Understanding the proton radius puzzle: Nuclear structure effects in light muonic atoms

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Abstract. We present calculations of nuclear structure effects to the Lamb shift in light muonic atoms. We adopt a modern *ab-initio* approach by combining state-of-the-art nuclear potentials with the hyperspherical harmonics method. Our calculations are instrumental to the determination of nuclear charge radii in the Lamb shift measurements, which will shed light on the proton radius puzzle.

1 Introduction

The proton radius extracted from recent high-precision measurements of the Lamb shift (2S-2P atomic transitions) in muonic hydrogen atom μH [1, 2] displays 7 standard deviations from results determined from eH spectroscopy or e-p elastic scattering. This discrepancy raises intriguing questions about lepton universality and challenges theories of standard model physics. To resolve the proton radius puzzle, the CREMA collaboration at Paul Scherrer Institute have performed new measurements of the Lamb shifts in other light muonic atoms (μD, μ^3He$^+$ and μ^4He$^+$) [3], with results yet to be published. These measurements aim to extract the nuclear charge radii with extremely high accuracy, which will be benchmarked with current and future spectroscopy measurements in electronic atoms [4–6] and electron-nucleus scattering data [7, 8].

The accuracy of the determined nuclear charge radius depends not only on the experimental precision of the Lamb shift, but also on the theoretical calculations. The theories relate the Lamb shift $\delta_{LS} \equiv E(2S) - E(2P)$ in a muonic atom/ion $\mu X^{(Z-1)+}$ to the charge radius $\langle r_{ch}^2 \rangle$ of a nucleus $_A^Z$X by

$$\delta_{LS} = \delta_{\text{QED}} + \mathcal{A}_{\text{OPE}} \langle r_{ch}^2 \rangle + \delta_{\text{TPE}}, \tag{1}$$

where δ_{QED} is the contribution from the quantum-electrodynamics (QED) including the photon vacuum polarization, muon self energy, and relativistic recoil corrections. The other two terms in Eq. (1) are corrections due to nuclear structure. The term linear to $\langle r_{ch}^2 \rangle$ is dominated by one-photon exchange

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between the muon and the nucleus (Fig. 1), where $\mathcal{A}_{\text{OPE}} \approx m_r^3 (Z\alpha)^4/12$ with $m_r = m_\mu M_X/(m_\mu + M_X)$ denoting the reduced mass in the muon (m_μ) and nucleus (M_X) system. δ_{TPE} represents the two photon-exchange (TPE) contribution (Fig. 1), which is a combination of elastic (δ_{Zem}) and inelastic (δ_{pol}) parts. δ_{Zem} is proportional to the third electric Zemach moment $\langle r_{ch}^3 \rangle_{(2)}$ by [9]

$$\delta_{\text{Zem}} = -m_r^4 (Z\alpha)^5 \langle r_{ch}^3 \rangle_{(2)}/24 \ . \tag{2}$$

The inelastic part δ_{pol} is the nuclear polarizability effect, where the nucleus in the muonic atom is virtually excited in the intermediate states by exchanging two photons with the muon.
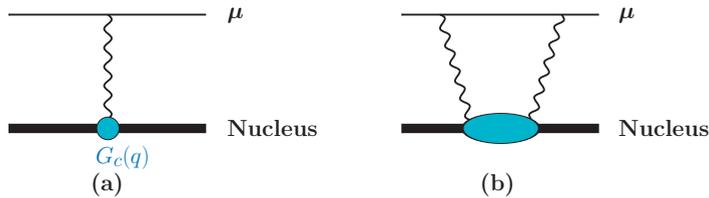

Figure 1. The muon-nucleus one-photon (a) and two-photon (b) exchange processes: The bulb in (a) indicates the insertion of nuclear electric form factor. The bulb in (b) denotes the excitation of nucleus in the intermediate states between two photons.

The bottleneck in accurately extracting the nuclear charge radius in Lamb shift measurements is the theoretical uncertainty in determining δ_{TPE}, especially δ_{pol}. To determine $\langle r_{ch}^2 \rangle$ of ^{3}He and ^{4}He from measurements of their muonic ions to a 3×10^{-4} accuracy, δ_{pol}, which strongly relies on our knowledge of nuclear structures and dynamics, has to be known within a 5% accuracy [3].

δ_{pol} can be represented as a combination of energy-dependent sum rules of a series of nuclear response functions. One way to calculate δ_{pol} is to use inputs from measurements of photo-absorption cross sections and quasielastic scattering data. However, the applications of this approach in μD [10] and $\mu\,^4$He$^+$ [11–13] do not provide the accuracy desired by the Lamb shift measurements. The high-accuracy requirement can be achieved alternatively through calculations using state-of-the-art nuclear potentials. For example, the studies of nuclear structure effects in μD using Argonne v_{18} (AV18) potentials and pionless effective field theory improved the accuracy to a ~ 1% level [14–16], which is one order of magnitude more accurate than previous methods.

We calculated the nuclear structure effects in μD using chiral effective field theory (χEFT) potentials [17]. Combining the modern inter-nucleon potentials with few-body methods, *i.e.*, effective interaction hyperspherical harmonics expansion (EIHH) [18] and Lanczos sum rules (LSR) [19], we also calculated δ_{TPE} in $\mu\,^4$He$^+$ [20, 21], $\mu\,^3$He$^+$ and $\mu\,^3$H [22]. By comparing the results using different potential models, we show that the uncertainty in δ_{TPE} is dominated by nuclear physics. Our results are key to the determination of $\langle r_{ch}^2 \rangle$ from future muonic atom measurements.

