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Clothed Particles in Quantum Electrodynamics and Quantum Chromodynamics

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Abstract. The notion of clothing in quantum field theory (QFT), put forward by Greenberg and Schweber and developed by M. Shirokov, is applied in quantum electrodynamics (QED) and quantum chromodynamics (QCD). Along the guideline we have derived a novel analytic expression for the QED Hamiltonian in the clothed particle representation (CPR). In addition, we are trying to realize this notion in QCD (to be definite for the gauge group $SU(3)$) when drawing parallels between QCD and QED.

1 Introductory Remarks

As before in [1] - [4], by using the instant form of relativistic dynamics we start from a total Hamiltonian $H = H(\overset{\circ}{\alpha}) = H_0(\overset{\circ}{\alpha}) + V_0(\overset{\circ}{\alpha})$, where unperturbed Hamiltonian $H_0(\overset{\circ}{\alpha})$ and interaction term $V_0(\overset{\circ}{\alpha})$ depend on the creation and destruction operators of "bare" particles (e.g., bosons and fermions) with trial masses and coupling constants. Here $\overset{\circ}{\alpha}$ denotes the set of all such operators. Following [2] we introduce specific auxiliary transformations, reminiscent of the canonical transformations (in particular, the Bogoliubov ones) in the theory of superfluidity and superconductivity, that convert the primary bare bosons and fermions into some intermediate-level particles in a new α - representation with observable (in general, *a priori* given [1]) masses (cf. the terminology adopted in our survey [1]). As a result, the original Hamiltonian gets the form $H \equiv H(\alpha) = H_F(\alpha) + M_{ren}(\alpha) + V(\alpha) \equiv H_F(\alpha) + H_I(\alpha)$, where we meet the free Hamiltonian H_F expressed in terms of operators α and the operator M_{ren} which is contained the corresponding mass counterterms. Note that transition $\overset{\circ}{\alpha} \to \alpha$ remains the primary interaction intact and exemplifies one of nonequivalent representations of canonical commutation relations in QFT.

Just this form is subject to the similarity transformation $\alpha = W(\alpha_c)\alpha_c W^\dagger(\alpha_c)$ with the operator $W(\alpha_c) = W(\alpha) = \exp R$, $R = -R^\dagger$ that connects the bare particles representation (BPR) α and the clothed particles representation α_c. In its turn, transition $\alpha \to \alpha_c$ is a key point of the method of unitary clothing transformations (shortly, UCT method) for different approximations in our calculations with the Hamiltonian $H = W(\alpha_c)H(\alpha_c)W^\dagger(\alpha_c)$, the Lorentz boosts, the Nöther currents, etc. Physical constraints imposed upon W and mathematical aspects of the UCT method have been exposed in [1] and [4].

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[1] Some extension of our approach for field models such as QCD

2 Clothed Particles in QED

In the Coulomb gauge (CG) the interaction Hamiltonian of the spinor QED is given by

$$V_{qed} = \int d\vec{x} V_{qed}(\vec{x}) = -\int d\vec{x} j^k(\vec{x}) a_k(\vec{x}) + V_{Coul}, \tag{1}$$

with the electron-positron current $j^\mu(\vec{x}) = e\bar{\psi}(\vec{x})\gamma_\mu\psi(\vec{x})$ and the Coulomb part,

$$V_{Coul} = \frac{1}{2}\int d\vec{x}\int d\vec{y}\frac{j^0(\vec{x})j^0(\vec{y})}{4\pi|\vec{x}-\vec{y}|}. \tag{2}$$

