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Di-jet production and angular correlations in DIS at NLO

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Abstract. Angular correlations are a sensitive probe of the dynamics of QCD at high energy. In particular azimuthal angular correlations between two hadrons produced in Deeply Inelastic Scattering (DIS) of a virtual photon on a hadron or nucleus offer the best environment in which to investigate high gluon density (gluon saturation) effects expected to arise at small x. Here we give a progress report on our derivation of Next to Leading Order (NLO) corrections to di-jet (di-hadron) production in DIS.

1 Introduction

Behavior of hadronic cross sections at high energy is one of the most interesting aspects of QCD where significant progress has been made in the last few years. Perturbative QCD calculations show a growth of hadronic cross sections with energy which would eventually violate unitarity bounds due to the untamed growth of parton distribution functions of a hadron with $1/x$. This fast rise of parton (specially gluon) distribution functions with decreasing x was one of the two surprises seen in DIS experiments at HERA [1]. Even before this experimental observation, theoretical considerations of gluon radiation in pQCD had predicted a power-like growth of gluon distribution function $xG(x, Q^2) \sim x^{-\delta}$ with $\delta \sim 0.3-0.5$ [2]. It was suggested that due to the large number of radiated gluons into a small area they would eventually overlap in the transverse plane and "recombine" [3, 4]. This lead to the idea of gluon saturation and its subsequent evolution into a Color Glass Condensate (CGC) [5]. It has been applied to high energy collisions involving at least one hadron. The two main physics effects encoded in the CGC formalism are *i*) multiple scattering (in the target frame) and *ii*) non-linear evolution of a hadronic or nuclear wave function with x (equivalently with energy or rapidity) [6, 7]. The first effect leads to a broadening of the transverse momentum spectra which appears as the so-called Cronin peak (enhancement) in pA collisions. The second effect due to non-linear evolution leads to a suppression of the transverse momentum spectra in pA as compared to a pp collisions, when properly normalized. Depending on the center of mass energy of the collision and the rapidity of the produced particles one or the the other effect will dominate. These two effects distinguish the CGC approach from that of collinear factorization based pQCD approaches and lead to perturbative unitarization of hadronic cross sections (at fixed impact parameter).

The CGC formalism, when applied to single inclusive particle production [8] in a high energy proton-nucleus (pA) collision, predicts an enhancement of the transverse momentum spectra in pA

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collisions as compared to that in a proton-proton collision (normalized) in the mid rapidity region. This is knows as the Cronin effect. More significantly the CGC formalism successfully predicted a suppression of the transverse momentum spectra in pA collisions as one moved from the mid rapidity to the forward rapidity region [9]. This is due to non-linear evolution of the nucleus wave function with x which reduces the number of partons in the nucleus. This theoretical prediction was subsequently verified experimentally and was considered a major indication of presence saturation effects in the target nucleus wave function.

More significantly, a disappearance of the away side peak (at 180 degrees) in di-hadron production in the forward rapidity region of proton-nucleus collisions was predicted [10]. The effect is two-fold, first there is broadening of the away side peak due to multiple scatterings and second, there is the reduction in the number of partons in the target wave function due to the non-linear evolution with x [11]. The disappearance of the away side was subsequently observed in the forward proton (deuteron)- gold collisions at RHIC and was a significant indication of dominance of saturation dynamics in the forward rapidity region at RHIC. It should however be noted that approaches based on collinear factorization in pQCD which include a modification of nuclear parton distribution unctions (nuclear shadowing) and some cold matter energy loss have also been able to fit the forward rapidity data at RHIC [12].

Despite the many successes of CGC formalism in describing particle production at RHIC, there are still some aspects that need improvement. Often particle production in the CGC formalism is know only at the Leading Order (LO) level. To be able to make truly quantitative predictions one need a NLO description [13]. In this work we take the first step in this direction by considering three parton production in DIS using the CGC formalism [14]. This will be useful in two aspects; first it gives us another (and less inclusive) particle production channel in which the CGC formalism can be applied and tested. While in di-hadron angular correlation studies one fixed the kinematics of the leading hadron and varies the momenta of the away side hadron, here one can vary the momenta of two away side hadrons which provides an additional handle on probing the dynamics of gluon saturation. Second, upon integrating out one of the three final state particles it will give us the real part of the NLO di-hadron production cross section. Including the virtual corrections then will give us the complete NLO di-hadron production cross section which can then be used to investigate azimuthal angular correlations between the produced hadrons.

