

## New advances in the statistical parton distributions approach <sup>\*</sup>

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**Abstract.** The quantum statistical parton distributions approach proposed more than one decade ago is revisited by considering a larger set of recent and accurate Deep Inelastic Scattering experimental results. It enables us to improve the description of the data by means of a new determination of the parton distributions. This global next-to-leading order QCD analysis leads to a good description of several structure functions, involving unpolarized parton distributions and helicity distributions, in terms of a rather small number of free parameters. There are many serious challenging issues. The predictions of this theoretical approach will be tested for single-jet production and charge asymmetry in  $W^\pm$  production in  $\bar{p}p$  and  $pp$  collisions up to LHC energies, using recent data and also for forthcoming experimental results.

### 1 Introduction

Deep Inelastic Scattering (DIS) of leptons and nucleons is indeed our main source of information to study the internal nucleon structure in terms of parton distributions. Several years ago a new set of parton distribution functions (PDFs) was constructed in the framework of a statistical approach of the nucleon [1]. For quarks (antiquarks), the building blocks are the helicity dependent distributions  $q^\pm(x)$  ( $\bar{q}^\pm(x)$ ). This allows to describe simultaneously the unpolarized distributions  $q(x) = q^+(x) + q^-(x)$  and the helicity distributions  $\Delta q(x) = q^+(x) - q^-(x)$  (similarly for antiquarks). At the initial energy scale  $Q_0^2$ , these distributions are given by the sum of two terms, a quasi Fermi-Dirac function and a helicity independent diffractive contribution. The flavor asymmetry for the light sea, *i.e.*  $\bar{d}(x) > \bar{u}(x)$ , observed in the data is built in. This is simply understood in terms of the Pauli exclusion principle, based on the fact that the proton contains two up-quarks and only one down-quark. We predict that  $\bar{d}(x)/\bar{u}(x)$  must remain above one for all  $x$  values and this is a real challenge for our approach, in particular in the large  $x$  region which is under experimental investigation at the moment. The flattening out of the ratio  $d(x)/u(x)$  in the high  $x$  region, predicted by the statistical approach, is another interesting challenge worth mentioning. The chiral properties of QCD lead to strong relations between  $q(x)$  and  $\bar{q}(x)$ . For example, it is found that the well established result  $\Delta u(x) > 0$  implies  $\Delta \bar{u}(x) > 0$  and similarly  $\Delta d(x) < 0$  leads to  $\Delta \bar{d}(x) < 0$ . This earlier prediction was confirmed by

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recent polarized DIS data and it was also demonstrated that the magnitude predicted by the statistical approach is compatible with recent BNL-RHIC data on  $W^\pm$  production [2]. In addition we found the approximate equality of the flavor asymmetries, namely  $\bar{d}(x) - \bar{u}(x) \sim \Delta\bar{u}(x) - \Delta\bar{d}(x)$ . Concerning the gluon, the unpolarized distribution  $G(x, Q_0^2)$  is given in terms of a quasi Bose-Einstein function, with only *one free parameter*. The new analysis of a larger set of recent accurate DIS data leads to the emergence of a large positive gluon helicity distribution, giving a significant contribution to the proton spin, a major point which was emphasized in a recent letter [3].

It is crucial to note that the quantum-statistical approach differs from the usual global parton fitting methodology for the following reasons:

- i) It incorporates physical principles to reduce the number of free parameters which have a physical interpretation
- ii) It has very specific predictions, so far confirmed by the data
- iii) It is an attempt to reach a more physical picture on our knowledge of the nucleon structure, the ultimate goal being to solve the problem of confinement
- iv) Treating simultaneously unpolarized distributions and helicity distributions, a unique situation in the literature, has the advantage to give access to a vast set of experimental data, in particular up to LHC energies

