

## Properties of ${}^4\text{He}$ and ${}^6\text{Li}$ with improved chiral EFT interactions

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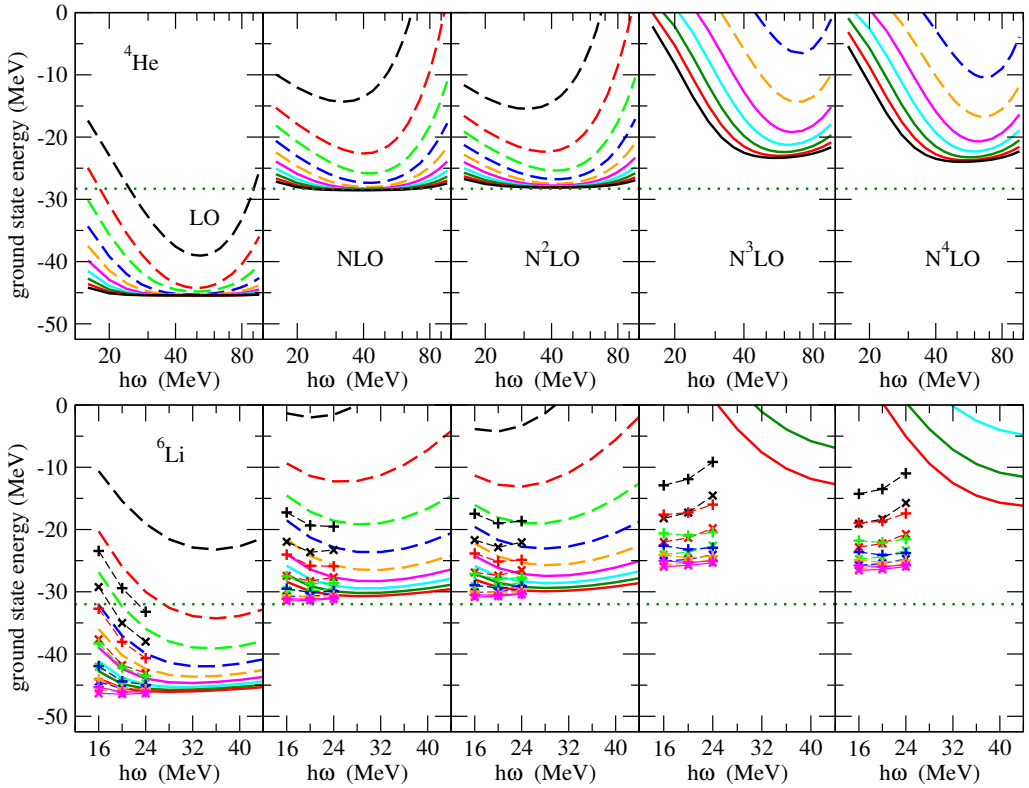
**Abstract.** We present recent results for  ${}^4\text{He}$  and  ${}^6\text{Li}$  obtained with improved NN interactions derived from chiral effective field theory up to  $\text{N}^4\text{LO}$ . The many-body calculations are performed order-by-order in the chiral expansion. At  $\text{N}^3\text{LO}$  and  $\text{N}^4\text{LO}$  additional renormalization using the Similarity Renormalization Group is adopted to improve numerical convergence of the many-body calculations. We discuss results for the ground state energies, as well as the magnetic moment and the low-lying spectrum of  ${}^6\text{Li}$ .

### 1 Nuclear potential from chiral EFT

Chiral effective field theory (EFT) allows one to derive nuclear forces (and corresponding electroweak currents) in a systematical way [1]. In particular, the leading order (LO) and next-to-leading order (NLO) contributions to the interaction are given solely by nucleon-nucleon (NN) operators while three-nucleon forces (3NFs) appear first at next-to-next-to-leading order ( $\text{N}^2\text{LO}$ ) [2] in the chiral expansion. Four-nucleon forces are even more suppressed and start contributing at  $\text{N}^3\text{LO}$ . The chiral power counting thus provides a natural explanation of the observed hierarchy of nuclear forces.

In Refs. [3, 4] a new generation of chiral EFT NN potentials up to  $\text{N}^4\text{LO}$  was developed, using local regulators for the long-range terms and with reduced finite-cutoff artefacts by employing a novel ultraviolet regularization scheme; in addition, a procedure to estimate the theoretical uncertainties due to the truncation of the chiral expansion was proposed (see also [5]). Here we present results for  ${}^4\text{He}$  and  ${}^6\text{Li}$  obtained with these improved chiral NN potentials using a regulator of  $R = 1.0$  fm [6].

