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Effective Screened Coulomb Potential from e^-- e^- to ^{208}Pb -^{208}Pb Systems

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Abstract. We confirmed that the Coulomb phase shifts can not be given with a screened Coulomb potential by a monotonic increase of the range R. However, several screening "discrete range-bands" reproduce the Coulomb phase shift in a wide energy region. If the range-band of a proton-proton system for example is obtained, then the range-bands are generalized by three universal arguments: the universal range $\mathcal{R}(k)(\equiv kR)$, the Sommerfeld parameter $\eta(k)(\equiv Z_1Z_2e^2\nu/k)$, and the universal asymptotic phase $\mathcal{F}(k)(\equiv 2kr_c)$. Finally, the phase shifts of any other systems from $e^-{-}e^-$ to heavy ions such as ^{208}Pb$-^{208}$Pb can be reproduced automatically by an individual range $R = \mathcal{R}(k)/k$ on the universal range-bands $\mathcal{R}$(k).

1 Introduction

The screened Coulomb potential can not reproduce the Coulomb *phase shift* by the monotonic increase-range method, although the screened Coulomb potential itself reaches the Coulomb potential in the infinite range [1][2]. However, the method is still adopted as the second best way in a wide field from the electron-electron system to the heavy ion-systems. In this paper, we would like to present a method to avoid the Coulomb singularity by using a universal "range-bands".

2 A Phase Function in the Asymptotic Region

The Coulomb phase shift is analytically obtained in configuration space and expressed by

$$\sigma_L(k) = \arg\Gamma(L+1+i\eta(k)), \tag{1}$$

where $\eta(k) = Z_1Z_2e^2\nu/k$ is the Sommerfeld parameter, L the two-body angular momentum, and $\nu = m_1m_2/(m_1+m_2)$ reduced mass, respectively.
The phase shift appears in the asymptotic Coulomb wave function,

$$w_L^{(\pm)}(kr) = \exp\Big[\pm i\{kr - \pi L/2 - \eta(k)\ln 2kr + \sigma_L(k)\}\Big], \tag{2}$$

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where the r-dependent phase: $\eta(k)\ln 2kr$ differs from the one obtained from short-ranged potentials,

$$h_L^{(\pm)}(kr) = \exp\Big[\pm i\{kr - \pi L/2 + \delta_L^R(k)\}\Big], \tag{3}$$

where $\delta_L^R(k)$ is the phase shift with short range potential.

Let us define a phase function by the ratio between both wave functions in the region $r \geq r_c$, where r_c is a starting point of the asymptotic region,

$$\mathcal{Y}_L^{\pm}(kr) \equiv \ln y_L^{\pm}(kr) \equiv \ln\Big[\frac{h_L^{(\pm)}(kr)}{w_L^{(\pm)}(kr)}\Big] = \pm i\Big(\eta(k)\ln 2kr + \delta_L^R(k) - \sigma_L(k)\Big). \tag{4}$$

The phase function satisfies a differential equation,

$$2i\Big(k - \frac{\eta(k)}{r}\Big)\frac{d\mathcal{Y}_L(kr)}{dr} = V^R(r) - V^C(r), \tag{5}$$

where $V^C(r)$ and $V^R(r)$ are the Coulomb and the screened Coulomb potentials, respectively.
The solution of the equation is obtained by the integral from r_c to ∞ with a Yukawa-type screened Coulomb potential

$$\exp\Big[-\eta(k)/\mathcal{R}(k)\Big]Ei(-t_c) - \ln\Big|\frac{\mathcal{F}(k) - 2\eta(k)}{\mathcal{F}(k)}\exp\Big[\pm 2m\pi/\eta(k)\Big]\Big| = 0, \quad (m = 0, 1, 2, \ldots) \tag{6}$$

with the integral exponential function, and the universal asymptotic phase $\mathcal{F}(k) \equiv 2kr_c$,

$$Ei(-t_c) \equiv -\int_{t_c}^{\infty}\exp(-t)\frac{dt}{t}\ ; \qquad t_c = \frac{\mathcal{F}(k) - 2\eta(k)}{2\mathcal{R}(k)}, \tag{7}$$

where $\mathcal{F}(k)$ is a variational parameter to minimize $\mathcal{R}(k)$ at a fixed $\eta(k)$ in Eq.(6). Therefore, a unique screened Coulomb potential is given by the permitted range $\mathcal{R}(k) = \mathcal{R}_n(k)$: $(n = 0,\ 1,\ 2, \ldots)$ where n indicates phase shift regions: $n = 0$ for $\sigma_0 < 0$, and $(n-1)\pi \leq \sigma_0 < n\pi$ for $1 \leq n$.
However, the minimization procedure is very sensitive in the numerical calculation, therefore we will refer the analytic phase shift of Eq.(1) and obtain the $\mathcal{R}(k) - \eta(k)$ relation with the universal variables, instead of $R - k$ relation, which is shown for the $L = 0$ case in Figure 1.
Fortunately, the σ_L cases for $1 \leq L$ are automatically obtained by the $\mathcal{R}(k) - \eta(k)$ relation.

