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# Searching saturation effects in inclusive and exclusive $eA$ processes

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**Abstract.** The searching of saturation effects in inclusive and exclusive $eA$ collisions is reviewed. In particular, we present a comparison between the linear and non-linear predictions for the nuclear structure functions as well as for the Deep Virtual Compton Scattering and vector meson production in future $eA$ colliders.

## 1 Introduction

The study of the hadronic structure in the non-linear regime of the Quantum Chromodynamics (QCD) is one of the main goals of the future Electron - Ion Colliders (EIC) [1]. It is expected that the resulting experimental data will be able to determine the presence of gluon saturation effects, the magnitude of the associated non-linear corrections and what is the correct theoretical framework for their description [2]. This expectation is easily understood, if we analyse the Bjorken-$x$ and nuclear mass number $A$ behaviours of the nuclear saturation scale, $Q_{s,A}$, which determines the onset of non-linear effects in the QCD dynamics [2]. Assuming that the saturation scale of the nucleus is enhanced with respect to the nucleon one by the *oomph* factor $A^{\frac{1}{3}}$, it can be expressed by the parametrization $Q^2_{s,A} = A^{\frac{1}{3}} \times Q^2_0 (\frac{x_0}{x})^{\lambda}$. As a consequence nuclei are an efficient amplifier of the non-linear effects. In Fig. 1 we present the theoretical expectations for the saturation scale as a function of $x$ and $A$, considering the parameters $Q^2_0 = 1.0$ GeV$^2$, $x_0 = 1.632 \times 10^{-5}$ and $\lambda = 0.2197$. We can observe that, while in the proton case we need very small values of $x$ to obtain large values of $Q^2_s$, in the nuclear case a similar value can be obtained for values of $x$ approximately two orders of magnitude greater. Consequently, the parton density that is accessed in electron - nucleus ($eA$) collisions would be equivalent to that obtained in an electron - proton collider at energies that are at least one order of magnitude higher than at HERA.

The enhancement of the non-linear effects with the nuclear mass number has motivated the development of an intense phenomenology about the implications of these effects in inclusive and diffractive observables which could be measured in the Electron - Ion Colliders [3–22]. In this contribution we review the results obtained in Refs. [11, 18] which have indicated that the analysis of inclusive observables, as e.g. the nuclear structure functions $F^A_2$ and $F^A_L$, probably will not be the best way to obtain a signature of the non-linear effects due to the large uncertainty present in the collinear predictions at small $x$. As a alternative we discuss the possibility to constrain these effects by the analysis

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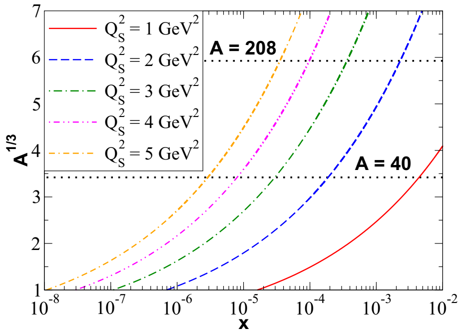


**Figure 1.** Nuclear saturation scale as a function of the Bjorken $x$ and nuclear mass number $A$.

of the logarithmic $Q^2$ slope of the nuclear structure functions, proposed originally in Refs. [3, 4]. Moreover, we will discuss the results from Refs. [12, 15, 21, 22] which demonstrated that, in contrast, diffractive events in $eA$ collisions can be considered a smoking gun for the gluon saturation physics. In particular, we will present our recent results for the nuclear deeply virtual Compton scattering (DVCS) and vector meson production in $eA$ processes.

## 2 Saturation physics in inclusive $eA$ processes

In the color dipole formalism the nuclear $F_2$ structure function is given by $F_2^A(x,Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}}\sigma_{tot}$, with

$$\sigma_{tot} = \sigma_T + \sigma_L \text{ and } \sigma_{T,L} = \int d^2r\, dz\, |\Psi_{T,L}(r,z,Q^2)|^2\; \sigma_{dip}^A(x,r), \tag{1}$$

where $\Psi_{T,L}$ is the light-cone wave function of the virtual photon and $\sigma_{dip}^A$ is the dipole nucleus cross section describing the interaction of the $q\bar{q}$ dipole with the nucleus target. In equation (1) $r$ is the transverse separation of the $q\bar{q}$ pair and $z$ is the photon momentum fraction carried by the quark . Moreover, the nuclear longitudinal structure function is given by $F_L^A(x,Q^2) = (Q^2/4\pi^2\alpha_{em})\,\sigma_L$ .