2 Three parton production in DIS at small x

We start by considering $\gamma^\star \to q\,\bar{q}\,g$ at high energy. There are two diagrams as shown in figure 1 which can be understood as a two step process. First the virtual photon splits into a quark anti-quark pair that interacts with the target and then either one of the quark or anti-quark radiates a gluon. This is shown in the first diagram. The second diagram corresponds to the virtual photon splitting into a quark anti-quark pair, one of which radiates a gluon and the three final state partons interact with the target. Alternatively one can think of the gluon radiation happening either before or after the scattering from the target. Finally, only radiation from a quark (top line) is shown so that there are two additional diagrams where the gluon is radiated from the anti-quark (bottom line). These additional diagram will give results which can be obtained from the diagrams shown by a trivial change of momenta.

It should be noted that there is no radiation during the passage of the quark anti-quark pair through the target due to the eikonal kinematics (target moves with the speed of light). Furthermore it can be shown that all diagrams where only one (either one or two) of the lines in the first (second) diagram interacts with the target cancel amongst each other.

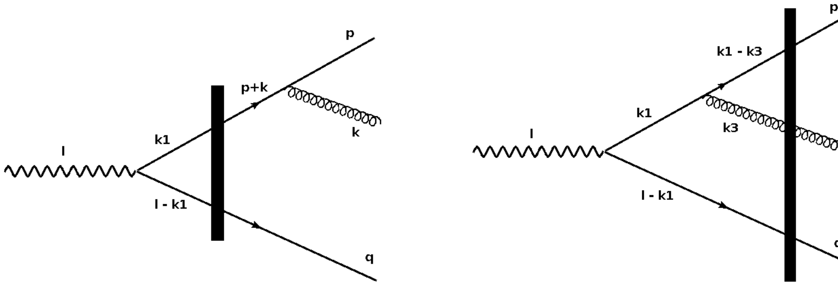

Figure 1. Leading Order 3-parton production diagrams corresponding to $\mathcal{A}_1$ (left) and $\mathcal{A}_2$ (right). The solid thick line represents interactions with the target (shock wave). The diagrams where the anti-quark radiates the gluon ($\mathcal{A}_3$ and $\mathcal{A}_4$) are not shown.

The Leading Order (LO) amplitude can be written as

$$\mathcal{A}^a = \mathcal{A}_1^a + \mathcal{A}_2^a + \mathcal{A}_3^a + \mathcal{A}_4^a \tag{1}$$

with

$$\begin{aligned}
\mathcal{A}_1^a &= (ie)(ig)\bar{u}(p)\,\gamma^\mu\, t^a\, S_F(p+k,k_1)\gamma^\nu\, S_F(l-k_1,q)\left[S_F^0(q)\right]^{-1}\, v(q)\,\epsilon_\nu^\star(l)\,\epsilon_\mu(k)\,\frac{d^4k_1}{(2\pi)^4}\\
\mathcal{A}_2^a &= (ie)(ig)\bar{u}(p)\,\left[S_F^0(p)\right]^{-1}\, S_F(p,k_1-k_3)\gamma^\lambda\, t^c\, S_F^0(k_1)\gamma^\nu\, S_F(l-k_1,q)\,\left[S_F^0(q)\right]^{-1}\, v(q)\\
&\quad \left[G_\lambda{}^\delta\right]^{ca}(k_3,k)\left[G_\delta^{0\mu}(k)\right]^{-1}\,\epsilon_\nu^\star(l)\,\epsilon_\mu(k)\,\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}\\
\mathcal{A}_3^a &= (ie)(ig)\bar{u}(p)\,\left[S_F^0(p)\right]^{-1}\, S_F(p,k_1)\,\gamma^\nu S_F(l-k_1,q+k)\,\gamma^\mu\, t^a\, v(q)\,\epsilon_\nu^\star(l)\,\epsilon_\mu(k)\,\frac{d^4k_1}{(2\pi)^4}\\
\mathcal{A}_4^a &= (ie)(ig)\bar{u}(p)\,\left[S_F^0(p)\right]^{-1}\, S_F(p,k_1)\,\gamma^\nu\, S_F^0(l-k_1)\gamma^\lambda\, t^c\, S_F(l-k_1-k_3,q)\,\left[S_F^0(q)\right]^{-1}\\
&\quad \left[G_\lambda{}^\delta\right]^{ca}(k_3,k)\left[G_\delta^{0\mu}(k)\right]^{-1}\,\epsilon_\nu^\star(l)\,\epsilon_\mu(k)\,\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}
\end{aligned} \tag{2}$$