2 Nuclear structure effects in μD

δ_{TPE} in μD was calculated by Pachucki [14] with the implementation of AV18 nuclear potentials [23]. The dominant contribution is the sum rule of the electric-dipole response function, with corrections from the higher-multipole response function sum rules, relativistic and Coulomb distortion effects, *etc*. This calculation achieved a ~ 1% accuracy by systematically analyzing neglected higher-order effects in atomic physics. Our recent work [17] provided an independent derivation of the formalism

for δ_{TPE}, which is consistent with Ref. [14].[1] The later work by Pachucki *et al.* [16] carefully analyzed the relativistic-recoil effects in δ_{TPE}, which indicate, overall, a few parts per mil correction.

To fully analyze the uncertainty in nuclear physics, we used in Ref. [17] not only the AV18 potential but also χEFT with a series of parameterizations [24, 25]. We studied the order-by-order chiral convergence and the regulation-cutoff dependence. Our study shows a 0.6% uncertainty from potential variations, which must be included in the total uncertainty of δ_{TPE} in μD. With a further investigation, we include an additional correction $\delta_{\text{Zem}}^{hadr}$ to our results in Ref. [17] to be consistent with Refs. [15, 16]. This term is proportional to the proton Zemach moment, its contribution to δ_{TPE} in $\mu X^{(Z-1)+}$, is related to the elastic TPE effects in muonic hydrogen $\delta_{\text{Zem}}(\mu H)$ by

$$\delta_{\text{Zem}}^{hadr}(\mu X) = -[Z m_r(\mu X)]^4 \alpha^5 \langle r_p^3 \rangle_{(2)}/24 = [Z m_r(\mu X)/m_r(\mu H)]^4 \delta_{\text{Zem}}(\mu H) \qquad (3)$$

The hadronic polarizability in μX can be approximated from its effects in μH by

$$\delta_{\text{pol}}^{hadr}(\mu X) \approx Z^3(Z+N) [m_r(\mu X)/m_r(\mu H)]^3 \delta_{\text{pol}}^{hadr}(\mu H) . \qquad (4)$$

However, this estimate is based on the hadronic polarizability effect involving a single nucleon, but does not include the interference with nucleon-nucleon interactions. A more accurate calculation of $\delta_{\text{pol}}^{hadr}$ in μD is done by Carlson *et al.* by using high-Q^2 part of the deuteron scattering data [10]. The combination of $\delta_{\text{Zem}}^{hadr}(\mu X)$ and $\delta_{\text{pol}}^{hadr}(\mu X)$ yields a total hadronic TPE contributions in μD to be $\delta_{\text{TPE}}^{hadr} = -47(10)\,\mu$eV. Through collaborative communications among independent groups, δ_{TPE} in μD is agreed to be 1.709 ± 0.020 meV, which is adopted in a recent review by the CREMA collaboration [26] and will be used for extracting the deuteron charge radius in their Lamb shift measurement.

3 Nuclear structure effects in μ^4He$^+$, μ^3He$^+$ and μ^3H

To extend the calculations to other light muonic atoms, we applied the EIHH method to expand the nuclear Hamiltonians and wave functions in the hyperspherical harmonic basis. This method is suitable for Hamiltonians with local (AV18) or non-local (χEFT) potentials. The energy-dependent sum rules of the response functions are obtained by implementing the LSR method [19].

We calculated the nuclear structure effects to the Lamb shift in μ^4He$^+$, including the inelastic part of δ_{TPE} due to nuclear excitations ($\delta_{\text{pol}}^{nucl}$) [20] and the elastic part due to nuclear charge distributions (δ_{Zem}) [21]. We have recently improved the treatment of nucleon size effects and we show in Table 1 updated results of $\delta_{\text{pol}}^{nucl}$ for μD and μ^4He$^+$.[2] For the hadronic TPE effects, rigorous calculations using scattering data are not yet available. Therefore, we estimate such effects in μ^4He$^+$ using Eqs. (3) and (4). We find $\delta_{\text{pol}}^{hadr}(\mu^4He^+) \approx -0.34$ meV and $\delta_{\text{Zem}}^{hadr}(\mu^4He^+) \approx -0.54(12)$ meV with the error dominated by the uncertainty in neutron polarizability. δ_{TPE} in μD and μ^4He$^+$ are summarized in Table 1.

Table 1. Two-photon exchange contributions to the Lamb shift in light muonic atoms in units of meV.

	δ_{Zem}	$\delta_{\text{pol}}^{nucl}$	atomic error	$\delta_{\text{TPE}}^{hadr}$	δ_{TPE}
μD	-0.424(3)	-1.245(5)	±0.018	-0.047(10)	-1.716(20)
μ^4He$^+$	-6.29(28)	-2.36(10)	±0.10	-0.89(12)	-9.54(34)

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[1]Small discrepancies between Refs. [14, 17] appear in the reduced-mass–dependent coefficients of higher-multipole contributions. These are resolved later by Pachucki *et al.* in Ref. [16], which agrees with Ref. [17].

[2]The *np* correlated nucleon-size corrections to TPE were calculated in [20] with an inaccurate assumption that proton and neutron behave the same except for Coulomb. With a more rigorous calculation, we provide updated results.

We have recently extended our studies of TPE effects to μ^3He$^+$ and μ^3H. These muonic atoms of mirror isotopes, which contain unequal neutrons and protons, may provide additional information for the puzzle by probing the differences between muon-proton and muon-neutron interactions. The calculations of δ_{TPE} in μ^3He$^+$ and μ^3H will be detailed in an upcoming paper [22].

4 Conclusion

We illustrate *ab-initio* calculations of TPE contributions to the Lamb shift in light muonic atoms originated from nuclear structure. We provide the results for different muonic hydrogen and helium isotopes, which are important for determining the nuclear charge radii in Lamb shift measurements.

Acknowledgements

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