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The proton–deuteron scattering length in pionless EFT

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Abstract. We present a fully perturbative calculation of the quartet-channel proton–deuteron scattering length ($^4a_{p\text{–}d}$) up to next-to-next-to-leading order (NNLO) in pionless effective field theory. In particular, we use a framework that consistently extracts the Coulomb-modified effective range function for a screened Coulomb potential in momentum space. We find a natural convergence pattern as we go to higher orders in the EFT expansion. Our NNLO result of (10.9 ± 0.4) fm agrees with older experimental determinations but deviates from more recent calculations, which find values around 14 fm. To resolve this discrepancy, we discuss the scheme dependence of Coulomb subtractions in a three-body system.

1 Introduction

The quartet-channel proton–deuteron scattering length $^4a_{p\text{–}d}$ is a fundamental observable in the nuclear three-body system. The most recent determination of this quantity, carried out by Black *et al.* [1], extracts a value of $^4a_{p\text{–}d} = (14.7 \pm 2.3)$ fm. While this falls in line with theoretical extractions of the quantity based on potential-model calculations [2–4] that find values close to about 13.8 fm, it deviates quite significantly from older experimental results for $^4a_{p\text{–}d}$ between 11 and 12 fm [5, 6]. In this work, we present a theoretical calculation of $^4a_{p\text{–}d}$ using pionless effective field theory (pionless EFT). This EFT is tailored specifically for the systematic description of few-nucleon systems at very low energies. It only includes short-range contact interactions between nucleons [7, 8] and is constructed to reproduce the effective range expansion [9] in the two-body system. Observable are determined as an expansion in the parameter $Q/\Lambda_{\not\pi}$, where $Q \sim \gamma_d \approx 45$ MeV is the typical momentum scale set by the deuteron binding momentum, and $\Lambda_{\not\pi} = \mathcal{O}(m_\pi)$ is the natural breakdown scale set by the left-out pion physics; it can alternatively be written in terms of the N–N scattering lengths and effective ranges.

We obtain here a fully perturbative result up to next-to-next-to-leading order (N^{2}LO). The key feature of our approach is a consistent numerical calculation of the Coulomb-modified effective range function that takes into account the screening of the Coulomb interaction by introducing a small photon mass in the momentum-space Skorniakov-Ter-Martirosian (STM) equation. This method can also be applied to other systems of charged particles, *e.g.*, to halo nuclei, using the EFT constructed to describe such systems [10, 11]. We argue that the Coulomb-modified scattering length in a three-body system like *p–d* depends on the convention used for the subtraction of Coulomb effects. If we use a field-theoretical Coulomb subtraction scheme based on diagrammatic methods, our result agrees quite well with the older experimental determinations. With a two-body subtraction scheme that mimics the approach taken by configuration-space potential model calculations, we can achieve good agreement with 13.8 fm. For a more detailed discussion of the results presented here we refer the reader to Ref. [12].

2 Formalism

For the purpose of this paper, the relevant part of the pionless EFT Lagrangian is

$$\mathcal{L} = N^\dagger\left(\mathrm{i}D_0 + \frac{\boldsymbol{D}^2}{2M_N}\right)N - d^{i\dagger}\left[\sigma_d + \left(\mathrm{i}D_0 + \frac{\boldsymbol{D}^2}{4M_N}\right)\right]d^i + y_d\left[d^{i\dagger}\left(N^T P^i_d N\right) + \text{h.c.}\right] + \mathcal{L}_{\text{photon}}\,, \tag{1}$$

including a nucleon field N (doublet in spin- and isospin space) and a single auxiliary dibaryon field d^i corresponding to the deuteron-channel ($^3S_1$). The electromagnetic sector is included via the covariant derivative $D_\mu = \partial_\mu + \mathrm{i}eA_\mu\hat{Q}$ (with the charge operator $\hat{Q}$ and the photon field A_μ) and a kinetic term for the photons included in $\mathcal{L}_{\text{photon}}$. It suffices here to keep only so-called *Coulomb photons*, which simply correspond to a static potential between charged particles. More details of the formalism can be found, for example, in Ref. [13] and earlier references therein.

The full leading-order dibaryon propagator $\Delta_d(p_0, \mathbf{p})$ is obtained by dressing the bare expression with nucleon bubbles to all orders and matching $-y_d^2\Delta_d(p_0 = k^2/M_N, \mathbf{p} = \mathbf{0})$ to the effective range expansion around the deuteron pole, $k\cot\delta_d = -\gamma_d + \frac{\rho_d}{2}(k^2 + \gamma_d^2) + \cdots$, for renormalization [7, 8]. For the parameters, we use $\gamma_d = \sqrt{M_N E_d} = 45.7022(1)$ MeV [14] and $\rho_d = 1.765(4)$ fm [15]. To perform a strictly perturbative calculation up to N^{2}LO as described in Ref. [16], it is convenient to define $D_d(E; q) \equiv (-\mathrm{i}) \cdot \Delta_d\left(E - q^2/(2M_N), q\right)$ and expand this as