## 2 Review of the statistical parton distributions

Let us now recall the main features of the statistical approach for building up the PDFs, as opposed to the standard polynomial type parameterizations of the PDF, based on Regge theory at low  $x$  and on counting rules at large  $x$ . The fermion distributions are given by the sum of two terms, a quasi Fermi-Dirac function and a helicity independent diffractive contribution, at the input energy scale  $Q_0^2 = 1\text{GeV}^2$ ,

$$xq^h(x, Q_0^2) = \frac{A_q X_{0q}^h x^{b_q}}{\exp[(x - X_{0q}^h)/\bar{x}] + 1} + \frac{\tilde{A}_q x^{\tilde{b}_q}}{\exp(x/\bar{x}) + 1}, \quad (1)$$

$$x\bar{q}^h(x, Q_0^2) = \frac{\bar{A}_q (X_{0q}^{-h})^{-1} x^{\tilde{b}_q}}{\exp[(x + X_{0q}^{-h})/\bar{x}] + 1} + \frac{\tilde{A}_q x^{\tilde{b}_q}}{\exp(x/\bar{x}) + 1}. \quad (2)$$

We note that the universal diffractive term is absent in the quark helicity distribution  $\Delta q$  and in the quark valence contribution  $q - \bar{q}$ .

In Eqs. (1,2) the multiplicative factors  $X_{0q}^h$  and  $(X_{0q}^{-h})^{-1}$  in the numerators of the non-diffractive parts of the  $q$ 's and  $\bar{q}$ 's distributions, imply a modification of the quantum statistical form, we were led to propose in order to agree with experimental data. The presence of these multiplicative factors was justified in our earlier attempt to generate the transverse momentum dependence (TMD) [4, 5]. The parameter  $\bar{x}$  plays the role of a *universal temperature* and  $X_{0q}^\pm$  are the two *thermodynamical potentials* of the quark  $q$ , with helicity  $h = \pm$ . They represent the fundamental characteristics of the model. Notice the change of sign of the potentials and helicity for the antiquarks <sup>1</sup>.

For a given flavor  $q$  the corresponding quark and antiquark distributions involve *eight* free parameters:  $X_{0q}^\pm, A_q, \bar{A}_q, \tilde{A}_q, b_q, \tilde{b}_q$  and  $\tilde{b}_q$ . It reduces to *seven* since one of them is fixed by the valence sum rule,  $\int (q(x) - \bar{q}(x))dx = N_q$ , where  $N_q = 2, 1, 0$  for  $u, d, s$ , respectively.

<sup>1</sup> At variance with statistical mechanics where the distributions are expressed in terms of the energy, here one uses  $x$  which is clearly the natural variable entering in all the sum rules of the parton model.

For the light quarks  $q = u, d$ , the total number of free parameters is reduced to *eight* by taking, as in Ref. [1],  $A_u = A_d$ ,  $\tilde{A}_u = \tilde{A}_d$ ,  $\tilde{A}_u = \tilde{A}_d$ ,  $b_u = b_d$ ,  $\tilde{b}_u = \tilde{b}_d$  and  $\tilde{b}_u = \tilde{b}_d$ . For the strange quark and antiquark distributions, the simple choice made in Ref. [1] was improved in Ref. [6], but here they are expressed in terms of *seven* free parameters.

For the gluons we consider the black-body inspired expression

$$xG(x, Q_0^2) = \frac{A_G x^{b_G}}{\exp(x/\bar{x}) - 1}, \quad (3)$$

a quasi Bose-Einstein function, with  $b_G$  being the only free parameter, since  $A_G$  is determined by the momentum sum rule. In our earlier works [1, 7], we were assuming that, at the input energy scale, the helicity gluon distribution vanishes, so

$$x\Delta G(x, Q_0^2) = 0. \quad (4)$$

However as a result of the present analysis of a much larger set of very accurate unpolarized and polarized DIS data, we must give up this simplifying assumption. We are now taking

$$x\Delta G(x, Q_0^2) = \frac{\tilde{A}_G x^{\tilde{b}_G}}{(1 + c_G x^{d_G})} \cdot \frac{1}{\exp(x/\bar{x}) - 1}. \quad (5)$$

To summarize the new determination of all PDFs involves a total of *twenty one* free parameters: in addition to the temperature  $\bar{x}$  and the exponent  $b_G$  of the gluon distribution, we have *eight* free parameters for the light quarks ( $u, d$ ), *seven* free parameters for the strange quarks and *four* free parameters for the gluon helicity distribution. These parameters have been determined from a next-to-leading order (NLO) QCD fit of a large set of accurate DIS data, unpolarized and polarized structure functions [8].