The quark or gluon propagators in the background field (shock wave) are defined as

$$\begin{aligned}
S_F(p,q) &\equiv S_F^0(2\pi)^4\delta^4(p-q) + S_F^0\tau_F(p,q)S_F^0(q)\\
G_{\mu\nu}^{ab}(p,q) &\equiv G_{\mu\nu}^{0\,ab}(p)\,\delta^{ab}\,(2\pi)^4\delta^4(p-q) + G^0{}_{\mu\lambda}(p)\tau_g^{ab}(p,q)G^{0,\lambda}{}_\nu(q)
\end{aligned} \tag{3}$$

where $S_F^0, G_{\mu\nu}^{0,ab}$ are the free fermion and gluon propagators,

$$S_F^0(p) = \frac{i(\not p + m)}{p^2-m^2+i\epsilon} \qquad G_{\mu\nu}^{0,ab}(k) = \frac{id_{\mu\nu}(k)}{k^2+i\epsilon}\,\delta^{ab} \tag{4}$$

with the projector

$$d_{\mu\nu}(k) \equiv \left[-g_{\mu\nu} + \frac{k_\mu n_\nu + k_\nu n_\mu}{n\cdot k}\right] \tag{5}$$

and τ_F, τ_g contain the all-order scattering from the target color field, defined via

$$\begin{aligned}
\tau_F(p,q) &\equiv (2\pi)\delta(p^- - q^-)\gamma^- \int d^2x_t e^{i(p_t-q_t)\cdot x_t}\left\{\theta(p^-)\left[V(x_t)-1\right] - \theta(-p^-)\left[V^\dagger(x_t)-1\right]\right\}\\
\tau_g(p,q) &\equiv -(2\pi)\delta(p^- - q^-)2p^- \int d^2x_t e^{i(p_t-q_t)\cdot x_t}\left\{\theta(p^-)\left[U(x_t)-1\right] - \theta(-p^-)\left[U^\dagger(x_t)-1\right]\right\}
\end{aligned}$$

where V, U are Wilson lines in the fundamental and adjoint representation of $SU(N_c)$,

$$V(x_t) \equiv \hat{P}exp\,\{ig \int dx^- A^+_a(x^-, x_t)\, t_a\}$$
$$U(x_t) \equiv \hat{P}exp\,\{ig \int dx^- A^+_a(x^-, x_t)\, T_a\}$$

and t_a and T_a are the corresponding generators. $\epsilon_\nu(l), \epsilon_\mu(k)$ are the polarization vectors of the incoming virtual photon and the outgoing gluon respectively. We will set the transverse momentum of the virtual photon equal to zero so that $l^+ = -Q^2/2l^-$ without any loss of generality.

Noticing that the couplings, the spinors and the polarization vectors will be common to all 4 diagrams, we define the reduced amplitude A via

$$\mathcal{A} \equiv -eg\, \bar{u}(p)\, [A]^{\mu\nu}\, v(q)\, \epsilon^\star_\nu(l)\, \epsilon_\mu(k) \tag{6}$$