$$\begin{aligned} D_d(E;q) = D_d^{(0)}(E;q) + D_d^{(1)}(E;q) + D_d^{(2)}(E;q) + \cdots = -\frac{4\pi}{M_N y_d^2}\frac{1}{-\gamma_d + \sqrt{3q^2/4 - M_N E - \mathrm{i}\varepsilon}} \\ \times\left[1 + \frac{\rho_d}{2}\frac{\left(3q^2/4 - M_N E - \gamma_d^2\right)}{-\gamma_d + \sqrt{3q^2/4 - M_N E - \mathrm{i}\varepsilon}} + \left(\frac{\rho_d}{2}\frac{\left(3q^2/4 - M_N E - \gamma_d^2\right)}{-\gamma_d + \sqrt{3q^2/4 - M_N E - \mathrm{i}\varepsilon}}\right)^2 + \cdots\right]. \end{aligned} \tag{2}$$

The deuteron wavefunction renormalization is defined as the residue of Δ_d at the bound-state pole, and its perturbative expansion can be read off from Eq. (2).

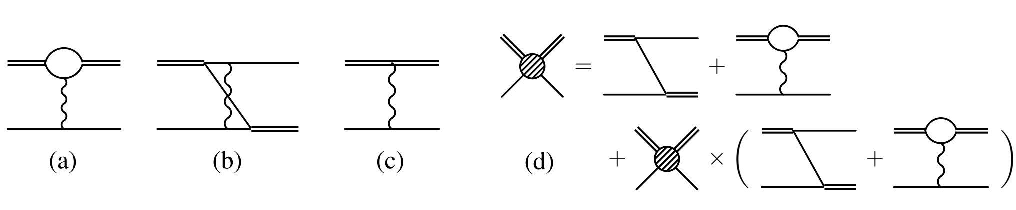

Figure 1. $\mathcal{O}(\alpha)$ diagrams involving Coulomb photons (a,b,c), and integral equation for the full (*i.e.* strong + Coulomb) scattering quartet-channel amplitude $\mathcal{T}_{\text{full}}$ (d). A single line represents a nucleon whereas the deuteron is expressed as a double line.

In Fig. 1(a-c) we show the $\mathcal{O}(\alpha)$ Feynman diagrams that are relevant for our *p–d* scattering calculation. With the operation $A \otimes B \equiv \frac{1}{2\pi^2}\int_0^\Lambda \mathrm{d}q\, q^2\, A(\ldots, q)$, where Λ denotes a momentum cutoff used for the numerical solution, the integral equation represented by Fig. 1(d) can be written as

$$\mathcal{T}_{\text{full}} = -M_N y_d^2\left(K_s - \tfrac{1}{2}K_{\text{bub}}\right) + \mathcal{T}_{\text{full}} \otimes \left[M_N y_d^2\, D_d\left(K_s - \tfrac{1}{2}K_{\text{bub}}\right)\right]\,, \tag{3}$$

where $K_s(E;k,p)$ corresponds to the one-nucleon exchange diagram and $K_{\rm bub}(E;k,p)$ is given by Fig. 1(a). Explicit expressions for these kernel functions, as well as for $K_{\rm box}(E;k,p)$ and $K_{\rho_d}(E;k,p)$, represented by Figs. 1(b) and (c), respectively, are given in Ref. [12]. The latter two play a role as we go to higher orders in the perturbative expansion, as discussed below. The equation for the pure Coulomb amplitude $\mathcal{T}_{\rm c}$ is obtained by omitting K_s in Eq. (3). From the on-shell amplitudes $\mathcal{T}_{\rm full}(E_k;k,k)$ and $\mathcal{T}_{\rm c}(E_k;k,k)$ with $E_k = 3k^2/(4M_N) - \gamma_d^2/M_N$ we obtain the full and pure Coulomb phase shifts, respectively, and then calculate the Coulomb-subtracted $\delta_{\rm diff}(k) = \delta_{\rm full}(k) - \delta_{\rm c}(k)$. This is then used in the Coulomb-modified effective range expansion (ERE),

$$C_{\eta,\lambda}^2\, k\cot\delta_{\rm diff}(k) + \gamma_{p\text{–}d}\, h_\lambda(\eta) = -\frac{1}{{}^4a_{p\text{–}d}} + O(k^2)\ ,\ \eta = \frac{\gamma_{p\text{–}d}}{2k}\ ,\ \gamma_{p\text{–}d} = 4\alpha M_N/3\,, \tag{4}$$

to determine the scattering length ${}^4a_{p\text{–}d}$. The λ here denotes a small photon mass that is used to regulate the otherwise singular photon-exchange diagrams. It has to be taken to zero in order to obtain physical results. A key feature of our approach is that we keep a nonzero photon mass throughout the calculation and consistently calculate the screened Coulomb functions as