Evidently, the corresponding interaction density $V_{qed}(x)$ in the Dirac(D) picture is not the Lorentz scalar and we cannot use the so-called Belinfante ansatz to construct the boost generator $\vec{N}$, i.e., put $\mathbf{N}_{qed} = -\int \mathbf{x} V_{qed}(\mathbf{x}) d\mathbf{x}$. Therefore one has to seek other ways to provide the relativistic invariance (RI) in Dirac sense (see, e.g., [4]).
Our departure point is the particle number representation with the Fourier expansions

$$a^\rho(\boldsymbol{x}) = \int \frac{d\boldsymbol{k}}{\sqrt{2(2\pi)^3|\boldsymbol{k}|}}\sum_\sigma [e^\rho(\boldsymbol{k},\sigma)c(\boldsymbol{k},\sigma) + e^{\rho*}(\boldsymbol{k},\sigma)c^\dagger(\boldsymbol{k},\sigma)]exp(i\boldsymbol{kx}), \tag{3}$$

$$\psi(\boldsymbol{x}) = \int d\boldsymbol{p}\sqrt{\frac{m}{(2\pi)^3 E_{\boldsymbol{p}}}}\sum_\mu [\bar{u}(\boldsymbol{p},\mu)b(\boldsymbol{p},\mu) + \upsilon(-\boldsymbol{p},\mu)d^\dagger(-\boldsymbol{p},\mu)]exp(i\boldsymbol{kx}), \tag{4}$$

where $E_{\boldsymbol{p}} = \sqrt{\boldsymbol{p}^2 + m^2}$ fermion energy on its mass shell and the creation(destruction) operators meet commutation relations $\{b(\boldsymbol{p},\mu), b^\dagger(\boldsymbol{p}',\mu')\} = \{d(\boldsymbol{p},\mu), d^\dagger(\boldsymbol{p}',\mu')\} = \delta_{\mu\mu'}\delta(\boldsymbol{p}-\boldsymbol{p}')$, $[c(\boldsymbol{k},\sigma), c^\dagger(\boldsymbol{k}',\sigma')] = \delta_{\sigma\sigma'}\delta(\boldsymbol{k}-\boldsymbol{k}')$.
After the first clothing transformation $W^{(1)} = \exp[R^{(1)}]$ ($R^{(1)\dagger} = -R^{(1)}$), which eliminates from V_{qed}, the primary interaction $V^{(1)}$, with its generator $R^{(1)}$ being determined by the equation

$$V^{(1)} + [R^{(1)}, H_F] = 0 \tag{5}$$

we find following [1] the clothed-particle interaction operators responsible for all relevant 2→2 processes (stemmed from $[R^{(1)}, V^{(1)}]$ commutator), the operators responsible for all 2→3 processes (from $[[R^{(1)}, [R^{(1)}, V^{(1)}]]$ commutator) and so on. I would like to show one of them.

2.1 $e^-e^- \to e^-e^-$ process

It is the case where we need to evaluate the $[R^{(1)}, V^{(1)}]$ commutator. The corresponding interaction operator $V(ee \to ee)$ for the two clothed electrons is

$$V(ee \to ee) = \frac{1}{2}[R^{(1)}, V^{(1)}](e^-e^- \to e^-e^-) + V_{Coul}(e^-e^- \to e^-e^-), \tag{6}$$

where the first term is the relevant part of $[R^{(1)}, V^{(1)}]$, and after a simple algebra one can get

$$V(ee \to ee) = \int\sum_\mu V(1',2';1,2)b^\dagger(1')b^\dagger(2')b(1)b(2)d\boldsymbol{p'}_1 d\boldsymbol{p'}_2 d\boldsymbol{p}_1 d\boldsymbol{p}_2, \tag{7}$$

$$V(1',2';1,2) = \frac{e^2}{(2\pi)^3}\frac{{m_e}^2}{\sqrt{E_{\vec{p}'_1}E_{\vec{p}'_2}E_{\vec{p}_1}E_{\vec{p}_2}}}\upsilon(1',2';1,2)\delta(\vec{P}'-\vec{P}), \tag{8}$$