### 3 A selection of results

Some selected experimental tests for the unpolarized PDFs have been considered from  $\mu N$  and  $eN$  DIS, for which several experiments have yielded a large number of data points on the structure functions  $F_2^N(x, Q^2)$ ,  $N$  stands for either a proton or a deuterium target. We have used fixed target measurements which probe a rather limited kinematic region in  $Q^2$  and  $x$  and also HERA data which cover a very large  $Q^2$  range and probe the very low  $x$  region, dominated by a fast rising behavior, consistent with our diffractive term (See Eq. (1)).

For illustration of the quality of our fit and, as an example, we show in Fig. 1, our results for  $F_2^p(x, Q^2)$  on different fixed proton targets, together with H1 and ZEUS data. We note that the analysis of the scaling violations leads to a gluon distribution  $xG(x, Q^2)$ , in fairly good agreement with our simple parameterization (See Eq. (3)).

We now turn to the important issue concerning the asymmetries  $A_1^{p,d,n}(x, Q^2)$ , measured in polarized DIS. We recall the definition of the asymmetry  $A_1(x, Q^2)$ , namely

$$A_1(x, Q^2) = \frac{[g_1(x, Q^2) - \gamma^2(x, Q^2)g_2(x, Q^2)]}{F_2(x, Q^2)} \frac{2x[1 + R(x, Q^2)]}{[1 + \gamma^2(x, Q^2)]}, \quad (6)$$

where  $g_{1,2}(x, Q^2)$  are the polarized structure functions,  $\gamma^2(x, Q^2) = 4x^2 M_p^2 / Q^2$  and  $R(x, Q^2)$  is the ratio between the longitudinal and transverse photoabsorption cross sections. When  $x \rightarrow 1$  for  $Q^2 = 4 \text{ GeV}^2$ ,  $R$  is the order of 0.30 or less and  $\gamma^2(x, Q^2)$  is close to 1, so if the  $u$  quark dominates, we

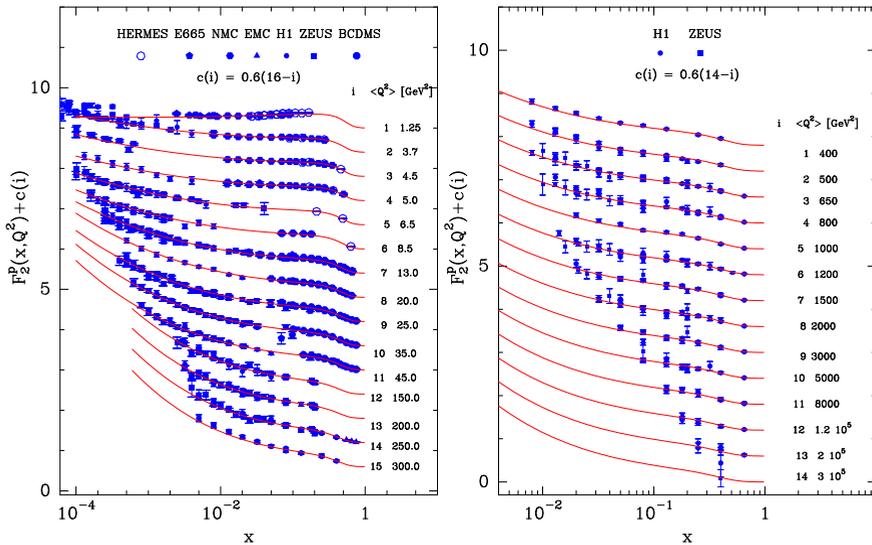
have  $A_1 \sim 0.6\Delta u(x)/u(x)$ . Therefore it is unlikely to find  $A_1 \rightarrow 1$ , as required by the counting rules prescription, which we don't impose. We display in Fig. 2 the world data on  $A_1^p(x, Q^2)$  (*Left*) and  $A_1^n(x, Q^2)$  (*Right*), with the results of the statistical approach at  $Q^2 = 4\text{GeV}^2$ , up to  $x = 1$ . Indeed we find that  $A_1^{p,n} < 1$ .