3 Numerical Results

We found five discrete $\mathcal{R}(k) - \eta(k)$-bands which reproduce the Coulomb phase shift for $\sigma_0 \leq 4\pi$.
1) The lowest $\mathcal{R}(k) - \eta(k)$-band (black circle in Figure 1) represents $\sigma_0(k) < 0$ in the higher energies.
2) The second band (black nabla sign) recreates $0 \leq \sigma_0(k) \leq \pi$.
3) The third band (black square sign) gives $\pi \leq \sigma_0(k) \leq 2\pi$.
4) The fourth band (white circle sign) offers $2\pi \leq \sigma_0(k) \leq 3\pi$.
5) The fifth band (black delta sign) displays $3\pi \leq \sigma_0(k) \leq 4\pi$.

In order to represent $\sigma_0(k) = n\pi$ $(n = 0, 1, 2, \cdots)$, the zero range potential $V^{R=0}(r)$ is required, however, it seems to be unusual. Therefore, we should jump to another nearby band before arriving at $\sigma_0(k) = n\pi$ at which the nearby band has a finite range (see Figure 1).

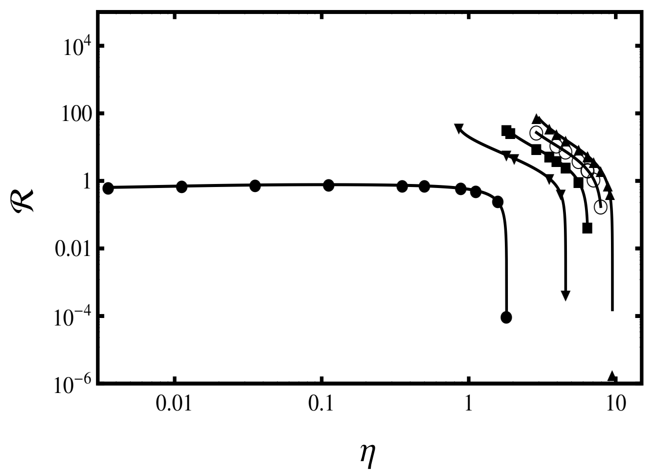

Figure 1. Arrowed universal ranges $\mathcal{R}(k)$ are denoted by five different signs: black circle, black nabla, black square, white circle, and black delta. The universal ranges $\mathcal{R} = \mathcal{R}(k) = \mathcal{R}_n(k)$ $(n = 0, 1, 2, 3, 4)$ are fitted as the functions of $\eta = \eta(k)$.
1) $\mathcal{R}_0 = 0$ at $\sigma_0(k) = 0$, $\mathcal{R}_0 = (37.283 + 5708.9\eta - 3166.9\eta^2)/(64.616 + 7062.0\eta - 2564.0\eta^2)$,
2) $\mathcal{R}_1 = 0$ at $\sigma_1(k) = \pi$,
$\mathcal{R}_1 = (-259.88 + 447.042\eta - 142.14\eta^2 + 12.414\eta^3)/(10.301 - 38.589\eta + 36.964\eta^2 - 5.8202\eta^3)$,
3) $\mathcal{R}_2 = 0$ at $\sigma_2(k) = 2\pi$,
$\mathcal{R}_2 = (42.289 - 42.199\eta + 26.9977\eta^2 - 3.3228\eta^3)/(0.050814 + 0.32437\eta - 0.91936\eta^2 + 0.57813\eta^3)$,
4) $\mathcal{R}_3 = 0$ at $\sigma_3(k) = 3\pi$,
$\mathcal{R}_3 = (516.55 - 217.77\eta + 40.497\eta^2 - 2.6540\eta^3)/(-22.633 + 19.254\eta - 4.6868\eta^2 + 0.50235\eta^3)$,
5) $\mathcal{R}_4 = 0$ at $\sigma_0(k) = 4\pi$,
$\mathcal{R}_4 = (-520.31 + 230.76\eta - 18.004\eta^2 - 0.056522\eta^3)/(15.524 - 11.918\eta + 2.1584\eta^2 + 0.034126\eta^3)$.
All bands become zero at $\sigma_0(k) = n\pi$ $(n = 0, 1, 2, \cdots)$. Such regions should be replaced by the finite range upper nearby-band. Each band can derive the Coulomb phase shifts very well (see Figure 2).