$$v(1',2';1,2) = \frac{1}{2}\frac{\bar{u}(1')\gamma^{\mu}u(1)\bar{u}(2')\gamma_{\mu}u(2)}{(p_1'-p_1)^2} + \frac{1}{2}\frac{\bar{u}(1')(\not{P}'-\not{P})u(1)\bar{u}(2')(\not{P}'-\not{P})u(2)}{\boldsymbol{k}^2}\frac{1}{(p_1'-p_1)^2} + $$
$$+\frac{1}{2}\frac{\bar{u}(1')\gamma^{0}u(1)\bar{u}(2')\gamma^{0}u(2)}{\boldsymbol{k}^2}\frac{(p_1'-p_1)^2-(p_2'-p_2)^2}{(p_1'-p_1)^2}, \quad (9)$$

where $P' = p_1' + p_2'$, $P = p_1 + p_2$ and $\boldsymbol{k} = \vec{p}_1' - \vec{p}_1 = \vec{p}_2 - \vec{p}_2'$. In eq. (7) the symbol $\sum_{\mu}$ denotes the summation over fermion polarization projections so $1 = \{\vec{p}_1, \mu_1\}$, etc. For brevity, we are omitting the lower index c at operators in the r.h.s. of the expression (7). One should note that the contribution from the noncovariant Coulomb interaction to the matrix elements by eqs. (8) - (9) on the energy shell $P_0' = P_0$ is canceled with the respective counterpart in the r.h.s. of eq. (6). An important point is that in the CPR such a cancellation (cf. our results [3] in mesodynamics) takes place directly in the Hamiltonian .

3 QCD Hamiltonian in CG. Some Parallels.

At the beginning , trying to extend our approach for describing different processes in quark-gluon systems we are following a remarkable survey [5], where a close similarity of QCD to QED when introducing the Yukawa-type vertex couplings between the interacting fields becomes popular even for pedestrians including myself. As a result, we find there the QCD Hamiltonian

$$H_{qcd} \equiv P^0_{qcd} = \int d\vec{x}(F^{0\lambda}_a F^a_{\lambda 0} + \frac{1}{4}F^{\mu\nu}_a F^a_{\mu\nu} + \frac{1}{2}[i\bar{q}\gamma_0\tilde{D}^0 q + h.c.]) \quad (10)$$

with the tensor $F^{\mu\nu}_a = \partial^\mu A^\nu_a - \partial^\nu A^\mu_a + igf^{ars}A^\mu_r A^\nu_s \equiv \overset{\circ}{F}{}^{\mu\nu}_a + g\,(A^\mu \times A^\nu)_a$ of color-vector (gluon) potentials A^μ_a (gluon indices $a(rs)$ change from 1 to $n_c^2 - 1$) and the covariant color-derivative $n_c \otimes n_c$ matrices $\tilde{D}^0_{cc'} = \delta_{cc'}\partial^0 - ig\tilde{A}^0_{cc'}$ (color indices c and c' run from 1 to n_c) versus the QED Hamiltonian

$$H_{qed} \equiv P^0_{qed} = \int d\vec{x}(F^{0\lambda}F_{\lambda 0} + \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}[i\bar{\psi}\gamma_0 D^0\psi + h.c.]) \quad (11)$$

with the colorless counterparts. Recall also that in case of the $SU(3)$ model with the conserved color currents

$$J^\mu_a = j^a_\mu + gf^{ars}F^{\mu\lambda}_r A^s_\lambda, \quad (12)$$

where the quark current $j^a_\mu = g\bar{q}\gamma_\mu T_a q$, the $3 \otimes 3$ matrices T_a are related to the Gell-Mann matrices $T_a = \frac{1}{2}\lambda_a$.

