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The general configuration-space Faddeev formalism for studying pd scattering

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Abstract. The configuration-space Faddeev equations are derived for p-d scattering taking into account the difference in interaction between the participant particles. Appropriate modifications have been made in the well-known configuration-space equations for n-d scattering. To show the effect of these modifications, the s-wave calculations are performed for bound state and scattering problems. We model the charge symmetry breaking effect for ^{3}H and ^{3}He with a modified Malfliet-Tjon MT I-III potential. Results obtained for elastic n-d and p-d scattering at E_{lab}=14.1 MeV are compared with our prediction (Ref. [1]) and those of the Los-Alamos/Iowa group (Ref. [2]) .

1 Introduction

The isotopic formalism was developed for the study of neutron-deuteron scattering in the framework of the configuration space Faddeev equations (Ref. [3]). Charge-independence breaking in the three-nucleon system was investigated in elastic neutron-deuteron and breakup prosesses (Ref. [4]). Here we study proton-deuteron scattering. Presence of the electromagnetic interaction requires one to consider the neutron and proton to be different particles and precludes literal use of the isotopic formalim of (Ref. [3]). So the FNNM equations used in (Ref. [5]) have to be changed.

Taking the neutron as particle 1 and protons as particles 2 and 3 we have the requirement $\Psi(1,2,3) = -\Psi(1,3,2)$. To satisfy this condition, we present Ψ in terms of Faddeev components as

$$\Psi(1,2,3) = \Phi_1(1,2,3) + \Phi_2(2,3,1) - \Phi_2(3,2,1), \tag{1}$$

where it is understood that in $\Phi(i,k,l)$ particles are grouped as $i + (kl)$ and Φ_1 is antisymmetric in the last pair of arguments: $\Phi_1(1,2,3) = -\Phi_1(1,3,2)$. As function $\Phi_2(2,3,1)$ has no definite properties under interchange $3 \leftrightarrow 1$ we encounter permutations which are not cyclic $P_{12}(231) = (321)$, $P_{13}(123) = (321)$. In terms of these operators and operators $P^{\pm}$ we obtain for the independent components Φ_1 and Φ_2 a system

$$\Big(E - \Delta - v_1(2,3)\Big)\Phi_1(1,2,3) = v_1(2,3)\Big(P^{-}\Phi_2(1,2,3) - P_{13}\Phi_2(1,2,3)\Big),$$

$$\Big(E - \Delta - v_2(3,1)\Big)\Phi_2(2,3,1) = v_2(3,1)\Big(P^{+}\Phi_1(2,3,1) - P_{12}\Phi_2(2,3,1)\Big). \tag{2}$$

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Here $v_1(2,3)$ is a sum of the Coulomb and nuclear potentials and $v_2(3,1)$ is the pure nuclear potential:

$$v_1(2,3) = v_c(2,3) + v_{pp}(2,3), \quad v_2(3,1) = v_{pn}(3,1). \tag{3}$$

The final pair of equations is essentially the same as for the *nd* case with changes described above.

$$\Big[E + \frac{\hbar^2}{m}(\partial^2_{x_1} + \partial^2_{y_1}) - v^{\lambda l}_\alpha(x_1,y_1)\Big]\Phi^{\lambda_0,s_0,M_0}_{1,\alpha}(x_1,y_1) = \sum_\beta \Big[v_c(x_1) + v_{pp}(x_1)\Big]_{\alpha\beta}\Big[\Phi^{\lambda_0 s_0,M_0}_{1,\beta}(x_1,y_1)+$$

$$\frac{1}{2}\int_{-1}^{1} du \sum_\gamma \Big(g^{(-)}_{\beta\gamma}(y_1/x_1,u)\Phi^{\lambda_0,s_0,M_0}_{2,\gamma}(x_2,y_2) - g^{(13)}_{\beta\gamma}(y_1/x_1,u)\Phi^{\lambda_0,s_0,M_0}_{3,\gamma}(x_3,y_3)\Big), \tag{4}$$

where $(x_2,y_2) = P^-(x_1,y_1)$ and $(x_3,y_3) = P_{13}(x_1,y_1)$.