The main input for the calculations of inclusive observables in the dipole picture is $\sigma_{dip}^A(x,r)$ which is determined by the QCD dynamics at small $x$. In the eikonal approximation it is given by:

$$\sigma_{dip}^A(x,r) = 2\int d^2b\, \mathcal{N}^A(x,r,b) \tag{2}$$

where $\mathcal{N}^A(x,r,b)$ is the forward scattering amplitude for a dipole with size $r$ and impact parameter $b$ which encodes all the information about the hadronic scattering, and thus about the non-linear and quantum effects in the hadron wave function. In this contribution we assume that the forward dipole-nucleus amplitude was parametrized as follows

$$\mathcal{N}^A(x,r,b) = 1 - \exp\left[-\frac{1}{2}A\, T_A(b)\, \sigma_{dip}^p(x,r^2)\right], \tag{3}$$

where $T_A(b)$ is the nuclear profile function, which is obtained from a 3-parameter Fermi distribution for the nuclear density normalized to unity. The above equation sums up all the multiple elastic rescattering diagrams of the $q\bar{q}$ pair and is justified for large coherence length, where the transverse

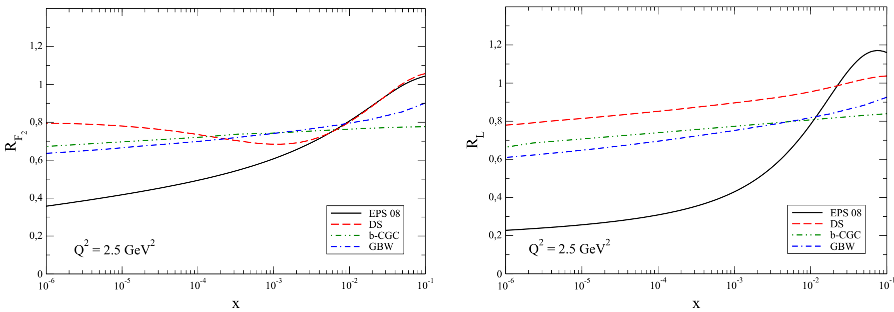


**Figure 2.** Comparison between the predictions for $R_{F_2} \equiv 2F_2^A/AF_2^D$ and $R_L \equiv 2F_L^A/AF_L^D$ of the saturation models (b-CGC and GBW) and the collinear ones (DS and EPS08) for $Q^2 = 2.5$ GeV$^2$ and $A = Pb$.

separation $r$ of partons in the multiparton Fock state of the photon becomes a conserved quantity, *i. e.* the size of the pair $r$ becomes eigenvalue of the scattering matrix. In [11] we have considered that $\sigma_{dip}^p$ is given by the bCGC model [23], which describes the *ep* HERA data.

In Fig. 2 we present our predictions for the $x$ dependence of the ratios $R_{F_2}$ and $R_{F_L}$ for $Q^2 = 2.5$ GeV$^2$ and $A = Pb$. The saturation models predict a similar behaviour in the small $x$ region. For comparison we also show the predictions obtained using the collinear factorization and nuclear parton distributions resulting from the global analysis of the nuclear experimental data using the DGLAP evolution equation. Here we consider two different sets of nuclear parton distributions: the DS and EPS 08 nuclear parametrizations. It is important to emphasize that the collinear predictions *do not* include dynamical effects associated to non-linear (saturation) physics, since they are based on the linear DGLAP dynamics. Therefore, the comparison between the saturation and collinear predictions could, in principle, tell us if the observable considered can be used to discriminate between linear and non-linear dynamics. The results shown in Fig. 2 (left panel) demonstrate that this is not the case of the nuclear structure function. Although the predictions of the two collinear models are similar for large $x$ ($\geq 10^{-2}$), where there exist experimental data, the predicted behaviour at small $x$ is very distinct. This difference is a consequence of the choice of different experimental data used in the global analysis and of the assumptions related to the behaviour of the nuclear gluon density. The difference between the collinear predictions is so large at small $x$ that it is not possible to extract any information about the presence or not of new QCD dynamical effects from the study of the nuclear structure function. Let us now consider the behaviour of the nuclear longitudinal structure function. Our results are show in Fig. 2 (right panel). The predictions of saturation models are, as in the case of $R_{F_2}$, contained within the uncertainty range of the collinear models. On the other hand, the difference between the two collinear models is bigger. In this case the results for large $x$ are distinct too. This is due to the strong dependence of $F_L^A$ on the nuclear gluon distribution, which is very different in the two models considered. The conclusion that we can draw in Ref. [11] is that the two dipole models yield similar predictions for the structure functions and they fall inside the range of predictions of the EPS and DS parametrizations. The latter differ among each other because of the large freedom inherent to global data analysis. Although $F_2$ and $F_L$ are sensitive to saturation effects, it is not yet possible to draw any firm conclusion concerning the QCD dynamics from inclusive quantities. It implies that in order to discriminate the saturation effects we should consider less inclusive observables.