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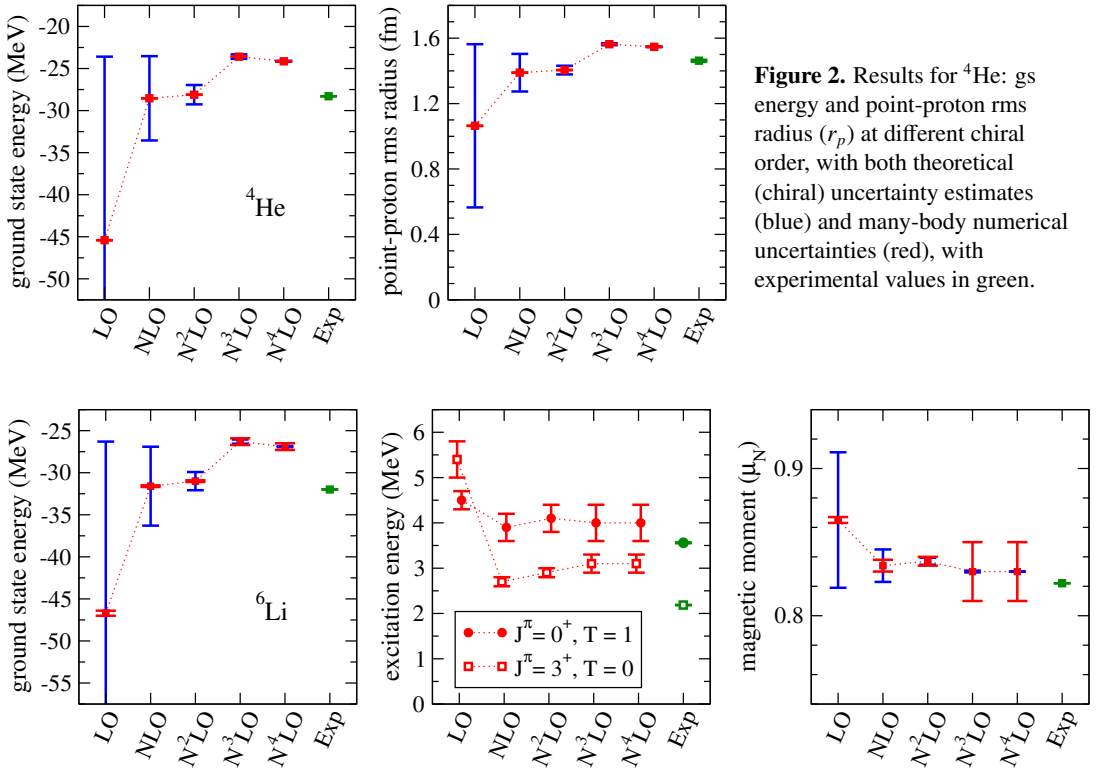
**Figure 1.** Ground state (gs) energy of  ${}^4\text{He}$  (top) and  ${}^6\text{Li}$  (bottom) as function of the HO parameter  $\hbar\omega$  for  $N_{\max} = 2$  (dashed black line) to  $N_{\max} = 20$  (solid black line,  ${}^4\text{He}$ ) and to  $N_{\max} = 18$  (solid red line,  ${}^6\text{Li}$ ) at LO, NLO,  $N^2\text{LO}$ ,  $N^3\text{LO}$ , and  $N^4\text{LO}$ . For  ${}^6\text{Li}$  we also show results with SRG renormalization (crosses and pluses) from  $N_{\max} = 2$  (black) to  $N_{\max} = 12$  (magenta). The horizontal dotted green line indicates the experimental values.

## 2 No-Core Configuration Interaction calculations

In No-Core Configuration Interaction (NCCI) calculations of nuclei [7] the wavefunction  $\Psi$  of a nucleus consisting of  $A$  nucleons is expanded in an  $A$ -body basis of Slater determinants  $\Phi_k$  of single-particle wavefunctions  $\phi_{nljm}(\vec{r})$ . Conventionally, one uses a harmonic oscillator (HO) basis with energy parameter  $\hbar\omega$  for the single-particle wavefunctions. A convenient and efficient truncation of the complete (infinite-dimensional) basis is a truncation on the total number of HO quanta: the basis is limited to many-body basis states with  $\sum_A N_i \leq N_0 + N_{\max}$ , with  $N_0$  the minimal number of quanta for that nucleus and  $N_{\max}$  the truncation parameter. Thus the many-body Schrödinger equation becomes a large sparse matrix eigenvalue problem with even (odd) values of  $N_{\max}$  providing results for natural (unnatural) parity. Convergence toward the exact results is obtained with increasing  $N_{\max}$ , and is marked by approximate  $N_{\max}$  and  $\hbar\omega$  independence.