$$\Big[E + \frac{\hbar^2}{m}(\partial^2_{x_2} + \partial^2_{y_2}) - v^{\lambda l}_\alpha(x_2,y_2)\Big]\Phi^{\lambda_0,s_0,M_0}_{2,\alpha}(x_2,y_2) = \sum_\beta \Big[v_{pn}(x_2)\Big]_{\alpha\beta}\Big[\Phi^{\lambda_0 s_0,M_0}_{2,\beta}(x_2,y_2)+$$

$$\frac{1}{2}\int_{1}^{1} du \sum_\gamma \Big(g^{(+)}_{\beta\gamma}(y_2/x_2,u)\Phi^{\lambda_0,s_0,M_0}_{1,\gamma}(x_1,y_1) - g^{(12)}_{\beta\gamma}(y_2/x_2,u)\Phi^{\lambda_0,s_0,M_0}_{3,\gamma}(x_3 y_3)\Big), \tag{5}$$

where $(x_1,y_1) = P^+(x_2,y_2)$ and $(x_3,y_3) = P_{12}(x_2,y_2)$. In these formulas the multi index $\alpha = \{l,\sigma,j,s,\lambda\}$, $g^{(\pm)}_{\alpha\alpha'}$ and $g^{(ik)}_{\alpha\alpha'}$ are representatives of the operators $2P^\pm$ and $2P_{ik}$ in the MGL basis (Ref. [6]).

2 s-wave approach. Elastic scattering

In the *s*-wave approach there exists a single equation in the spin-quartet case for quantum state $\alpha = \{0,1,1,3/2,0\}$ and our new results for n-d and p-d elastic amplitudes at E_{lab}=14.1 MeV calculated with the Malfliet-Tjon MT-I-III potential do not practically differ from our predictions (Ref. [1]) and those of the Los-Alamos/Iowa group (Ref. [2]) and we do not present them here. However in the spin-doublet case there exist three equations for quantum states $\alpha_1 = \{0,0,0,1/2,0\}$ and $\alpha_2 = \{0,1,1,1/2,0\}$, one for Φ_{1,α_1} and two for Φ_{2,α_i}, (i=1,2). For the ppn system, a set of equations is writen as:

$$\Big[E + \frac{\hbar^2}{2m}(\partial^2_{x_1} + \partial^2_{y_1}) - v^{00}_q\Big]\Phi_{1,\alpha_1}(x_1,y_1) = [v^c(x_1) + v_{pp}(x_1)]_{\alpha_1\alpha_1}\Big[\Phi_{1,\alpha_1}(x_1,y_1) + \frac{1}{2}\int_{-1}^{1} du h^0_{0000}(y_1/x_1,u)$$

$$\times\Big[\Big(-\frac{1}{2}\Phi_{2,\alpha_1}(x_2,y_2) - \frac{1}{2}\Phi_{2,\alpha_1}(x_3,y_3)\Big) + \Big(\frac{\sqrt{3}}{2}\Phi_{2,\alpha_2}(x_2,y_2) + \frac{\sqrt{3}}{2}\Phi_{2,\alpha_2}(x_3,y_3)\Big)\Big]\Big] \tag{6}$$

$$\Big[E + \frac{\hbar^2}{2m}(\partial^2_{x_2} + \partial^2_{y_2}) - v^{00}_q\Big]\Phi_{2,\alpha_1}(x_2,y_2) = [v_{pn}(x_2)]_{\alpha_1\alpha_1}\Big[\Phi_{2\alpha_1}(x_2,y_2) + \frac{1}{2}\int_{-1}^{1} du h^0_{0000}(y_2/x_2,u)$$

$$\Big(-\frac{1}{2}\Phi_{1,\alpha_1}(x_1,y_1) - \frac{1}{2}\Phi_{2,\alpha_1}(x_3,y_3) - \frac{\sqrt{3}}{2}\Phi_{2,\alpha_2}(x_3,y_3)\Big)\Big] \tag{7}$$

