{2M} B_7^a \\
 & + \left[M g^{\mu\nu} B_8^a + \frac{P^\mu P^\nu}{M} B_9^a + \frac{\Delta^\mu \Delta^\nu}{M} B_{10}^a + \frac{N^\mu N^\nu}{M} B_{11}^a + \frac{P^\mu N^\nu}{M} B_{12}^a + \frac{P^\nu N^\mu}{M} B_{13}^a \right] \frac{i\sigma^{N\Delta}}{2M^2} \\
 & + \frac{P^\mu \Delta^\nu}{M} B_{14}^a + \frac{P^\nu \Delta^\mu}{M} B_{15}^a + \frac{\Delta^\mu N^\nu}{M} B_{16}^a + \frac{\Delta^\nu N^\mu}{M} B_{17}^a + \frac{M}{2} i\sigma^{\mu\nu} B_{18}^a + \frac{\Delta^\nu i\sigma^{\mu\Delta}}{2M} B_{19}^a \\
 & + \frac{P^\mu i\sigma^{\nu N}}{2M} B_{20}^a + \frac{P^\nu i\sigma^{\mu N}}{2M} B_{21}^a + \frac{N^\mu i\sigma^{\nu N}}{2M} B_{22}^a + \frac{N^\nu i\sigma^{\mu N}}{2M} B_{23}^a \\
 & + \left[\frac{P^\mu \Delta^\nu}{M} B_{24}^a + \frac{P^\nu \Delta^\mu}{M} B_{25}^a + \frac{\Delta^\mu N^\nu}{M} B_{26}^a + \frac{\Delta^\nu N^\mu}{M} B_{27}^a \right] \frac{i\sigma^{N\Delta}}{2M^2}. \tag{4}
 \end{aligned}$$

For convenience, we introduced the notation $i\sigma^{\mu b} \equiv i\sigma^{\mu\alpha} b_\alpha$ and the factors of i such that the real part of the scalar functions is η -even while the imaginary part is η -odd

$$X_j^a(\xi, t; \eta) = X_j^{e,a}(\xi, t) + i\eta X_j^{o,a}(\xi, t) \tag{5}$$

as a consequence of time-reversal symmetry. The hermiticity property then implies that the real part of B_j^a with $j \geq 14$ is ξ -odd and the imaginary part is ξ -even. For the other functions, it is the opposite. The fact that only 32 independent structures exist can be obtained from a naive simple counting: the EMT $T_a^{\mu\nu}$ has $4 \times 4 = 16$ components; the target state polarizations $\pm S$ and $\pm S'$ bring another factor of $2 \times 2 = 4$; time-reversal and hermiticity having been used to fix the factors of i and the ξ -dependence, we are left with parity which reduces the number of independent polarization configurations by a factor 2. As announced, this leads to a total of 32 independent complex-valued amplitudes.

Manifestly, the EMTs $T_1^{\mu\nu}$ and $T_2^{\mu\nu}$ do not depend on N or η . All the corresponding scalar functions must then vanish except for $A_j^{e,a}(0, t)$ with $a = 1, 2$ and $j = 1, \dots, 5$, which are linearly related to the standard energy-momentum form factors [8, 14, 37] as follows

$$\begin{aligned}
 A_q(t) &= A_2^{e,1}(0, t), & A_G(t) &= A_2^{e,2}(0, t), \\
 B_q(t) &= A_4^{e,1}(0, t) + A_5^{e,1}(0, t) - A_2^{e,1}(0, t), & B_G(t) &= A_4^{e,2}(0, t) + A_5^{e,2}(0, t) - A_2^{e,2}(0, t), \\
 C_q(t) &= A_3^{e,1}(0, t), & C_G(t) &= A_3^{e,2}(0, t), \\
 \bar{C}_q(t) &= A_1^{e,1}(0, t) + \frac{t}{M^2} A_3^{e,1}(0, t), & \bar{C}_G(t) &= A_1^{e,2}(0, t) + \frac{t}{M^2} A_3^{e,2}(0, t), \\
 D_q(t) &= A_4^{e,1}(0, t) - A_5^{e,1}(0, t), & 0 &= A_4^{e,2}(0, t) - A_5^{e,2}(0, t).
 \end{aligned} \tag{6}$$

4 Linear and angular momentum constraints

The parametrization (4) is only constrained by space-time symmetries. We briefly present here the additional constraints arising from the conservation of total linear and angular momentum.

The average four-momentum in the LF form of dynamics can be obtained by contracting the EMT with $\frac{1}{2M^2} N_\mu$ in the forward limit $\Delta \rightarrow 0$

$$\langle p_a^v \rangle \equiv \frac{1}{2M^2} \langle P, S | T_a^{Nv}(0) | P, S \rangle = P^v A_2^{e,a} + N^v (A_1^{e,a} + B_2^{e,a}) + \delta^{a3} \frac{\eta}{2} \epsilon_T^{vS} (B_{18}^{o,3} - B_{20}^{o,3}). \quad (7)$$

Interestingly, the last term in Eq. (7) originating from the potential EMT $T_3^{\mu\nu}$ is η -odd and can be interpreted as the spin-dependent contribution to the momentum arising from initial and/or final-state interactions, see *e.g.* [38] and references therein. It comes with the structure $\epsilon_T^{vS} \equiv \epsilon^{\nu\mu\alpha\beta} S_\mu n_\alpha \bar{n}_\beta$ where \bar{n} is another lightlike four-vector satisfying $n \cdot \bar{n} = 1$ and such that $P^\mu = (P \cdot n) \bar{n}^\mu + (P \cdot \bar{n}) n^\mu$, which means that a transverse target polarization is required. The total four-momentum being P^v , we recover from the sum over all partons the well-known momentum constraints

$$\sum_{a=1,2} A_1^{e,a}(0,0) = \sum_{a=q,G} \bar{C}_a(0) = 0, \quad \sum_{a=1,2} A_2^{e,a}(0,0) = \sum_{a=q,G} A_a(0) = 1. \quad (8)$$