$$C_{\eta,\lambda}^2(k) = \left|1 + \frac{2M_N}{3\pi^2}\int_0^\Lambda \frac{\mathrm{d}p\, p^2\, Z_0\mathcal{T}_{\rm c}(E;p,k)}{p^2-k^2-\mathrm{i}\varepsilon}\right|^2\ ,\ h_\lambda(\eta) = \frac{3k^2}{2\alpha M_N}\frac{1}{\pi}\,\mathrm{P}\int_0^\Lambda \mathrm{d}q\,\frac{C_{\eta,\lambda}^2(q)}{(q+k)(q-k)}\,, \tag{5}$$

and then evaluate ${}^4a_{p\text{–}d}$ at $\lambda = 0$ by extrapolation. This approach is based on the general modified ERE derived by van Haeringen and Kok [17].

All quantities discussed here have perturbative expansions. Higher order corrections are generated primarily by the effective range, $\gamma_d\rho_d \sim Q/\Lambda_{\not\pi}$, as shown in Eq. (2). Similarly, Fig. 1(c) (K_{ρ_d}) is formally a range correction. The diagram shown in Fig. 1(b), $K_{\rm box}$, was counted as a higher-order correction by Rupak and Kong [18]. We present here results in this "RK" scheme as well as for an alternative "$O(\alpha)$ counting that includes this diagram already at LO, *i.e.*, iterating it together with the other diagrams shown in Fig. 1(d) [13].

3 Results and discussion

In Fig. 2 we show our results for ${}^4a_{p\text{–}d}$ as a function of the regulating photon mass λ. In the left panel, we use exactly the procedure described in the preceding section and find that the two counting schemes are practically indistinguishable at N^{2}LO. The very weak λ-dependence allows us to unambiguously extrapolate to $\lambda = 0$ and obtain, at N^{2}LO, ${}^4a_{p\text{–}d} = (10.9\pm0.4)$ fm. This agrees with older experimental determinations [5, 6] but deviates from the more recent determination of Black *et al.* [1] and potential-model calculations [2–4].

The conclusion, however, is not to dismiss the latter, but to carefully analyze where the discrepancy might come from. We note that in our EFT framework, it is natural to define the pure Coulomb sector by keeping only the diagrams without strong interaction between the proton and the deuteron, *i.e.*, Figs. 1(a) and (c). The leading diagram, Fig. 1(a), has some contributions from the short-range substructure of the deuteron because the photon only couples to a nucleon bubble. This is a three-body effect. Configuration-space potential-model calculations, on the other hand, subtract Coulomb effects purely at the two-body level [19]. We can mimic this scheme by defining the pure-Coulomb sector through a simple Yukawa potential, which keeps the photon-mass screening but contains no three-body effects. As shown in the right panel of Fig. 2, this procedure gives a result compatible with ${}^4a_{p\text{–}d} \approx 13.8$ fm, *i.e.*, good agreement with potential-model calculations, but it is somewhat unsatisfactory from the EFT perspective. We conclude that the Coulomb-modified scattering length should be called a subtraction-scheme-dependent quantity, at least for the scattering of composite particles.

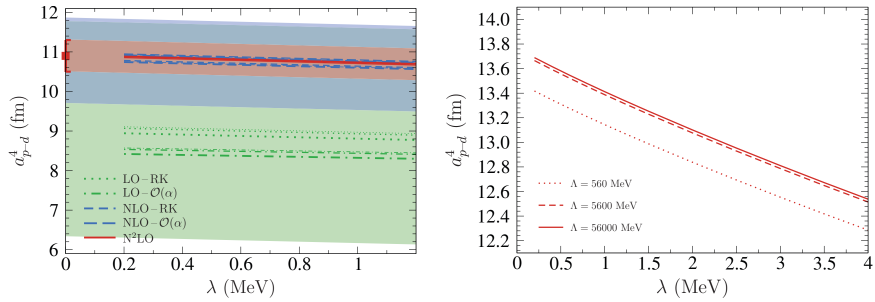

Figure 2. Photon-mass dependence and extrapolation of $^4a_{p\text{–}d}$. **Left panel:** Result with EFT subtraction scheme. Dotted lines: LO result, dashed lines: NLO result, solid lines: N^{2}LO result. Each calculation was performed at three different cutoffs, Λ = 140 MeV (thick lines), Λ = 280 MeV (medium lines), and Λ = 560 MeV (thin lines). "RK" and "$\mathcal{O}(\alpha)$" indicate the different Coulomb-counting schemes (see text). The bands (shown for the "RK" results only) reflect the expected EFT expansion uncertainty. **Right panel:** Result with a simple two-body Yukawa subtraction at N^{2}LO in the "$\mathcal{O}(\alpha)$" counting scheme.

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