$$\Big[E + \frac{\hbar^2}{2m}(\partial^2_{x_2} + \partial^2_{y_2}) - v^{00}_q\Big]\Phi_{2,\alpha_2}(x_2,y_2) = [v_{pn}(x_2)]_{\alpha_2\alpha_2}\Big[\Phi_{2\alpha_2}(x_2,y_2)$$

$$+\frac{1}{2}\int_{-1}^{1} du h^0_{0000}(y_2/x_2,u)\Big(\frac{\sqrt{3}}{2}\Phi_{1,\alpha_1}(x_1,y_1) - \frac{\sqrt{3}}{2}\Phi_{2,\alpha_1}(x_3,y_3) + \frac{1}{2}\Phi_{2,\alpha_2}(x_3,y_3)\Big)\Big]. \tag{8}$$

In the s-wave approach the functions $g^{(\pm)}_{\alpha\alpha'}$ and $g^{(ik)}_{\alpha\alpha'}$ are reduced to functions $h^0_{0000}(y_i/x_i, u) = x_i y_i/(x_k y_k)$. Here (x_k, y_k) are the coordinates of the integrand components $\Phi_{1,2}$ and $(k \neq i)$.

In the deuteron domain (x_2 finite, $y_2 \to \infty$) the asymptotic condition for the component Φ_2 corresponding to elastic channel:

$$\Phi^{0,1/2,1/2}_{2,\alpha_2}(x_2, y_2) \sim \Big\{\delta_{\sigma 1}\delta_{J1} e^{i\Delta^c_0} F^c_0(qy_2) + e^{-i\Delta^c_0}\Big(G^c_0(qy_2) + iF^c_0(qy_2)\Big) a^{1/2}_{01/2,01/2}\Big\}\psi_l(x_2). \tag{9}$$

In formula (9) $F_0(qy_2)$ and $G_0(qy_2)$ are the regular and irreqular Coulomb functions and $\psi_l(x_2)$ is the s - wave component of the deuteron wave function ($l = 0$). Amplitudes Φ_{1,α_1} and Φ_{2,α_1} have zero initial conditions and zero elastic asymptotics.

In the breakup domain we have for Φ_{1,α_1} and $\Phi_{2,\alpha_{1,2}}$ the asymptotics:

$$\Phi^{0\frac{1}{2}\frac{1}{2}}_{1,\alpha_1} \sim e^{-W_1(\theta_1)} A^{0\frac{1}{2}\frac{1}{2}}_{1,\alpha_1}(\theta_1),\ \ \Phi^{0\frac{1}{2}\frac{1}{2}}_{2,\alpha_{1,2}} \sim \mathcal{E}_0 A^{0\frac{1}{2}\frac{1}{2}}_{2,\alpha_{1,2}}(\theta_2),\ \ \mathcal{E}_0 = 2\pi \int_{-1}^{1} du e^{iW_2(\hat{\mathbf{x}}_2,\hat{\mathbf{y}}_2)},\ \ u = cos(\hat{\mathbf{x}}_2\hat{\mathbf{y}}_2). \tag{10}$$

In these formulas the Coulomb distorted phases W_1 and W_2 are as following

$$W_1(\theta) = -\frac{1}{2\sqrt{E}}\frac{me^2}{\hbar^2}\frac{\ln(2\sqrt{E}X)}{\cos\theta},\ \ \cos\theta = \frac{x_1}{X}, \tag{11}$$

and

$$W_2(\hat{\mathbf{x}}_2, \hat{\mathbf{y}}_2) = -\frac{1}{2\sqrt{E}}\frac{me^2}{\hbar^2}\frac{X}{|\mathbf{x_2}/2 + \sqrt{3}\mathbf{y_2}/2|}\ln(2\sqrt{E}X),\ \ X = \sqrt{x_i^2 + y_i^2}. \tag{12}$$