Finally one important outcome of this new analysis of the polarized DIS data in the framework of the statistical approach, is the confirmation of a large positive gluon helicity distribution, which gives a significant contribution to the proton spin [3].

The NLO QCD calculations at  $\mathcal{O}(\alpha_s^3)$  of the cross section for the production of a single-jet of rapidity  $y$  and transverse momentum  $p_T$ , in a  $pp$  or  $\bar{p}p$  collision, were done using a code based on a semi-analytical method within the "small-cone approximation", improved recently with a jet algorithm for a better definition <sup>2</sup>. In Fig. 3(*Left*) our results are compared with the data from STAR experiment at BNL-RHIC and this prediction agrees very well with the data.

Now we would like to test, in a pure hadronic collision, our new positive gluon helicity distribution, mentioned above. In a recent paper, the STAR experiment at BNL-RHIC has reported the observation, in single-jet inclusive production, of a non-vanishing positive double-helicity asymmetry  $A_{LL}^{jet}$  for  $5 \leq p_T \leq 30\text{GeV}$ , in the near-forward rapidity region [10]. We show in Fig. 3(*Right*) our prediction compared with these high-statistics data points and the agreement is very reasonable.

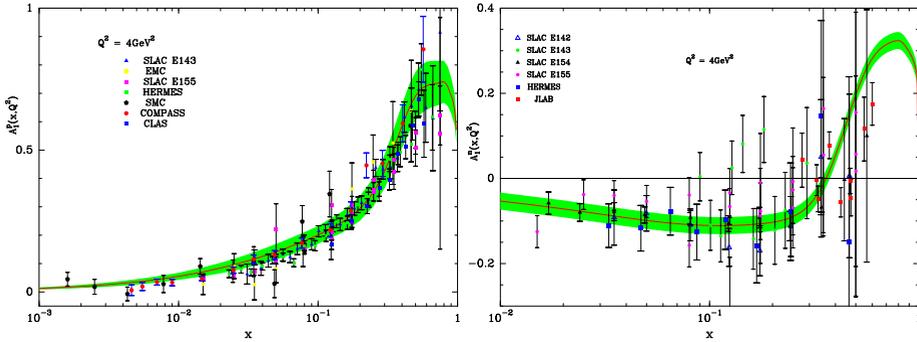
There are several data sets for the cross section of single-jet production, which allow us to test our predictions, in particular the results from ATLAS and CMS displayed in Fig. 4 at  $\sqrt{s} = 7\text{TeV}$ .