## 3 Results for ${}^4\text{He}$ and ${}^6\text{Li}$

Figure 1 clearly illustrates convergence patterns of NCCI calculations: as  $N_{\max}$  increases, we see a compression of successive  $N_{\max}$  curves, and the dependence on  $\hbar\omega$  near the variational minimum



**Figure 3.** Results for  ${}^6\text{Li}$ : gs energy, excitation energies of the two lowest excited states, and gs magnetic moment at different chiral order.

weakens. We also see a dramatic difference in convergence rate: the strongly bound nucleus  ${}^4\text{He}$  converges more rapidly than the weakly bound nucleus  ${}^6\text{Li}$ ; furthermore, up to  $\text{N}^2\text{LO}$  convergence is rapid, but at  $\text{N}^3\text{LO}$  and  $\text{N}^4\text{LO}$  our results for  ${}^6\text{Li}$  are far from being converged, even at  $N_{\text{max}} = 18$ .

In order to improve the convergence of the many-body calculations we apply the Similarity Renormalization Group (SRG) at the three-body level to 'soften' the chiral NN interaction [8–11]. **Indeed, at SRG evolution values of  $\alpha = 0.04 \text{ fm}^4$  and  $\alpha = 0.08 \text{ fm}^4$  we do find rapid convergence of the many-body calculation, and, including induced 3N interactions, only very weak dependence on the SRG evolution. Up to  $\text{N}^2\text{LO}$ , the SRG evolution produces results for  ${}^6\text{Li}$  to within a fraction of a percent of those without SRG; at  $\text{N}^3\text{LO}$  and  $\text{N}^4\text{LO}$  the results with the SRG evolution are significantly better converged than, and within the extrapolation uncertainties of, the unconverged results without SRG evolution.** Finally, as a cross-check we also confirm that, to within our estimated numerical accuracy, our results for  ${}^4\text{He}$  agree with results obtained in the Faddeev–Yakubovsky framework [6].

In figures 2 and 3 we summarize our results at different orders in the chiral expansion. In addition to the estimated numerical uncertainties in the many-body calculation, we also display the estimated theoretical chiral uncertainties following [3–6]. The chiral uncertainties decrease with increasing chiral order (as they should). However, the many-body numerical uncertainty increases with increasing chiral order, and at  $\text{N}^3\text{LO}$  and  $\text{N}^4\text{LO}$  our results for  ${}^6\text{Li}$  are dominated by the many-body uncertainties.

Starting at  $N^2\text{LO}$  there are 3NFs in the chiral expansion, which we have not taken into account in the present calculations – this is the main cause for the ‘jumps’ between  $N^2\text{LO}$  and  $N^3\text{LO}$  in figures 2 and 3. We are confident that incorporating consistent 3NFs, starting at  $N^2\text{LO}$ , will reduce (or even eliminate) these jumps.

The patterns for the energies are very similar to each other: we find large overbinding at LO, close to the experimental data at NLO and  $N^2\text{LO}$ , and at  $N^3\text{LO}$  and  $N^4\text{LO}$  a modest underbinding, indicative of the need for 3NFs. The radius  $r_p$  in  $^4\text{He}$  is correlated to its binding: far too small at LO, and slightly too large at  $N^3\text{LO}$  and  $N^4\text{LO}$ . Similar patterns are also found for the gs energy of  $^3\text{H}$  and the Nd total cross section at 10 MeV [6]. Up to  $N^2\text{LO}$  our results for  $^4\text{He}$  are also consistent, to within the estimated chiral uncertainties, with similar results obtained using Quantum Monte Carlo methods in combination with local chiral EFT NN potentials [12, 13] – **any differences can be attributed to differences in the NN potentials (e.g. different choices for the regulators).**

The binding energies of the lowest two excited states in  $^6\text{Li}$  follow the same pattern as the gs energy. Hence their excitation energies, depicted in figure 3, depend much less on the chiral order than one might naively expect based on the theoretical uncertainties in the binding energies. At present we are, however, unsure about the best way to estimate the chiral uncertainties in the excitation energies.

Finally, the gs magnetic moment of  $^6\text{Li}$ , calculated using the canonical one-body M1 operator, is reasonable well converged at  $N^2\text{LO}$ . Calculations incorporating meson-exchange currents [14] suggest that higher order chiral contributions to the M1 operator could very well lead to a correction of a few percent. The derivation and implementation of electroweak current operators consistent with these improved chiral EFT interactions is currently in progress.

## Acknowledgements

This work was performed by the LENPIC collaboration with support from: BMBF (contract 06DA7047I); DFG and NSFC (CRC 110); DFG (SFB 634, SFB/TR 16); ERC projects 259218 NUCLEAREFT and 307986 STRONGINT; EU (HadronPhysics3, Grant 283286); HIC for FAIR; Polish National Science Center DEC-2013/10/M/ST2/00420; US DOE DESC0008485, DE-FG-02-87ER40371, DE-SC0006758, DE-SC0008533; US NSF PHY-0904782, PHY-1306250. Supercomputer usage included: JSC-Jülich; PAS0680-Ohio SC; Edison-NERSC.

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