Thanks to the complete parametrization (4), we can easily compute the matrix elements of the corresponding OAM tensors $L_a^{\mu\nu\rho} = r^\nu T_a^{\mu\rho} - r^\rho T_a^{\mu\nu}$. Because of the explicit factors of r , the matrix elements of the generic LF OAM tensor need to be handled with care [14, 37]. For a longitudinally polarized target, we found a simple expression for the longitudinal component of OAM

$$\langle L_L^a \rangle \equiv \frac{\epsilon_{T\alpha\beta}}{2M^2} \left[i \frac{\partial}{\partial \Delta_\alpha} \langle p', S' | T_a^{N\beta}(0) | p, S \rangle \right]_{\Delta=0} = A_4^{e,a}(0,0). \quad (9)$$

Similarly, the quark and gluon spin contributions $S_1^{\mu\nu\rho} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \bar{\psi} \gamma_\beta \gamma_5 \psi$ and $S_2^{\mu\nu\rho} = -2 \text{Tr} [G^{\mu[\nu} A_{\text{phys}}^{\rho]}]$ can be expressed in terms of $L_4^{\mu\nu\rho}$ and $L_5^{\mu\nu\rho}$, respectively. We then found

$$\begin{aligned} \langle S_L^q \rangle &\equiv \frac{1}{2M^2} \langle P, S | \frac{1}{2} \epsilon_{T\alpha\beta} S_1^{N\alpha\beta}(0) | P, S \rangle = -\frac{1}{2} [A_4^{e,1}(0,0) - A_5^{e,1}(0,0)], \\ \langle S_L^G \rangle &\equiv \frac{1}{2M^2} \langle P, S | \frac{1}{2} \epsilon_{T\alpha\beta} S_2^{N\alpha\beta}(0) | P, S \rangle = -A_4^{e,5}(0,0), \end{aligned} \quad (10)$$

where Eq. (2) has been used to express $L_4^{\mu\nu\rho}$ in terms of $T_1^{\mu\nu}$. Adding together the spin and OAM contributions, we naturally recover the Ji relation for total angular momentum [8]

$$\begin{aligned} \langle J_L^q \rangle &= \langle S_L^q \rangle + \langle L_L^q \rangle = \frac{1}{2} [A_4^{e,1}(0,0) + A_5^{e,1}(0,0)] = \frac{1}{2} [A_q(0) + B_q(0)], \\ \langle J_L^G \rangle &= \langle S_L^G \rangle + \langle L_L^G \rangle = \frac{1}{2} [A_4^{e,2}(0,0) + A_5^{e,2}(0,0)] = \frac{1}{2} [A_G(0) + B_G(0)]. \end{aligned} \quad (11)$$

Finally, combining the fact that the total angular momentum is 1/2 and the momentum constraints (8), we recover also the anomalous gravitomagnetic moment sum rule [39, 40]

$$\sum_{a=1,2} [A_4^{e,a}(0,0) + A_5^{e,a}(0,0) - A_2^{e,a}(0,0)] = \sum_{a=q,G} B_a(0) = 0. \quad (12)$$

5 Link with measurable parton distributions

The matrix elements of the EMT we are interested in can easily be expressed in terms of the GTMD correlator [18]

$$\langle p', S' | T^{\mu\nu}(0) | p, S \rangle = \int d^4k k^\nu W_{S'S}^\mu. \quad (13)$$

By considering appropriate projections [21, 41, 42], we can at the end relate some of the scalar functions appearing in our generic parametrization (4) to measurable parton distributions, like *e.g.* Generalized Parton Distributions (GPDs) accessed in exclusive scatterings [43] and Transverse-Momentum dependent Distributions (TMDs) accessed in semi-inclusive scatterings [20]. The detailed relations between the EMT scalar functions and two-parton GPDs and TMDs of any twist can be found in [34].

Among the interesting results, let us just mention that we derived the following sum rules

$$\sum_{a=q,G} \int dx d^2k_T \frac{k_T^2}{2M^2} F^a(x, k_T^2) = 0 \quad (14)$$

with $F^a = f_{1T}^{\perp a}, f_L^{\perp a}, f_{3T}^{\perp a}$. They express the fact that the total transverse momentum (w.r.t. the target momentum) has to vanish. The leading-twist relation (*i.e.* the one involving the Sivers function $f_{1T}^{\perp a}$) is known as the Burkardt sum rule [44, 45]. The other three are new and much harder to test experimentally, but it would be very interesting to test them using phenomenological models, Lattice QCD and perturbative QCD.

6 Conclusions

It is possible to define a gauge-invariant canonical energy-momentum tensor once one relaxes the assumption of strict locality. We argued that the canonical energy-momentum tensor can be considered as a physical object and *a priori* measured experimentally *via* particular moments of parton distributions extracted from various physical processes.

We presented here a complete parametrization for the matrix elements of the generic light-front energy-momentum tensor and discussed some of the constraints arising from linear and angular momentum conservation. We showed that this energy-momentum tensor can be related to particular moments of the generalized and transverse-momentum dependent parton distributions. Among the interesting results, we rederived in a simpler way the Burkardt sum rule and obtained three new sum rules involving higher-twist distributions, all expressing basically the conservation of transverse momentum. We expect in a near future exciting new developments in these matters coming from new experimental data obtained in existing and future facilities, and explicit investigations using phenomenological models, Lattice QCD and perturbative QCD.

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