The two doublet amplitudes are given as follows

$$\mathcal{A}_{\alpha_1}(\theta_2) = A_{2,\alpha_1}(\theta_2) + \frac{1}{2}\int_{-1}^{1} du h^0_{0000}(y_2/x_2, u)$$
$$\Big(g_{11}A_{1,\alpha_1}(\theta_1) - g^{(12)}_{11}A_{2,\alpha_1}(\theta_3) - g^{(12)}_{12}A_{2,\alpha_2}(\theta_3)\Big) \tag{13}$$
$$\mathcal{A}_{\alpha_2}(\theta_2) = A_{2,\alpha_2}(\theta_2) + \frac{1}{2}\int_{-1}^{1} du h^0_{0000}(y_2/x_2, u)$$
$$\Big(g_{21}A_{1,\alpha_1}(\theta_1) - g^{(12)}_{21}A_{2,\alpha_1}(\theta_3) - g^{(12)}_{22}A_{2,\alpha_2}(\theta_3)\Big). \tag{14}$$

Equations (6 - 8) violate the isospin symmetry because of distinctions between nn, pp and np forces. The charge asymmetry is obtained by allowing the strengths of the central *nn* and *pp* forces to be different from *np* one. We modify MT-I-III potentials to reproduce singlet n-n and p-p scattering data by scaling the *pn* potential by factors 0.982 and 0.9745 for *nn* and *pp*, respectively. While MT-I-III uses for $a_{\rm np}$, $a_{\rm nn}$ and $a_{\rm pp}$ the value of scattering length is -23.5 fm, we modify the potential to produce their scattering lenghts to agreement with experimental data (Ref. [10]).

The accuracy of this adjustment procedure is checked by calculating binding energies for ^{3}H and ^{3}He using MT-I-III potential and its modifications. Our new results and previous ones from Ref. [7] and Ref. [8] obtained in isospin formalism are given in table 1. Our new results calculated applying a new set of three Faddeev equations with modified MT-I-III NN potentials are in a good agreement with those from Ref. [7] and Ref. [8]. Results for our Coulomb energy ΔB_c and CSB energy ΔB(CBS) are given in table 1.

In the *s*-wave approach the value of ΔB_c is 661 keV slightly different from the result of 693 keV (Ref. [9]). Our result for the charge-symmetry breaking energy ΔB(CSB) is 61 keV close to 71 keV evaluated by Miller et al. [10]. Our new results for phase shifts and elasticities for n-d

Table 1. $B(^3\text{H})$ and $B(^3\text{He})$ binding energies (in MeV), the Coulomb energy ΔB_c (in keV), CSB effect for energy ΔB(CSB) (in keV). The results of Ref. [7] (Ref. [8]) are given in brackets (square brackets). m_n (m_p) is the mass of neutron (proton).

		m_n	m_p	$B(^3\text{H})$	$B(^3\text{He})$	ΔB_c	ΔB(CSB)
MT-I-III		939.0	939.0	8.545			
		–	–	(8.535)	(7.868)		
		–	–	[8.54]	[7.88]		
		939.565	938.272	8.548	7.882		
Modified MT-I-III	nn	–	–	8.396	7.735	661	
	pp	–	–		7.674	–	61

Table 2. Spin-doublet case. n-d and p-d elastic shifts and inelasticities at E_{lab}=14.1 MeV.

	n - d			p - d		
	MT-I-III	[1]	modified MT-I-III	MT-I-III	[1]	modified MT-I-III
$\delta(deg)$	106.16	105.47	105.56	111.05	108.06	110.76
η	0.4653	0.4649	0.4744	0.533	0.4929	0.536

and p-d breakup scattering at E_{lab}=14.1 MeV calculated assuming the neutrons and protons to be distinguishable particles are presented in table 2. One concludes that CSB is visible in phase shifts and inelasticities for neutron-deuteron and proton-deuteron scattering at E_{lab}=14.1 MeV. However these results have been obtained applying partly artificial procedure for constructing *s*-wave singlet *nn* and *pp* components of MT-I-III potential. Therefore we are currently extending our studies using the charge dependent AV18 NN potential.

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