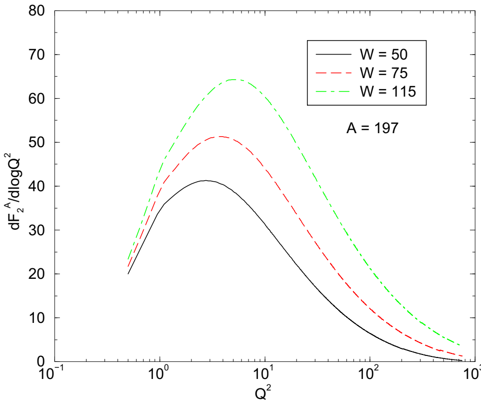


**Figure 3.** The logarithmic $Q^2$ slope of $F_2^A$ as function of the variable $Q^2$ at different values of $W$ and $A = 197$.

An alternative to search saturation effects in $eA$ processes is the analysis of the logarithmic slope of the nuclear structure function proposed in Refs. [3, 4]. In Refs. [3, 4] we considered that the Bjorken variable $x$ and the photon virtuality $Q^2$ are related by the expression $x = Q^2/W^2$, where $W$ is the $\gamma^* h$ c.m. energy, and calculate the $x$ and $Q^2$ dependences for $W$ and $A$ fixed. The Fig. 3 shows the logarithmic $Q^2$ slope of $F_2^A$ plotted for different values of $W^2$ as a function of $Q^2$. The remarkable property of the plot is the presence of a distinct maximum for slope which depends of the center of mass energy. Moreover, as shown in Ref. [4], the maximum also is dependent on the atomic number. At fixed $A$, if we increase the value of the energy, the maximum value of the slope occur at larger values of $Q^2$. The remarkable properties of the collisions with nuclei targets are the large values of $Q^2$ for the turnover of $F_2^A$ slope and the large shift of the turnover at large values of $Q^2$ with the growth of the energy. In Ref. [4] we verified that, at fixed $W$, the turnover is displaced at larger values of $x$ and $Q^2$ if we increase the number of nucleons $A$. These behaviours can be understood intuitively. The turnover is associated with the regime in which the partons in the nucleus form a dense system with mutual interactions and recombinations, with a transition between the linear and non linear regime at the saturation scale. As the partonic density growth at larger values of the number of nucleons $A$ and smaller values of $x$ we have that, at fixed $A$, the saturation scale $Q_s^2$ will increase at small values of $x$, since this is directly proportional to the gluon distribution. Moreover, at fixed energy $W$, the same density at $A = 12, 32, 197$ will be obtained at larger values of $x$ and $Q^2$. These properties of the high density effects are verified in the $F_2^A$ slope. The main result from the analysis presented in Ref. [4] is that the saturation should occur already at rather small distances (large $Q_s^2$) well below where soft dynamics is supposed to set in, justifying the use of perturbative QCD to approach a highly dense system.

## 3 Saturation effects in exclusive $eA$ processes

An alternative to search the saturation effects can be the study of observables measured in diffractive deep inelastic scattering (DDIS), since the total diffractive cross section is much more sensitive to large-size dipoles than the inclusive one. Basically, the saturation effects screen large-size dipole (soft) contributions, so that a fairly large fraction of the cross section is hard and hence eligible for

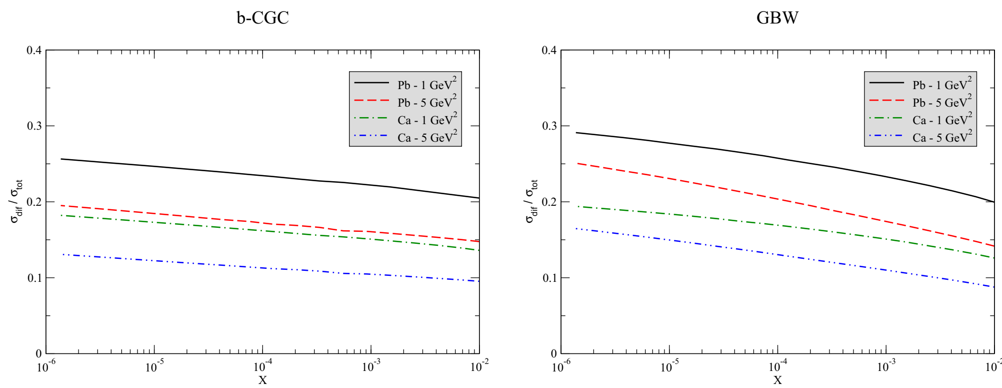


**Figure 4.** Ratio of diffractive to total cross sections, $R_\sigma$, as a function of $x$ with the b-CGC and GBW models. Long dashed and solid lines are for Pb targets at $Q^2 = 5.0$ and $1.0$ GeV$^2$, respectively. Dot-dot-dash and dot-dash lines are the same for Ca targets.