The operators H_{qcd} and H_{qed} are gauge independent, i.e., remain intact under local gauge transformations by formula (2.22) in [5], that has much in common with the Fock-Weyl criterion. From practical viewpoint it is convenient to employ the CG in which $\partial^k A^a_k = 0$ so Gauss' law takes place for each of the gluon field components. Then Hamiltonian H_{qcd} can be expressed through the longitudinal and transverse color electric fields to get the division $H_{qcd} = H_0(\alpha) + V(\alpha) +$ mass and vertex counterterms, $V(\alpha) = \int d\vec{x}\, V_{qcd}(\vec{x})$ with the interaction density

$$V_{qcd}(\vec{x}) = -j^k_a(\vec{x})A^a_k(\vec{x}) - \frac{1}{2}F^{a(l)}_{k0}F^{k0(l)}_a + \frac{g}{2}\overset{\circ}{F}{}^a_{kn}(A^k \times A^n)^a + \frac{g^2}{4}(A_k \times A_n)^a(A^k \times A^n)^a, \quad (13)$$

and the constraint equation

$$\partial^k F^{a(l)}_{k0} + g(A^m \times F^{(l)}_{m0})^a = -j^a_0 + g(E_m \times A^m)^a. \quad (14)$$

At this point, we prefer to proceed with the set α composed of the creation and destruction operators (cf., [5]) in the expansions

$$A_a^\mu(\vec{x}) = \int \frac{d\vec{k}}{\sqrt{2(2\pi)^3|\vec{k}|}} \sum_\sigma \left[e^\mu(\vec{k},\sigma) c_a(\vec{k},\sigma) + e^{\mu^*}(-\vec{k},\sigma) c_a^\dagger(-\vec{k},\sigma) \right] \exp(i\vec{k}\vec{x}), \tag{15}$$

$$q_{fc}(\vec{x}) = \int d\vec{p} \sqrt{\frac{m}{(2\pi)^3 E_{\vec{p}}}} \sum_\mu \left[\bar{u}(\vec{p}\mu) b_{fc}(\vec{p}\mu) + v(-\vec{p}\mu) d_{fc}^\dagger(-\vec{p}\mu) \right] \exp(i\vec{p}\vec{x}), \tag{16}$$

with the flavor-color label fc, if we want, and canonical commutations

$$[c_a(\vec{k},\sigma), c_{a'}^\dagger(\vec{k}',\sigma')] = \delta(\vec{k}-\vec{k}')\delta_{\sigma\sigma'}\delta_{aa'},$$

$$\{b_{fc}(\vec{p}\mu), b_{f'c'}^\dagger(\vec{p}'\mu')\} = \{d_{fc}(\vec{p}\mu), d_{f'c'}^\dagger(\vec{p}'\mu')\} = \delta(\vec{p}-\vec{p}')\delta_{\mu\mu'}\delta_{ff'}\delta_{cc'}.$$

these preliminaries are aimed at reformulating the field model in terms of the "clothed " quarks whose masses, being some adjustable parameters, can be adopted, for instance, from the constituent quark models.

For the first time our current explorations are focused on studying the properties of positronium (quarkonium) states with instantaneous interactions between the clothed electrons and positrons (quarks and antiquarks). The corresponding 2→2 operators have the structure (in particular, for the annihilation processes $e^-e^+ \rightarrow \gamma\gamma$ and $q\bar{q} \rightarrow$ two gluons) similar to that by Eq. (7) with the c - number coefficients $V(1', 2'; 1, 2)$ that determine properly symmetrized kernels of the relevant sets of coupled equations for the states mentioned above (cf., Subsec 4,4 in [1]). By developing a recursive algebraic technique proposed in [2] we continue to calculate higher-order corrections to these coefficients that stem from the multiple commutations of UCT generators with the so-called bad terms in a primary Hamiltonian H that prevent the vacuum and clothed one-particle states to be the H eigenstates. As before (see. e.g., [4]), we work with nonlocal extensions of the available field interactions by introducing some covariant cutoff factors into their vertices to handle finite intermediate quantities for a permanent cancellation of charge and mass renormalization terms with their counterparts from more and more complex commutators in question. In this context, a separate challenge for us is to compare our results in mesodynamics, QED and QCD with another dressing procedure based on some renormalization group ideas (see surveys [6] and [7] and refs. therein) and do it in the instant! form of relativistic dynamics. I am very grateful to referee for this possibility to become familiar (unfortunately, for a very short time) with explorations in this direction.

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