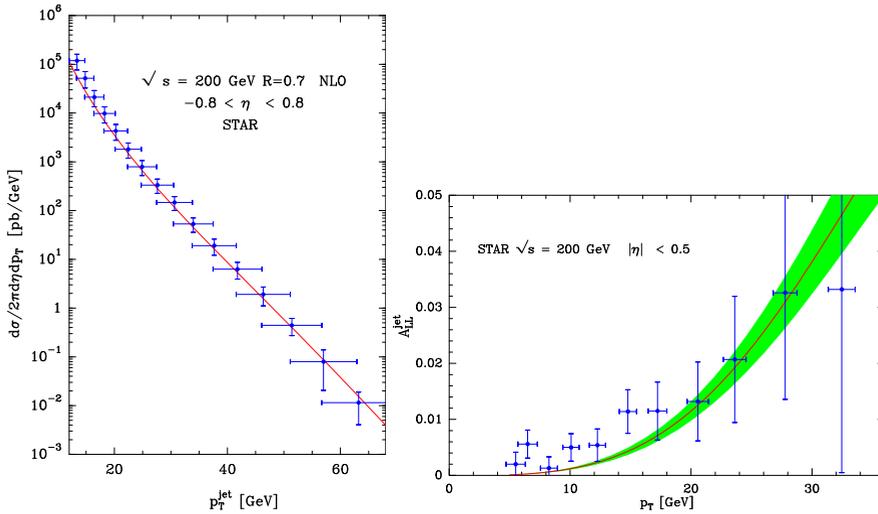
**Figure 1.**  $F_2^p(x, Q^2)$  as a function of  $x$  for fixed  $\langle Q^2 \rangle$  and data from HERMES, E665, NMC, EMC, H1, ZEUS, BCDMS. *Left*: The function  $c(i) = 0.6(16 - i)$ ,  $i = 1$  corresponds to  $\langle Q^2 \rangle = 1.25\text{GeV}^2$ . *Right*: The function  $c(i) = 0.6(14 - i)$ ,  $i = 1$  corresponds to  $\langle Q^2 \rangle = 400\text{GeV}^2$ . The curves are the results of the statistical approach.

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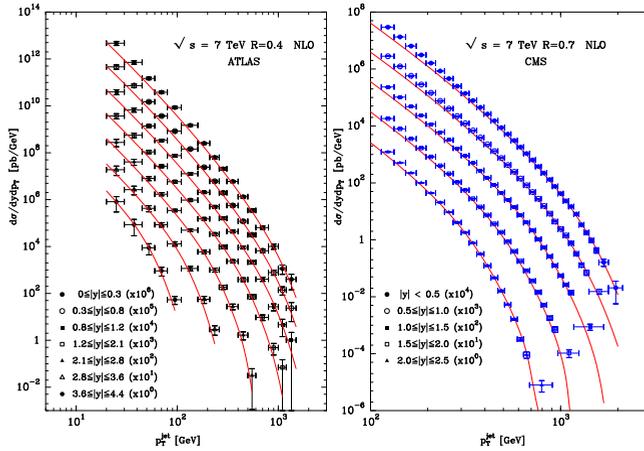
## Physics Opportunities at an Electron-Ion Collider



**Figure 2.** *Left* : Comparison of the world data on  $A_1^p(x, Q^2)$  with the result of the statistical approach at  $Q^2 = 4 \text{ GeV}^2$ , including the corresponding error band. *Right*: Comparison of the world data on  $A_1^n(x, Q^2)$  with the result of the statistical approach at  $Q^2 = 4 \text{ GeV}^2$ , including the corresponding error band.



**Figure 3.** *Left*: Double-differential inclusive single-jet cross section in  $pp$  collisions at  $\sqrt{s} = 200 \text{ GeV}$ , versus  $p_T^{jet}$ , with jet radius parameter  $R=0.7$ , for  $-0.8 < \eta < 0.8$ , from STAR data, obtained with an integrated luminosity of  $5.39 \text{ pb}^{-1}$  [9] and the prediction from the statistical approach. *Right*: Our predicted double-helicity asymmetry  $A_{LL}^{jet}$  for single-jet production at BNL-RHIC in the near-forward rapidity region, versus  $p_T$  and the data points from STAR [10], with the corresponding error band.



**Figure 4.** *Left:* Double-differential inclusive single-jet cross section in  $pp$  collisions at  $\sqrt{s} = 7\text{TeV}$ , versus  $p_T^{jet}$ , with jet radius parameter  $R = 0.4$ , for different rapidity bins from ATLAS [11] and the predictions from the statistical approach  
*Right:* Same from CMS [12], with  $R = 0.7$ .

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