a perturbative treatment. Here we estimate the ratio $R_\sigma = \sigma_{diff}/\sigma_{tot}$, with the total diffractive cross section given by

$$\sigma_{diff} = \sigma_L^D + \sigma_T^D \text{ and } \sigma_{L,T}^D = \frac{1}{4} \int d^2r\, dz\, |\Psi_{T,L}(r,z,Q^2)|^2 \int d^2b \left(\frac{d\sigma_{dip}^A}{d^2b}\right)^2 \quad (4)$$

where $d\sigma_{dip}^A/d^2b = 2\,\mathcal{N}^A(x,r,b)$, with $\mathcal{N}^A$ given by Eq. (3). In Fig. 4 we present the predictions obtained in Ref. [11] for the ratio $R_\sigma$ as a function of $x$ for two values of $Q^2$ and the atomic number considering the b-CGC and GBW models. These results indicated that the contribution of diffractive processes to the total cross section is about 20 % at large $A$ and small $Q^2$, being factor two larger than observed in $ep$ collisions at HERA. Consequently, the study of diffractive processes should be an easy task in an $eA$ collider, allowing for a detailed analysis of the QCD dynamics. In particular, this observation have motivated more detailed studies of the diffractive interactions, with special attention to exclusive diffractive vector meson production and the deeply virtual Compton scattering (DVCS), which are experimentally clean and can be unambiguously identified by the presence of a rapidity gap in final state. As these exclusive processes are driven by the gluon content of the target, with the cross sections being proportional to the square of the scattering amplitude, they are strongly sensitive to the underlying QCD dynamics.

In the color dipole approach the exclusive production $\gamma^* A \rightarrow EY$ ($E = \gamma, \rho, J/\Psi, ...$) in electron-nucleus interactions at high energies can be factorized in terms of the fluctuation of the virtual photon into a $q\bar{q}$ color dipole, the dipole-nucleus scattering by a color singlet exchange and the recombination into the exclusive final state $E$. This process is characterized by a rapidity gap in the final state. If the nucleus scatters elastically, $Y = A$, the process is called coherent production. On the other hand, if the nucleus scatters inelastically, i.e. breaks up ($Y = A'$), the process is denoted incoherent production. In Refs. [21, 22] we have presented a detailed analysis of the impact of the saturation effects in the nuclear DVCS processes and in the $\rho$ meson production considering coherent and incoherent $eA$ collisions. In particular, results for the distributions on the square of the momentum transfer ($t$) were presented, which are an important source of information about the spatial distribution of the gluons in a nucleus and about fluctuations of the nuclear color fields. In Fig. 5 we present a comparison

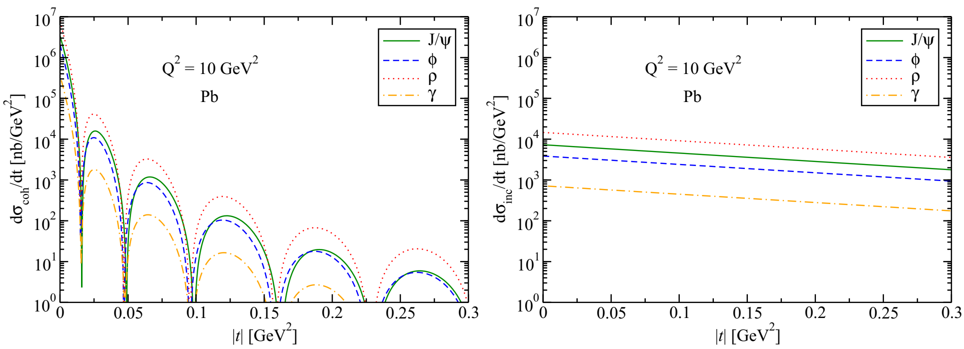


**Figure 5.** Comparison between the differential cross sections for coherent and incoherent interactions considering $Q^2 = 10$ GeV$^2$.

between our predictions for the $\rho$ production with those for other exclusive final states: $J/\Psi$, $\phi$ and $\gamma$. The results are presented for $Q^2 = 10$ GeV$^2$ and $A = Pb$. In Ref. [22] we have demonstrated that for $Q^2 = 0$ the differential cross sections decrease at heavier vector mesons and the position of the dips for the $J/\Psi$ production is slightly different of the other mesons, which is associated to the dominance of small dipoles in this final state. In contrast, the results for $Q^2 = 10$ GeV$^2$ presented in Fig. 5, indicate that the position of the dips becomes almost identical for all exclusive final states.