with

$$\begin{aligned}
A^{\mu\nu}_1 &= \gamma^\mu\, t^a\, S^0_F(p+k)\, \tau_F(p+k, k_1)\, S^0_F(k_1)\gamma^\nu\, S^0_F(l-k_1)\, \tau_F(l-k_1, q)\, \frac{d^4k_1}{(2\pi)^4} \\
A^{\mu\nu}_2 &= \tau_F(p, k_1-k_3)\, S^0_F(k_1-k_3)\gamma^\lambda\, t^a\, S^0_F(k_1)\, \gamma^\nu\, S^0_F(l-k_1)\, \tau_F(l-k_1, q)\, G^{0\mu}_\lambda(k_3)\tau^{ac}_g(k_3, k) \\
&\quad \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4} \\
A^{\mu\nu}_3 &= \tau_F(p, k_1)\, S^0_F(k_1)\, \gamma^\nu\, S^0_F(l-k_1)\, \tau_F(l-k_1, q+k) S^0_F(q+k)\, \gamma^\mu\, t^a\, \frac{d^4k_1}{(2\pi)^4} \\
A^{\mu\nu}_4 &= \tau_F(p, k_1)\, S^0_F(k_1)\, \gamma^\nu\, S^0_F(l-k_1)\, \gamma^\lambda\, t^c S^0_F(l-k_1-k_3)\tau_F(l-k_1-k_3, q)\, G^{0\,\mu}_\lambda(k_3)\tau^{ca}_g(k_3, k) \\
&\quad \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}
\end{aligned} \tag{7}$$

To proceed further we perform the momentum integrations in the amplitude; to be specific let's consider $A^{\mu\nu}_1$ given in Eq. (7). The integration over d^4k_1 can be performed in two different ways: either by introducing the standard Feynman parameters, or by taking advantage of the delta function in k^-_1 and performing the integration over k^+_1 using contour integration on the complex plane. Following the second choice we get

$$\begin{aligned}
A^{\mu\nu}_1 &= \frac{i}{2\, l^-}\frac{1}{(p+k)^2}\, \delta(l^- - p^- - q^- - k^-) \int d^2x_t\, d^2y_t e^{-i(p_t+k_t)\cdot x_t} e^{-iq_t\cdot y_t} V(x_t)\, V^\dagger(y_t)\, \gamma^\mu\, t^a\, i(\not{p} + \not{k}) \\
&\quad \gamma^-\, i\not{k}_1\, \gamma^\nu\, i(\not{l} - \not{k}_1)\, \gamma^-\, K_0\, [L\,(x_t - y_t)]
\end{aligned} \tag{8}$$

where K_0 is the modified Bessel function, $L^2 = \frac{q^-(p^-+k^-)}{l^-\, l^-} Q^2$ and $k^-_1 = p^- + k^- = l^- - q^-$, $k^+_1 = \frac{k^2_{1t} - i\epsilon}{2(p^-+k^-)}$ and all k_{1t} terms appearing above are to be understood as $-i\partial_{x_t - y_t}$ acting on the Bessel function K_0. We also note the presence of $\frac{1}{(p+k)^2}$ term above which will appear as a collinear divergence when one integrates over the parent quark or gluon in the final state in order to compute the NLO corrections to di-hadron production cross section. This will be identified as one of the contributions to the scale evolution of quark or gluon fragmentation function. Contribution of the $A^{\mu\nu}_3$ proceeds in exactly the same way. On the other hand one can not perform all the integrations in $A^{\mu\nu}_2$ and $A^{\mu\nu}_4$ where one is left with a one-dimensional integral which must be evaluated numerically.

The next step is to square the amplitude and to evaluate the Dirac traces that appear. This is done using FORM and the result is long. Furthermore, the result will involve trace of products of multiple

Wilson lines [15–17] whose evolution equation is known. As a last step one needs to specify the kinematics of the three produced hadrons and convolute the three final state partons with the corresponding parton-hadron fragmentation functions. To avoid issues dealing with di-hadron fragmentation functions we assume the produced hadrons are not too close in azimuthal angle so that using independent fragmentation functions may be justified. Alternatively one can use our results for three-jet (rather than hadron) production which will sidestep the issue of independent hadronization.

3 Summary

Three-parton (hadron) azimuthal angular correlations in DIS are a sensitive probe of gluon saturation dynamics in high energy QCD. They can also be used to calculate the real corrections to NLO corrections to di-hadron production cross section. Work in this direction is in progress and will be reported on elsewhere [14].

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