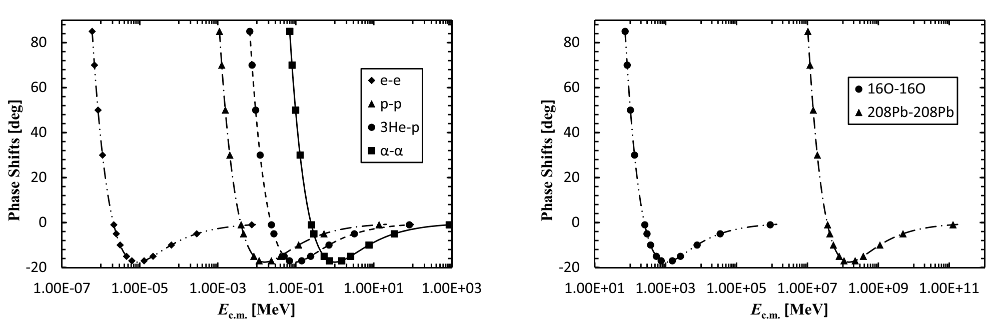

Figure 2. The Coulomb phase shifts in the S-wave are illustrated for several systems from e^--e^- to ^{208}Pb-^{208}Pb. These are calculated by the universal ranges which are given in Figure 1. Present screening range is only available for the Yukawa-type screened Coulomb potential which can reproduce the Coulomb phase shift not only for the wide energy range but also for any other system.

Table 1. $\sigma_0(k) - \eta(k) - R_n(k)$ relation which is seen in Figure 1. The Coulomb phase shift, $\sigma_0(k)$[deg], for the proton-proton system, with respect to the screening range bands $R_n(k)$[fm] ($n = 0, 1, 2, \cdots$), and the universal range bands $\mathcal{R}_n(k)$ (no dimension). The Sommerfeld parameter $\eta(k)$ as a function of the momentum, $k = \sqrt{2\nu E}$. E_{pp}[MeV] is the proton-proton c.m. energy. The energy for $\eta(k) = 1.81$ leads to $\sigma_0(k) = 0$[deg], the energy for $\eta(k) = 4.56$ shows almost $\sigma_0(k) = \pi$, $\eta(k) = 6.41$ corresponds nearly to $\sigma_0(k) = 2\pi$, $\eta(k) = 7.90$ means $\sigma_0 = 3\pi$, and $\eta(k) = 9.457$ gives practically $\sigma_0 = 4\pi$, respectively.

σ_0[deg]	−0.370	−3.66	−17.6	0.00	179.5
$\eta(k)$	0.0112	0.112	0.884	1.81	4.56
E_{pp}[MeV]	1.00×10^{2}	1.00×10^{0}	1.60×10^{-2}	3.81×10^{-3}	6.00×10^{-4}
R_0[fm]	4.60×10^{-1}	5.00×10^{-1}	3.10×10^{1}	1.04×10^{-4}	
$\mathcal{R}_0$	7.14×10^{-1}	7.76×10^{-1}	6.09×10^{-1}	9.61×10^{-6}	
R_1[fm]			1.84×10^{3}	6.08×10^{2}	1.10×10^{-1}
$\mathcal{R}_1$			3.72×10^{1}	5.84×10^{0}	4.18×10^{-4}
R_2[fm]			6.75×10^{4}	3.45×10^{3}	6.75×10^{2}
$\mathcal{R}_2$			1.37×10^{3}	3.32×10^{1}	2.57×10^{0}

σ_0[deg]	−1.01	167.6	0.00
$\eta(k)$	6.41	7.90	9.457
E_{pp}[MeV]	3.03×10^{-4}	2.00×10^{-4}	1.40×10^{-4}
R_2[fm]	1.60×10^{0}		
$\mathcal{R}_2$	4.33×10^{-2}		
R_3[fm]	7.76×10^{2}	8.00×10^{1}	
$\mathcal{R}_3$	9.16×10^{-5}	1.76×10^{-1}	
R_4[fm]	2.05×10^{3}	9.30×10^{2}	1.00×10^{-3}
$\mathcal{R}_4$	5.54×10^{0}	2.04×10^{0}	1.83×10^{-6}

4 Summary

One of the aims of this paper is to show that the usual screened Coulomb method is not correct, but the proper screening range-band exists. Second, if we adopt the universal range, the Sommerfeld parameter, and the universal asymptotic phase, then the Coulomb phase shifts of any two-charged particle systems can be obtained by the same universal range, and phase shifts of the higher partial wave are automatically given. An extension of this method to define the off-shell Coulomb amplitude will be done by using a so-called *Lemma* (see ref. [2]), where the universal range should be re-defined to satisfy an auxiliary phase shift, $\phi(k) \equiv \sigma(k) - \delta^R(k) = 0$, by solving the Schrödinger equation with an auxiliary potential $V^{\phi}(r) \equiv V^C(r) - V^R(t)$.

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