Finally, in order to estimate the magnitude of the non linear effects, we present in Fig. 6 a comparison between our predictions and those obtained disregarding the multiple scatterings of the dipole with the nuclei and the presence of the non linear effects in the nucleon. Basically, we will assume as input in our calculations the following models for the scattering amplitudes:

$$\mathcal{N}^A(x, r, b) = \frac{1}{2}\sigma_{dip}(x, r) A T_A(b) \tag{5}$$

with $\sigma_{dip}$ given by Eq. (2) and $\mathcal{N}^p(x, \boldsymbol{r}, \boldsymbol{b})$ given by the linear part of the bCGC model

$$\mathcal{N}^p(x, \boldsymbol{r}, \boldsymbol{b}) = \mathcal{N}_0 \left(\frac{r\, Q_s(b)}{2}\right)^{2\left(\gamma_s + \frac{\ln(2/rQ_s(b))}{\kappa \lambda Y}\right)} . \tag{6}$$

We can observe that the incoherent cross sections are not strongly modified by the non linear effects. In contrast, in the case of coherent interactions, the magnitude of the cross section and position of the dips are distinct in the non linear and linear predictions, which makes the analysis of this observable a sensitive probe of the non linear QCD dynamics.

## 4 Conclusions

The study of exclusive processes in deep inelastic scattering (DIS) processes have been one of the main focuses of hadronic physics in the last years. It is expected that analysis of these processes allow us to probe the QCD dynamics at high energies, driven by the gluon content of the target (proton or nucleus) which is strongly subject to non linear effects (parton saturation) effects. In particular, electron - nucleus collisions the gluon density probed is amplified due to the coherent contributions from many nucleons. In this contribution we have discussed the search of saturation effects in inclusive and exclusive $eA$ processes. These results demonstrated that although the inclusive observables

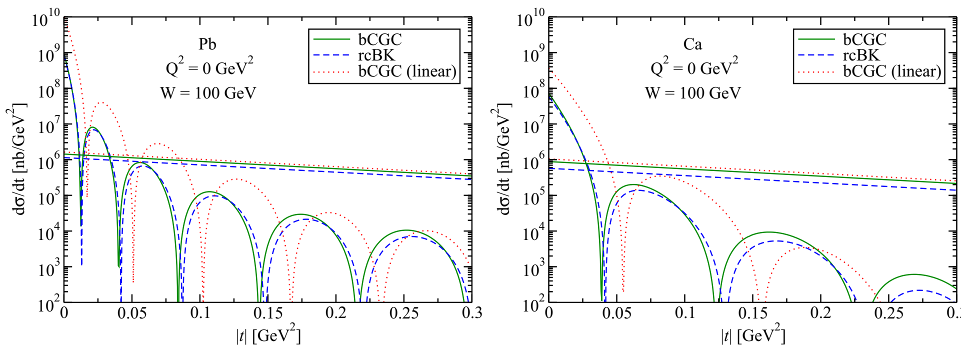


**Figure 6.** Comparison between non linear and linear predictions for the differential cross sections in coherent and incoherent interactions.

are sensitive to saturation effects, it is not yet possible to draw any firm conclusion concerning the QCD dynamics from inclusive quantities due to the large uncertainty present in the collinear predictions. In contrast, exclusive processes are promising observables to search saturation effects, due to the quadratic dependence on the forward scattering amplitude. In particular, the analysis of the nuclear DVCS and vector meson production demonstrated that the energy dependence of the differential cross sections are strongly modified with the increasing of the atomic mass number and that coherent cross section dominates at small $t$ and the incoherent one at large $t$. Moreover, the number of dips at small $t$ increases with the atomic number, with the position of the dips being almost independent of the model used to treat the dipole - proton interaction. Furthermore, the non linear effects implies different positions for the dips in comparison to the linear one. These results are robust predictions from the saturation physics, which can be used to challenge the treatment of the non linear QCD dynamics in the kinematical range of future electron - ion experiments.

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